(1:30)

FINAL EXAM WED 7/22 10:30 AM - 12:30 PM

BLACKBOARD. SIMILAR TO EXAM 2.

20 - 30 QUESTIONS

MEAN, MEDIAN, MODE

VARIANCE, STAND DEV. POP / SAMPLES

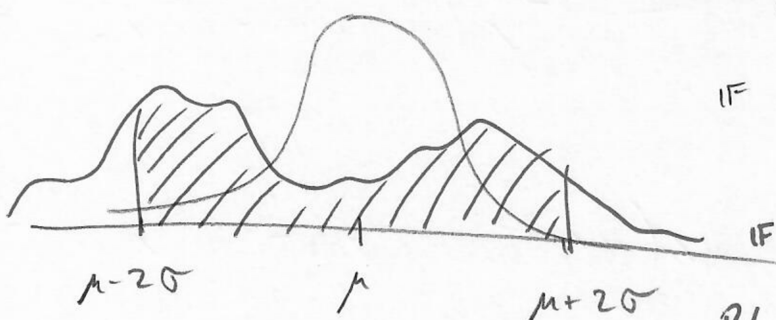
↓

CHEVYCHEV'S THM:

AT LEAST $\left(1 - \frac{1}{k^2}\right)$ OF MEASUREMENTS

LIE WITHIN k STAND. DEV.'S OF THE MEAN.

(REGARDLESS OF DISTRIBUTION)



IF NORMAL $\approx P(\mu - 2\sigma \leq x \leq \mu + 2\sigma)$

$\approx 95\%$

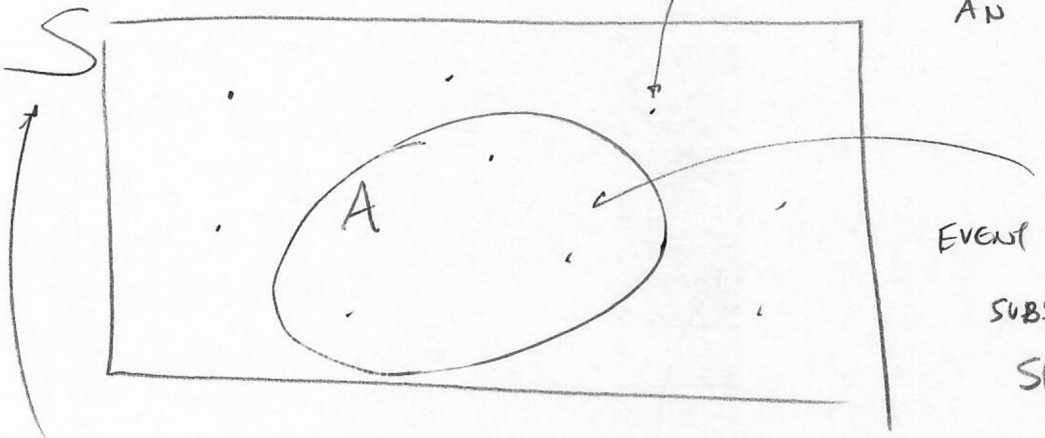
IF NOT

$P(\mu - 2\sigma \leq x \leq \mu + 2\sigma) \geq 1 - \frac{1}{2^2}$

$\geq .75$

PROBABILITY

SIMPLE EVENT IS AN OUTCOME OF AN EXPERIMENT



EVENT A IS A SUBSET OF THE SAMPLE SPACE S.

SAMPLE SPACE = COLLECTION OF ALL SIMPLE EVENTS

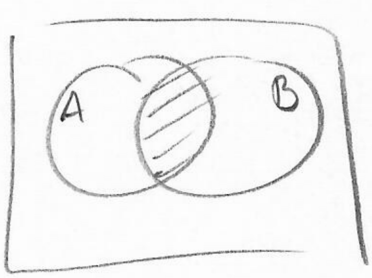
$$0 \leq \text{PROBABILITY} \leq 1$$

(MEASURE OF LIKELIHOOD)

