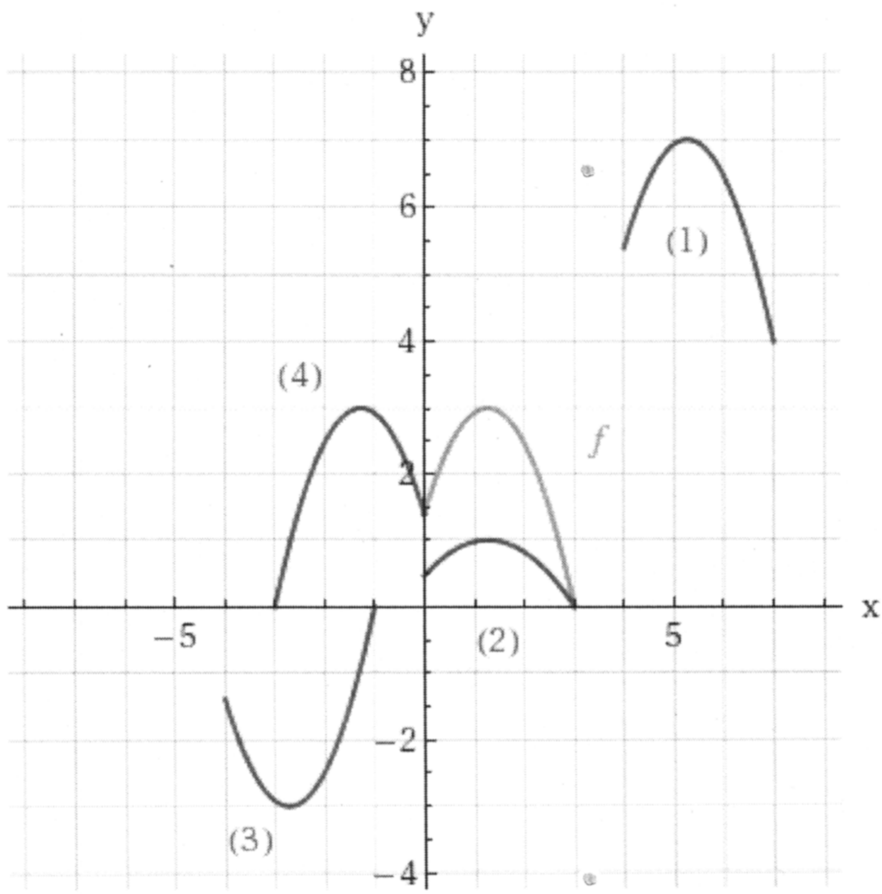
Name: *ANSWER KEY* 11/30/2016

Math 190-EF Exam 3

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Good luck!

1. The graph of $y = f(x)$ is given below. Match equation with its graph (1, 2, 3, or 4).



(a) (2 points) $y = \frac{1}{3}f(x)$ 2

(b) (2 points) $y = -f(x+4)$ 3

(c) (2 points) $y = f(x-4)+4$ 1

(d) (2 points) $y = f(-x)$ 4

2. Let functions f and g be defined as follows.

$$f(x) = x^2 - 3x + 1 \qquad g(x) = \sqrt{x} + 1$$

Find the following compositions.

(a) (4 points) $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x} + 1)$

$= \boxed{(\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 1}$

$= x + 2\sqrt{x} + 1 - 3\sqrt{x} - 3 + 1$

$= \boxed{x - \sqrt{x} - 1}$

(b) (4 points) $(g \circ f)(x)$

$(g \circ f)(x) = g(f(x)) = g(x^2 - 3x + 1)$

$= \boxed{\sqrt{x^2 - 3x + 1} + 1}$

3. Consider the following quadratic function.

$$q(x) = -x^2 - 4x - 3$$

(a) (4 points) Express $q(x)$ in *standard form.*

$$q(x) = -(x^2 + 4x) - 3$$

$$= -(x^2 + 4x + 4) - 3 + 4$$

$$\boxed{q(x) = -(x+2)^2 + 1}$$

(b) (3 points) Give the coordinates of the *vertex* of the graph $y = q(x)$.

