## §5.2 BINOMIAL RANDOM VARIABLES

$n$ TRIALS OF AN EXPERIMENT THAT RESULTS IN SUCCESS OR FAILURE.

TRIALS ARE IDENTICAL, INDEPENDENT.

$P(\text{SUCCESS}) = p$ , $P(\text{FAILURE}) = q = 1 - p$

$X$ = # SUCCESSES IN $n$ TRIALS.

ex. BB PLAYER MAKES 84% OF FREE THROWS.

$p = .84 \qquad q = .16$

IF BB PLAYER SHOOTS 8 FREE THROWS, FIND PROB THAT THEY MAKE 6.

$C^8_6$

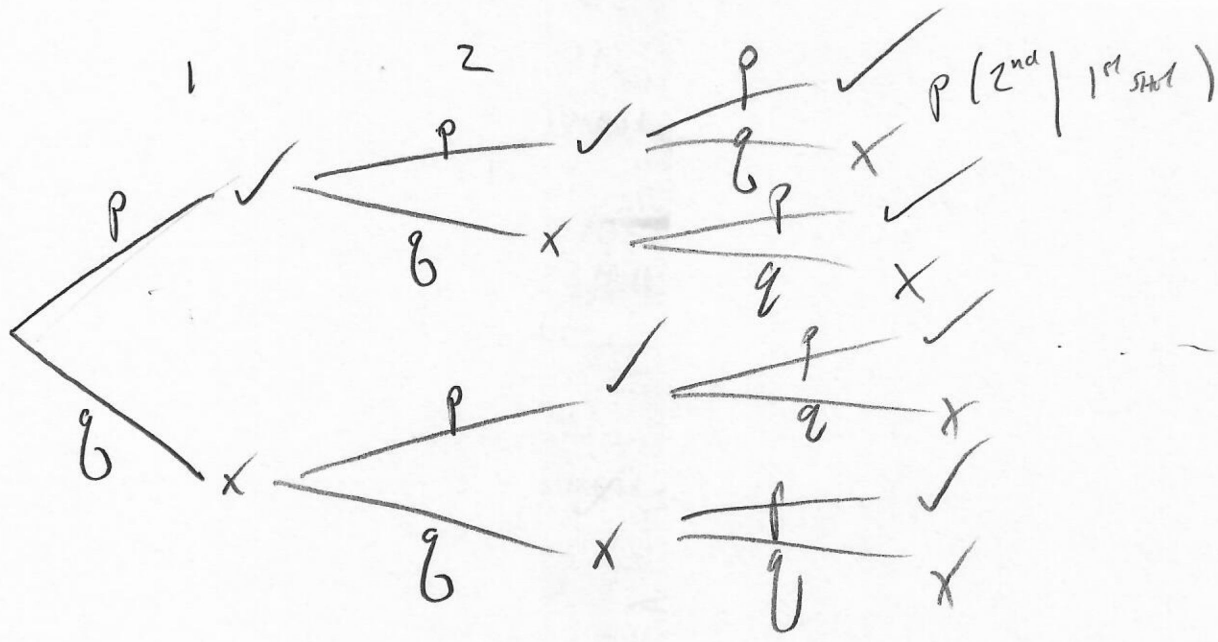
$$\left.\begin{array}{l} pppppp\,qq \\ pp\,q\,pppp\,q\,p \\ q\,p\,q\,ppppp \end{array}\right\}$$

SIMPLE EVENTS WITH SAME PROB

$C^8_6\, p^6 q^2$

$\vdots$

HOW MANY WAYS TO DO THIS?




IN GENERAL, P(k SUCCESSES IN n TRIALS)

$$= C^n_k \, p^k q^{n-k}$$

P(6 SHOTS IN 8 FREE THROWS) $= C^8_6 \, (.84)^6 (.16)^2$

$$= .2518$$

§9.3 PERFORMING AN EXPERIMENT, TRYING TO SHOW (ALTERNATIVE HYP.) POPULATION MEAN IS HIGHER / LOWER THAN AN ACCEPTED VALUE.

e.g. YOU WANT TO SHOW THE AVERAGE CCNY STUDENT GETS LESS THAN 7 HRS OF SLEEP PER NIGHT.

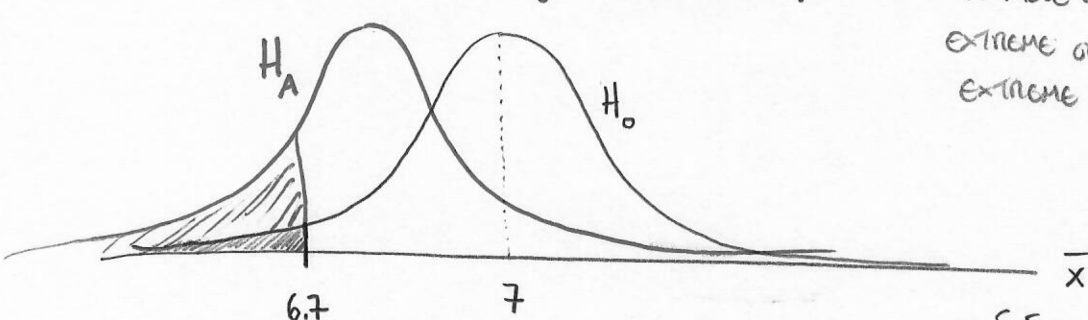
NULL $H_0$ $\mu = 7$

ALT $H_A$ $\mu < 7$ ← WANT TO SHOW.