SIMPLE EVENTS: $E_1, E_2, \ldots, E_n$

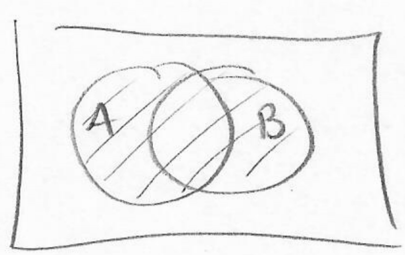
$$P(E_1) + P(E_2) + \ldots + P(E_n) = 1$$

INTERSECTION



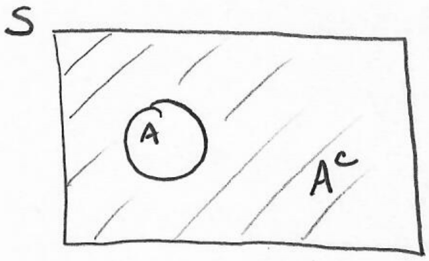
$A \cap B$

UNION



$A \cup B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



COMPLIMENTS:

A^c = { SIMPLE EVENTS NOT IN A }

↑ "NOT A" "A COMPLIMENT"

$$\boxed{P(A) + P(A^c) = 1}$$

$$\boxed{P(A) = 1 - P(A^c)}$$

$$P(S) = 1$$

$$P(A \cup A^c) = 1$$

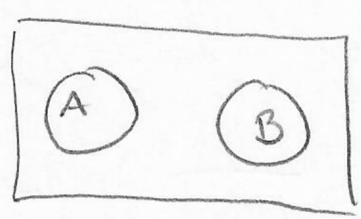
$$P(A) + P(A^c) - \underbrace{P(\underbrace{A \cap A^c}_{\text{EMPTY}})}_{0} = 1$$

$$P(\underbrace{z > 1.02}_{A}) = 1 - P(\underbrace{z \le 1.02}_{A^c})$$

A, B MUTUALLY EXCLUSIVE IF

$$P(A \cap B) = 0$$

(NO OVERLAP)



IF S CONSISTS OF FINITELY MANY SIMPLE EVENTS, ALL EQUALLY LIKELY, THEN FOR ANY EVENT $A \subset S$, WE HAVE

$$P(A) = \frac{\#A}{\#S}$$

\# OF SIMPLE EVENTS

(COUNTING)

K-STAGE EVENT: EXPERIMENT IS PERFORMED IN K STAGES, WHERE

1^{st} STAGE HAS n_1 POSSIBLE OUTCOMES,

2^{nd} n_2

⋮

k^{th} n_k

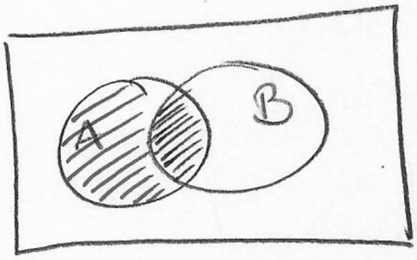
THEN TOTAL # POSSIBLE OUTCOMES IS $n_1 n_2 \cdots n_k$.

- \# WAYS TO CHOOSE & ARRANGE r OBJECTS FROM n OBJECTS IS

$$P^n_r = \frac{n!}{(n-r)!}$$

- \# WAYS TO CHOOSE r OBJECTS FROM n OBJECTS IS

$$C^n_r = \frac{n!}{(n-r)!\,r!}$$



$$P(A) = P(\underline{A \cap B}) + P(\underline{A \cap B^c})$$

	A	A^c
B	1 .35	2 .41
B^c	3 .12	4 .12

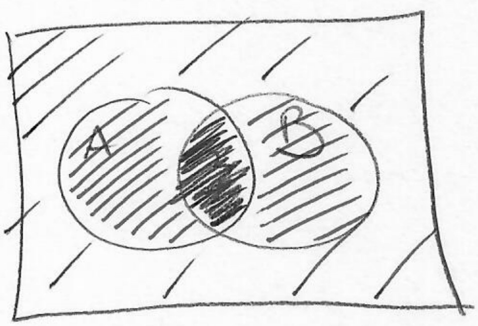
ADD UP TO 1.

