The City College of New York **Department of Mathematics**
Final Examination **Math 19000** **Fall 2016**

Your section: M190______ *ANSWER KEY*
Your instructor's name ______________________
Print your name: ______________________
Sign____________________________

Instructions

No Calculators. Show all work inside this booklet.
All electronic devices must be turned off and out of sight.
There are 4 parts, with 6 questions each. Answer 5 questions from each part, for a total of 20 questions.
Each question is worth 5 points.
In each part, cross out the question that you omit.
In exactly one box on each row of the chart below, **write the word OMIT** to show the problem you leave out. If you don't write the word OMIT, the first five questions will be graded. Otherwise do not write anything below.

#1	#2	#3	#4	#5	#6	**Part I Total**
#7	#8	#9	#10	#11	#12	**Part II Total**
#13	#14	#15	#16	#17	#18	**Part III Total**
#19	#20	#21	#22	#23	#24	**Part IV Total**

Exam Total:

1 Do five of the following six problems: 1–6

1. Use properties of exponents to simplify $(\frac{a^3b^{-2}}{a^{-3}b^2})^{-2}$. Write your answer with only positive exponents.

$$= \frac{(a^3)^{-2}(b^{-2})^{-2}}{(a^{-3})^{-2}(b^2)^{-2}} = \frac{a^{-6}b^4}{a^6b^{-4}} = \frac{b^4b^4}{a^6a^6} = \boxed{\frac{b^8}{a^{12}}}$$

or $$= \left(\frac{a^3a^3}{b^2b^2}\right)^{-2} = \left(\frac{a^6}{b^4}\right)^{-2} = \left(\frac{b^4}{a^6}\right)^2 = \frac{b^8}{a^{12}}$$

2. Expand and simplify $(5x-4)^2$.

$$= (5x-4)(5x-4)$$

$$= 25x^2 - 20x - 20x + 16$$

$$= \boxed{25x^2 - 40x + 16}$$

3. Perform the indicated operations and simplify as much as possible $\frac{\frac{3}{7}+\frac{1}{3}}{\frac{1}{21}+\frac{2}{14}}$.

$$= \frac{\frac{3}{7}+\frac{1}{3}}{\frac{1}{21}+\frac{1}{7}} \cdot \frac{21}{21} = \frac{\frac{3}{7}\cdot 21 + \frac{1}{3}\cdot 21}{\frac{1}{21}\cdot 21 + \frac{1}{7}\cdot 21}$$

$$= \frac{9+7}{1+3} = \frac{16}{4} = \boxed{4}$$

4. Simplify $(27x^{15})^{-\frac{2}{3}}$ and eliminate any negative exponents.

$$= \left(\left(\left(27x^{15}\right)^{1/3}\right)^{2}\right)^{-1}$$

$$= \left(\left(3x^{5}\right)^{2}\right)^{-1} = \left(9x^{10}\right)^{-1}$$

$$= \boxed{\frac{1}{9x^{10}}}$$

5. Solve $P = 2L + 2W$ for L.

$P - 2W = 2L$

$\boxed{L = \frac{P - 2W}{2}}$

6. Let $A(-6, -1)$ and $B(2, 7)$ be points in the plane.
   (a) Find the slope of the line that contains A and B.
   (b) Find the length of the segment AB.

(a) $m = \frac{x_2 - x_1}{y_2 - y_1} = \frac{2 - (-6)}{7 - (-1)} = \frac{8}{8} = \boxed{1}$

(b) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - (-6))^2 + (7 - (-1))^2}$

$= \sqrt{8^2 + 8^2} = \sqrt{64 \cdot 2}$

$= \boxed{8\sqrt{2}}$

2 Do five of the following six problems: 7–12

7. Solve for x in the equation $3(1 - x) = 5(1 + 2x) + 2$.

$3 - 3x = 5 + 10x + 2$

$-13x = 4$

$\boxed{x = -\frac{4}{13}}$

8. Solve the equation $x^2 - x = 56$.

$x^2 - x - 56 = 0$

$(x-8)(x+7) = 0$

$\boxed{x = 8, -7}$

9. Find the standard form of the equation of the circle having diameter with endpoints $(-11, 9)$ and $(7, -1)$.

CENTER $= \text{MIDPOINT} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (-2, 4)$