SAMPLE: $\underbrace{n = 50, \ \bar{x} = 6.7, \ s = .25}$

$p$-VALUE: HOW LIKELY ARE WE TO OBSERVE THIS IF $H_0$ IS TRUE?

A VALUE OF $\bar{x}$ AS EXTREME OR MORE EXTREME.




$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$

$\alpha = .1, .05, .02, .01$

$P(\bar{x} \leq 6.7) = P\left(z \leq \frac{6.7 - 7}{.25/\sqrt{50}}\right)$

GIVEN

$= P(z \leq -8.49) \approx 0$

SO UNLIKELY

$p$-VALUE OF 0

↓

WHEN $p$-VALUE $\leq \alpha$, WE REJECT $H_0$ IN FAVOR $H_A$.

§9.3 ONE-TAILED HYPOTHESIS TESTS ONLY.

$$\mu_1 - \mu_2 \approx \bar{X}_1 - \bar{X}_2 \pm z_{\alpha/2}\, S.E. \longrightarrow$$

$$\underline{\text{ERROR} \leq \text{BOUND}}$$

$$z_{\alpha/2}\ S.E. \leq .17$$

$$1.645 \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq .17$$

$$\left(\sqrt{\frac{27.8 + 27.8}{n}}\right)^2 \leq \left(\frac{.17}{1.645}\right)^2$$

$$\cancel{n}\ \frac{55.6}{\cancel{n}} \leq \left(\frac{.17}{1.645}\right)^2 n$$

$$\frac{55.6}{\left(\frac{.17}{1.645}\right)^2} \leq \frac{\cancel{\left(\frac{.17}{1.645}\right)^2} n}{\cancel{\left(\frac{.17}{1.645}\right)^2}}$$

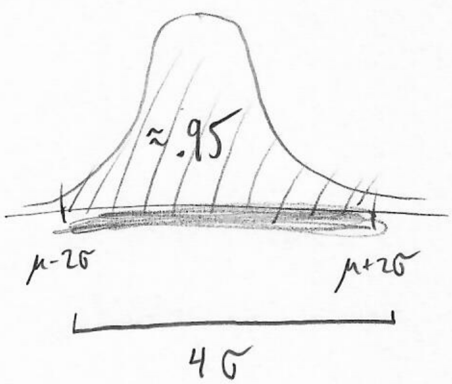
$$\boxed{5207 \leq n}$$

ESTIMATE $\mu_1 - \mu_2$

POP 1 — AMERICANS 10 YRS AGO

POP 2 — AMERICANS TODAY.

$$\mu_1 - \mu_2 \approx \bar{X}_1 - \bar{X}_2 \pm \underbrace{z_{\alpha/2}\, S.E}_{\text{ERROR} \le 5}$$




$$2.58 \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le 5$$

$n_1 = n_2 = n$

CRUDE ESTIMATE: $\sigma \approx \frac{\text{RANGE}}{4} = \frac{104}{4} = 26$

$$2.58 \sqrt{\frac{26^2 + 26^2}{n}} \le 5$$

$$\frac{2 \cdot 26^2}{n} \le \left(\frac{5}{2.58}\right)^2$$

$$359.98 = \frac{2 \cdot 26^2}{\left(\frac{5}{2.58}\right)^2} \le n \qquad \boxed{n \ge 360}$$

OBSERVATIONS: UNITS %

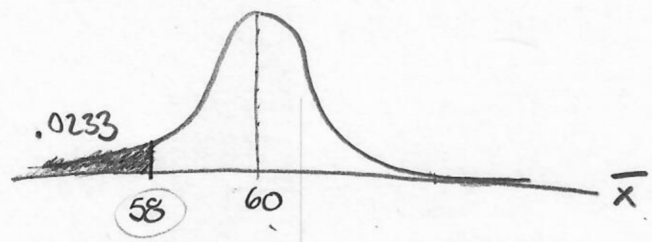
$H_0 = \mu = 60$

$H_A : \mu < 60$ (UNPROFITABLE)

p-VALUE $\overset{?}{\leq} \alpha$

IF YES $\Rightarrow$ REJECT $H_0$ IN FAVOR OF $H_A$

IF NO $\Rightarrow$ ACCEPT $H_0$




$n = 120$

$\sigma = 11$

p-VALUE : $P(\bar{x} \leq 58) = P\left(z \leq \frac{58-60}{11/\sqrt{120}}\right)$