$P(A \cap B)$ ✓

$P(A \cap B^c)$ ✓

$P(A^c \cap B)$ ✓

$P(A^c \cap B^c)$ ✓



$$P(A) = P(A \cap B) + P(A \cap B^c)$$
$$= .35 + .12 = .47$$

$$P(B) = .35 + .41 = .76$$

A,B MUT. EXCL.? NO

CONDITIONAL PROB:
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \left.\begin{aligned} P(A \cap B) &= P(B)P(A|B) \\ &= P(A)P(B|A) \end{aligned}\right\} \text{MULTIPLICATION RULE.}$$

	A	A^c
B	.2	.4
B^c	.3	.1

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.2}{.2 + .4} = \underline{\underline{.3333}}$$

$$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{.4}{.4 + .1} = \underline{\underline{.8}}$$

A, B ARE <u>INDEPENDENT</u> IF

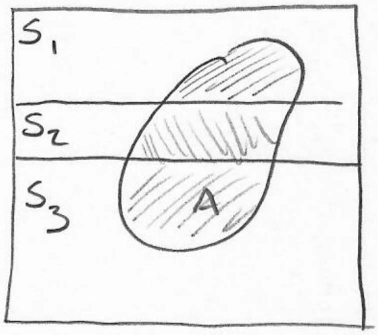
EITHER: $P(A|B) = P(A)$ * $\quad$ $P(A|B^c) = P(A)$

$P(B|A) = P(B)$ $\quad$ $P(B|A^c) = P(B)$

$$P(A \cap B) = P(A)P(B)$$

LAW OF -

Law of Total Probability.



$$P(A) = P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3)$$

$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)$$

WEIGHTED AVERAGE OF $P(A|S_i)$

Bayes' Rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \begin{cases} \dfrac{\cancel{P(B)}P(A|B)}{\cancel{P(B)}} & \text{OK...} \\ \dfrac{P(A)P(B|A)}{P(B)} \end{cases}$$

BAYES' RULE

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

COMBINE B.R. + L.O.T.P.

$$P(S_1)P(B|S_1) + \dots + P(S_n)P(B|S_n)$$

RAN. VAR. IS # ASSIGNED TO EACH SIMPLE EVENT.

DISTRIBUTION FOR DISCRETE RAN. VAR. X

X	ALL POSSIBLE VALUES OF R.V. X
$p(x)$	ALL CORRESPONDING PROB.

ex. 1,000 RAFFLE TIX SOLD FOR \$2 EACH

1 PRIZE \$500

3 PRIZE \$100

10 PRIZE \$50

REST \$0

x	498	98	48	-2
$p(x)$	.001	.003	.010	.986

$\Sigma p(x) = 1$

LET X = NET GAIN/LOSS FROM 1 RAFFLE TICKET

FIND PROB. DISTR.

LONG TERM AVERAGE

MEAN = EXPECTED VALUE FOR R.V. X

$= \Sigma \, x p(x)$

$$E[x] = (498)(.001) + (98)(.003) + 48(.010) + (-2)(.986)$$

IDENTICAL, INDEPENDENT

BINOMIAL R.V.

n TRIALS OF EXPERIMENT THAT RESULTS IN SUCCESS OR FAILURE

$$P(\text{SUCCESS}) = p \quad , \quad P(\text{FAILURE}) = q$$

X = # SUCCESSES IN n TRIALS

↳ RAN. VAR.

X	0	1	… k …	n
$p(x)$			$C^n_k \, p^k q^{n-k}$	

HYP. GEO. ex. 13 FRESH & 5 DEAD BATTERIES IN DRAWER.