RADIUS $= \frac{1}{2}$ DIAMETER $= \frac{1}{2}\sqrt{(-11-7)^2 + (9+1)^2}$

$= \frac{1}{2}\sqrt{18^2 + 10^2} = \frac{1}{2}\sqrt{324 + 100}$

$= \frac{1}{2}\sqrt{424} = \frac{1}{2} \cdot 2\sqrt{106} = \sqrt{106}$

CIRCLE : $\boxed{(x+2)^2 + (y-4)^2 = 106}$

10. Solve $\frac{1}{x} - \frac{1}{x-4} = 1$ for x.

$\left[\frac{1}{x} - \frac{1}{x-4} = 1 \right] x(x-4)$

LCD

$x - 4 - x = x(x-4) = x^2 - 4x$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$\boxed{x = 2}$

11. Given $f(x) = \sqrt{(\frac{x^2+1}{2})}$ and $g(x) = 3 + \sqrt{x}$. Compute and simplify $f \circ g(16)$.

$g(16) = 3 + \sqrt{16} = 3 + 4 = 7$

$f(g(16)) = f(7) = \sqrt{\frac{7^2+1}{2}} = \sqrt{25} = \boxed{5}$

12. Evaluate the function and simplify $h(3-2a)$ when $h(x) = \frac{x^2+9}{2}$.

$$h(3-2a) = \frac{(3-2a)^2+9}{2}$$

$$= \frac{9 - 12a + 4a^2 + 9}{2}$$

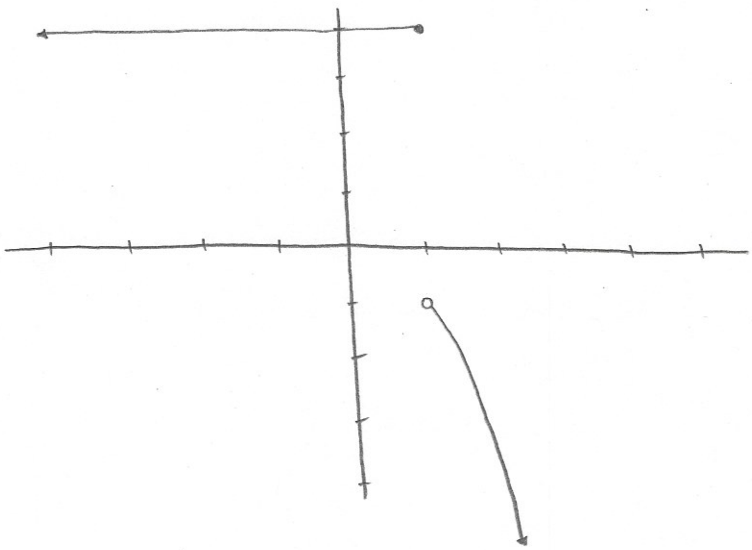
$$= \frac{2(9 - 6a + 2a^2)}{2}$$

$$= \boxed{2a^2 - 6a + 9}$$

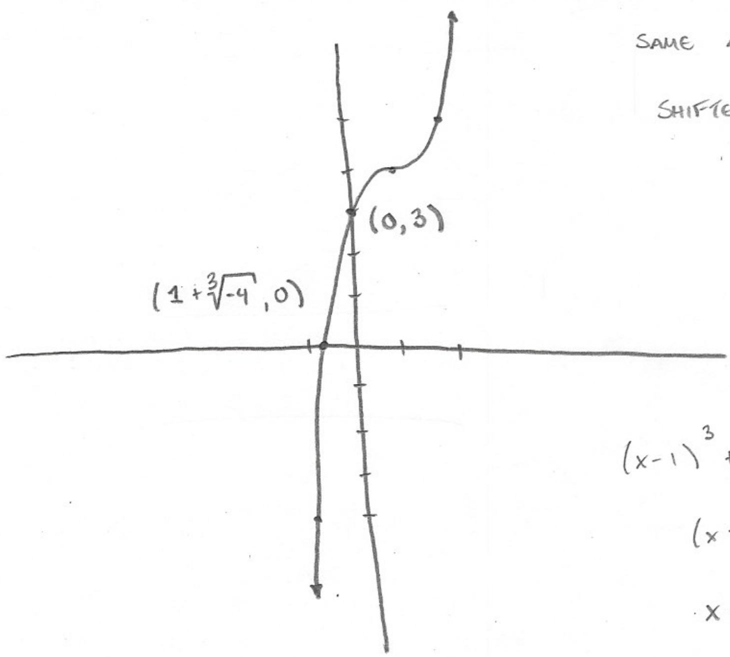
3 Do five of the following six problems: 13–18

13. Sketch a graph of the piecewise defined function

$$f(x) = \begin{cases} 4 & \text{if } x \le -1 \\ -x^2 & \text{if } x > -1 \end{cases}$$



14. Graph the function $r(x) = (x-1)^3+4$ by indicating how a more basic function has been shifted, reflected, stretched, or compressed. Label all x-intercepts and y-intercepts clearly on your graph.