$= P(z \leq -1.99) \approx .0233 \leq \alpha = .05$ ✓

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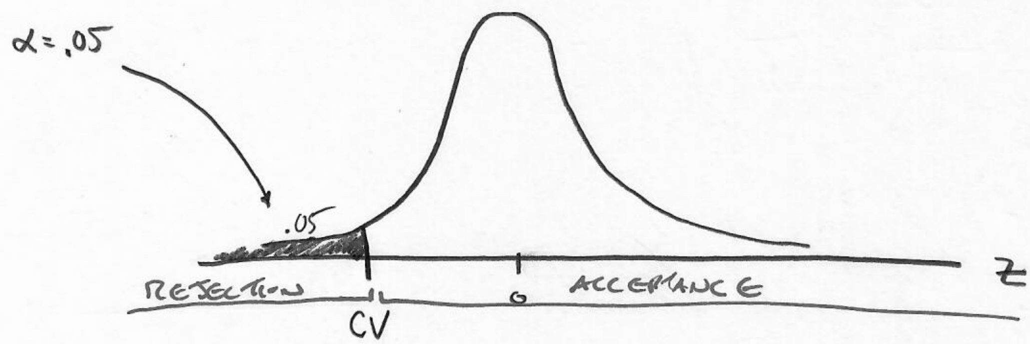



$\alpha$

CRITICAL VALUE : $-1.645$

$P(z \leq -1.645) \approx .05$

OUR OBSERVATION $\bar{x} = 58$ HAS z-VALUE $-1.99 \leq$ CRITICAL VALUE

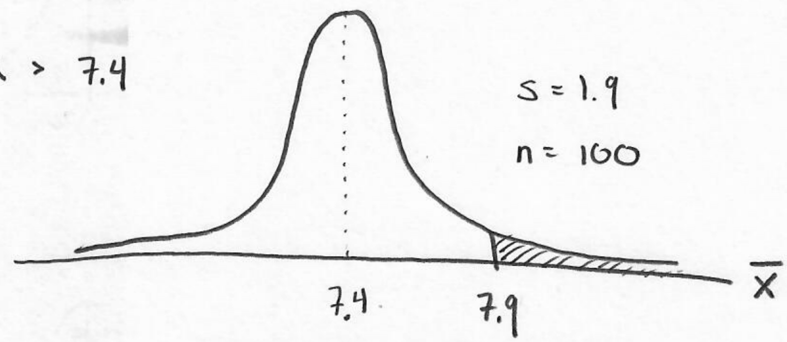
(i.e. REJECTION REGION)

REJECT $H_0$ IN FAVOR OF $H_A : \mu < 60$

FLIGHT IS UNPROFITABLE.

$H_0: \mu = 7.4$

$H_A: \mu > 7.4$




$s = 1.9$

$n = 100$

p-VALUE = PROB OF MAKING OBSERVATION AS EXTREME OR MORE EXTREME THAN THAT OBSERVED.

$$P(\overline{x} \geq 7.9) = P\left(z \geq \frac{7.9 - 7.4}{1.9/\sqrt{100}}\right)$$

$$= 1 - P(z \leq 2.63)$$

$$= 1 - .9957 = \underline{.0043}$$

REJECT WHEN p-VALUE $\leq \alpha$

$\alpha$ IS THE GREATEST p VALUE SUCH THAT WE REJECT THE $H_0$.

$\alpha = .1$

$.05$

$.02$

$.01$

WE REJECT $H_0$ AT LEVELS

$\underline{\alpha \geq .0043}$

§8.4

$$\mu \approx \boxed{\overline{X}} \pm \boxed{1.96\frac{\sigma}{\sqrt{n}}}$$

POINT ESTIMATE

$$p \approx \boxed{\hat{p}} \pm \boxed{1.96\sqrt{\frac{pq}{n}}}$$

"MARGIN OF ERROR"

§8.5

95% CONF. <u>INTERVAL</u>

90%, 98%, 99%

| CONF. COEFFICIENT | $z_{\alpha/2}$ |
|---|---|
| 99% | 2.58 |
| 98% | 2.33 |
| 95% | 1.96 |
| 90% | 1.645 |


CONF INT: $\boxed{\text{POPULATION PARAMETER} \approx \text{SAMPLE STAT} \pm z_{\alpha/2}\ \text{S.E.}}$

$$\boxed{\text{POPULATION PARAM} \approx \text{SAMPLE STAT.} \pm z_{\alpha/2}\ \text{S.E.}}$$

CONFIDENCE INTERVALS

| POPULATION PARAM | SAMPLE STAT | S.E. |
|---|---|---|
| $\mu$ | $\bar{x}$ | $\frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$ |
| $p$ | $\hat{p}$ | $\sqrt{\frac{pq}{n}} \approx \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$ |
| $\mu_1 - \mu_2$ | $\bar{x}_1 - \bar{x}_2$ | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ |
| $p_1 - p_2$ | $\hat{p}_1 - \hat{p}_2$ | $\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$ |