PICK 4 AT RANDOM. X = # FRESH

$$P(X = 2) = \frac{C^{13}_2 \cdot C^5_2}{C^{18}_4}$$

SIZE n

ERROR

$$\text{Pop. Param} \approx \text{Sample Statistic} \pm z_{\alpha/2}\ \text{S.E.}$$

Pop Param	Sample Statistic	S.E.	
μ	$\bar{x}$	$\frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$	
p	$\hat{p}$	$\sqrt{\frac{pq}{n}} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}}$	
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$\sigma_1 \approx s_1$ $\sigma_2 \approx s_2$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$	$p_1 \approx \hat{p}_1$ $p_2 \approx \hat{p}_2$

ERROR $\leq B$

$z_{\alpha/2}$ S.E. $\leq B$

$1.96 \sqrt{\frac{pq}{n}} \leq B$

IF NO ESTIMATES FOR p, q

SET $p = q = .5$

(WORST CASE SCENERIO)

$1.96 \sqrt{\frac{(.5)^2}{n}} \leq B$

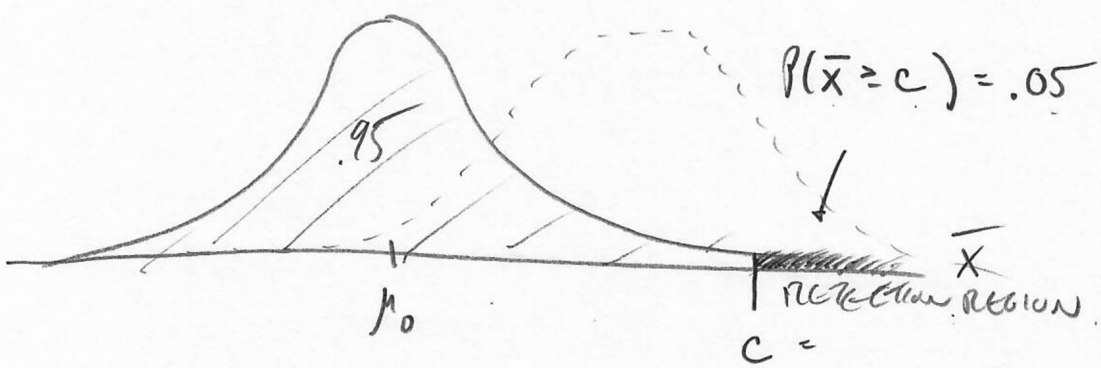
$n \geq \ldots$

SHOW $\mu > \mu_0$

$\alpha = .05$

$H_A: \mu > \mu_0$ SHOW MORE LIKELY

$H_0: \mu = \mu_0$ ASSUME

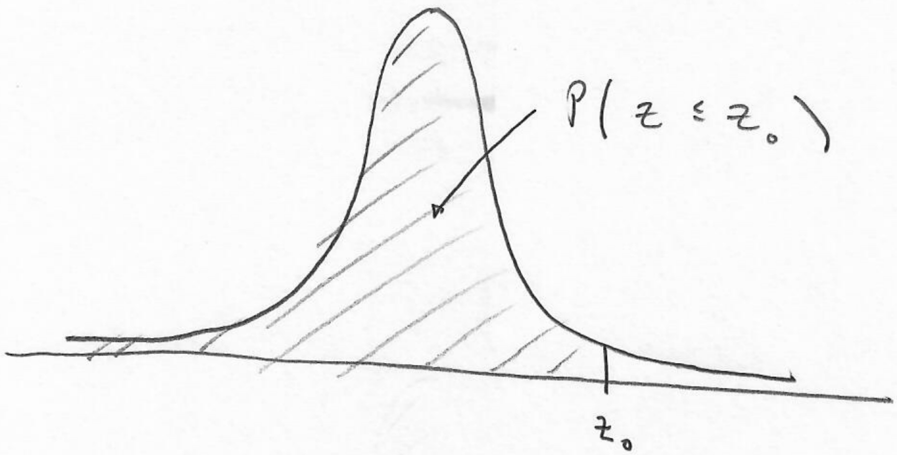


$n = 30$

$\bar{x} = \#$

$c = \mu_0 + z_\alpha$ S.E.

$c = \mu_0 + 1.645$ S.E.