SAME AS $y = x^3$ BUT
SHIFTED RIGHT 1
UP 4.

$(x-1)^3 + 4 = 0$

$(x-1)^3 = -4$

$x - 1 = \sqrt[3]{-4}$

$x = 1 + \sqrt[3]{-4}$

15. Determine whether $f(x) = -5x^2 - 40x - 81$ has either a maximum or a minimum or neither. If either a maximum or a minimum, find what that value is and where it occurs.

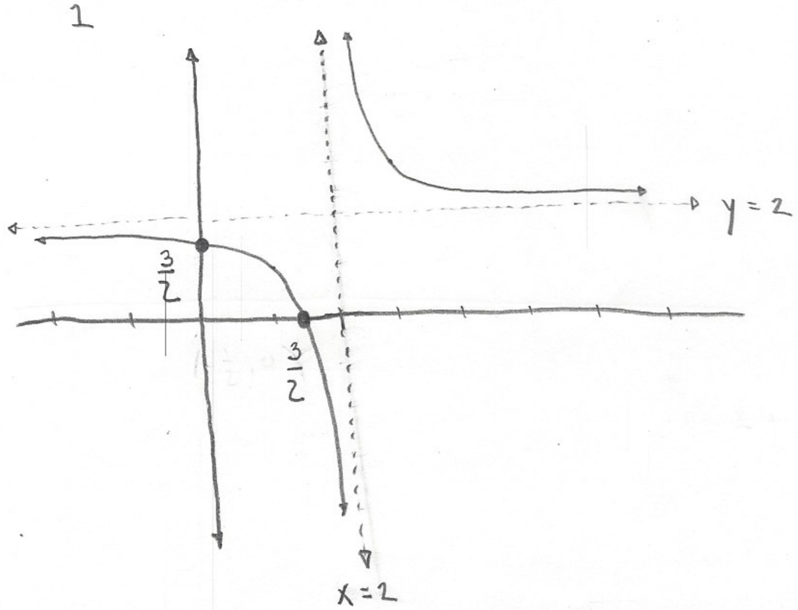
$$-\frac{b}{2a} = -\frac{(-40)}{2(-5)} = -4$$

$$f\left(-\frac{b}{2a}\right) = -5(-4)^2 - 40(-4) - 81$$

$$= -80 + 160 - 81 = \boxed{-1 \quad \text{MAXIMUM}}$$

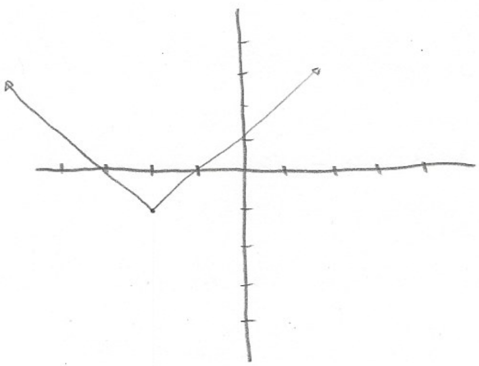
16. Use transformations of the graph $y = \frac{1}{x}$ to graph the rational function $g(x) = \frac{2x-3}{x-2}$. Label all asymptotes and intercepts clearly on your graph.

$$\begin{array}{r} 2 \\ x-2 \overline{)\,2x-3} \\ -(2x-4) \\ \hline 1 \end{array} \Rightarrow g(x) = 2 + \frac{1}{x-2}$$



17. Given $f(x) = |x+2| - 1$. Determine the intervals on which f is increasing and on which f is decreasing.

$y = f(x)$



INCREASING : $(-2, \infty)$

DECREASING : $(-\infty, -2)$

$\Rightarrow$ THE TWO NUMBERS CANNOT HAVE DIFFERENT SIGNS

18. The sum of two numbers is twice their difference. The larger number is 6 more than twice the smaller. Find the two numbers.