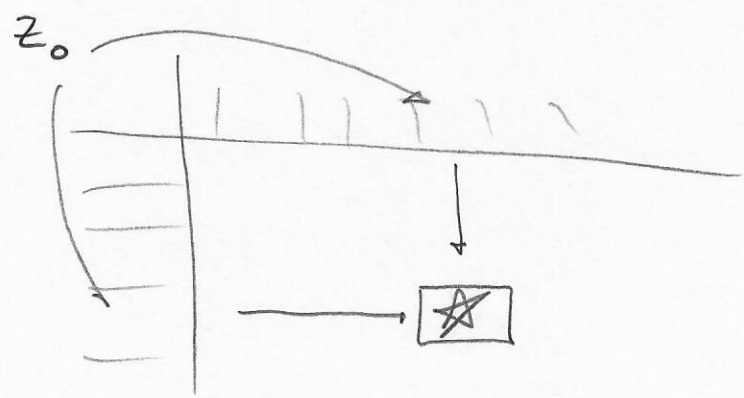


(1) GIVEN z_0, FIND $P(z \le z_0)$

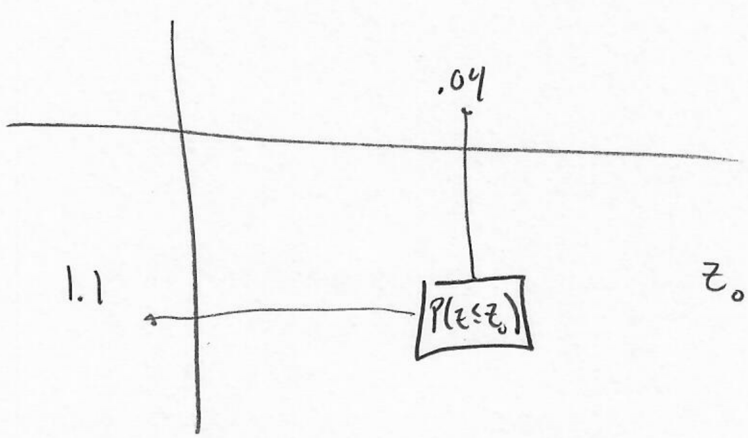
FIND z_0 IN

(2) GIVEN $P(z \le z_0)$, FIND z_0

(1)



(2)



$z_0 = 1.14$

DEF: A SAMPLE STATISTIC IS <u>UNBIASED</u> IF ITS MEAN VALUE (EXPECTED VALUE) IS EQUAL TO THE POPULATION PARAMETER.

i.e. $\bar{X}$ UNBIASED ESTIMATOR FOR μ

$\hat{p}$ UNBIASED ESTIMATOR FOR p

HOW MANY SAMPLES OF SIZE 100 ARE THERE FROM A POPULATION OF 350 MILLION?

$$C^{350 \text{ MILLION}}_{100}$$

IN ORDER FOR SAMPLE TO BE RANDOM ALL SAMPLES SHOULD BE EQUALLY LIKELY TO BE PICKED.

ex. GIVEN: 2% OF POPULATION HAS A DISEAS.

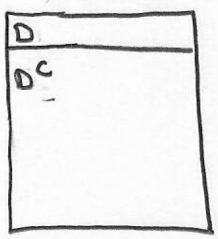
PROB OF TESTING POS GIVEN YOU HAVE THE DISEASE IS 90%

OPPOSITE CONDITIONAL PROB.

PROB OF TESTING NEG GIVEN YOU DO NOT HAVE THE DISEASE IS 85%.

FIND THE PROB THAT SOMEONE WHO TEST POSITIVE HAS THE DISEASE.

① NOTATION



LET D = DISEASE

D^c = NO DISEASE

T = TEST POSITIVE

T^c = TEST NEG.

② SUMMARIZE GIVEN DATA:

$P(D) = .02 \qquad P(D^c) = .98$

$P(T|D) = .9 \qquad P(T^c|D) = .1$

$P(T|D^c) = .15 \qquad P(T^c|D^c) = .85$

③ ANSWER QUESTION:

$$P(D|T) = \frac{P(D)P(T|D)}{P(T)}$$

BAYES' RULE

$$P(T) = \underbrace{P(D)P(T|D)}_{P(T\cap D)} + \underbrace{P(D^c)P(T|D^c)}_{P(T\cap D^c)}$$

L.O.T.P.

$$= \frac{(.02)(.9)}{(.02)(.9) + (.98)(.15)}$$

$\boxed{.1091}$