LET l = LARGER # $= 2s + 6$

s = SMALLER #

ASSUME $l, s \geq 0$

$l + s = 2(l - s)$

$2s + 6 + s = 2(2s + 6 - s)$

$3s + 6 = 2s + 12$

$\boxed{s = 6 \quad l = 18}$

OR

ASSUME $l, s \leq 0$

$l + s = 2(s - l)$

$2s + 6 + s = 2(s - (2s + 6))$

$3s + 6 = -2s - 12$

$5s = -18$

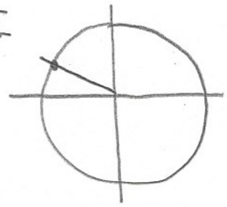
$s = -\frac{18}{5}$

$l = -\frac{6}{5}$

4 Do five of the following six problems: 19–24

19. Find the exact value of $\cos(-570°)$.

QII



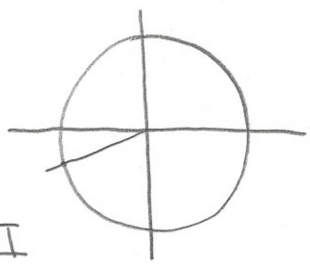
$-570° \cdot \frac{\pi}{180°} = -\frac{19\pi}{6}$

REFERENCE NUMBER $= \frac{\pi}{6}$

$\cos(-570°) = \cos\left(-\frac{19\pi}{6}\right) = \pm\cos\left(\frac{\pi}{6}\right) = \pm\frac{\sqrt{3}}{2}$

$= \boxed{-\frac{\sqrt{3}}{2}}$

20. Find the exact value of $\sin \frac{7\pi}{6}$.

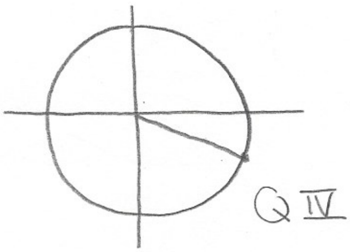


QIII

$\sin\left(\frac{7\pi}{6}\right) = \pm\sin\left(\frac{\pi}{6}\right) = \pm\frac{1}{2}$

$= \boxed{-\frac{1}{2}}$

21. Find the terminal point $P(x, y)$ on the unit circle determined by $t = -\frac{\pi}{6}$.



REFERENCE NUMBER $= \frac{\pi}{6}$

$$P = \left(\pm \cos \frac{\pi}{6}, \pm \sin \frac{\pi}{6} \right)$$

$$= \left(\pm \frac{\sqrt{3}}{2}, \pm \frac{1}{2} \right)$$

$$\boxed{P = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2} \right)}$$

22. The point P is on the unit circle. Find the y-coordinate of the point $P(x, y)$ if the x-coordinate of P is $x = -\frac{12}{13}$ and the y-coordinate is negative.

$$x^2 + y^2 = 1$$

$$\left(-\frac{12}{13}\right)^2 + y^2 = 1$$

$$y^2 = 1 - \frac{144}{169} = \frac{25}{169}$$

$$y = \pm \sqrt{\frac{25}{169}} = \pm \frac{5}{13}$$

$$\boxed{y = -\frac{5}{13}}$$

23. Solve the system

$$\begin{cases} x + 4y &= 8 \quad ① \\ 3x + 12y &= 24. \quad ② \end{cases}$$

or show that it has no solution. (If there is no solution, enter NO SOLUTION. If there are an infinite number of solutions, enter the general solution in terms of t, where t is any real number.)

②- 3① : $0 = 0$ TRUE! $\Rightarrow \infty$ MANY SOLUTIONS.

LET $x = t$. THEN

$t + 4y = 8$

$4y = 8 - t$

$y = \frac{8-t}{4}$

$\boxed{\left(t, \frac{8-t}{4}\right)}$

LET $y = t$. THEN

$x + 4t = 8$

$x = 8 - 4t$

OR $(8-4t, t)$

24. The system of equations has a unique solution. Find the solution using Gaussian elimination or Gauss Jordan elimination.

$$\begin{cases} x + y + z &= 10 \quad ① \\ 2x - 3y + 2z &= 20 \quad ② \\ 4x + y - 3z &= 5. \quad ③ \end{cases}$$

$x + y + z = 10 \quad \Rightarrow x = 5$

②-2①: $-5y = 0 \quad \Rightarrow y = 0$

③-4①: $-3y - 7z = -35 \quad \Rightarrow z = 5$

$\boxed{(5, 0, 5)}$