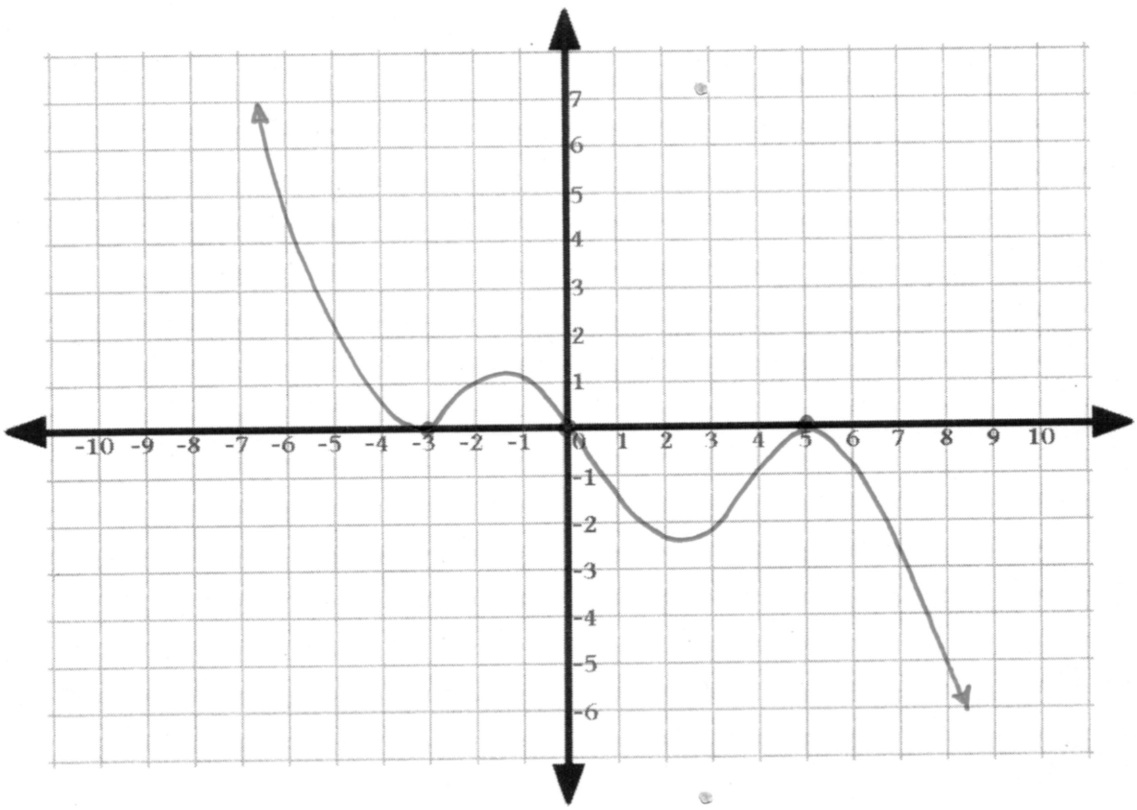
$$\boxed{(-2, 1)}$$

(c) (3 points) Find the minimum/maximum value of q and state whether it is a minimum or a maximum.

$$\boxed{\text{MAXIMUM VALUE : } 1}$$

4. (8 points) Graph the following polynomial in the coordinate plane below. Label all x-intercepts and y-intercepts and be sure your graph exhibits the proper end behavior.

$$P(x) = -2x^3(x+3)^4(x-5)^2$$



END BEHAVIOR: DEGREE $3 + 4 + 2 = 9$ ODD

LEAD COEFFICIENT -2 NEGATIVE

$\Rightarrow$

ROOTS/ZEROS: $x = -3$, MULTIPLICITY 4 (EVEN) $\rightarrow$ "BOUNCE"

$x = 0$, MULTIPLICITY 3 (ODD) $\rightarrow$ "CROSS"

$x = 5$, MULTIPLICITY 2 (EVEN) $\rightarrow$ "BOUNCE"

5. (8 points) Perform polynomial long division to find the quotient $Q(x)$ and the remainder $R(x)$ such that

$$\frac{x^7 - 4x^5 - x^4 + 4x^2 + 1}{x^3 - 1} = Q(x) + \frac{R(x)}{x^3 - 1}.$$

$$x^4 - 4x^2$$

$$x^3 - 1 \;\big)\; x^7 + 0x^6 - 4x^5 - x^4 + 0x^3 + 4x^2 + 0x + 1$$

$$-\left(x^7 \qquad - x^4\right)$$

$$-4x^5 + 0x^4 + 0x^3 + 4x^2 + 0x + 1$$

$$-\left(-4x^5 \qquad + 4x^2\right)$$

$$\boxed{1}$$

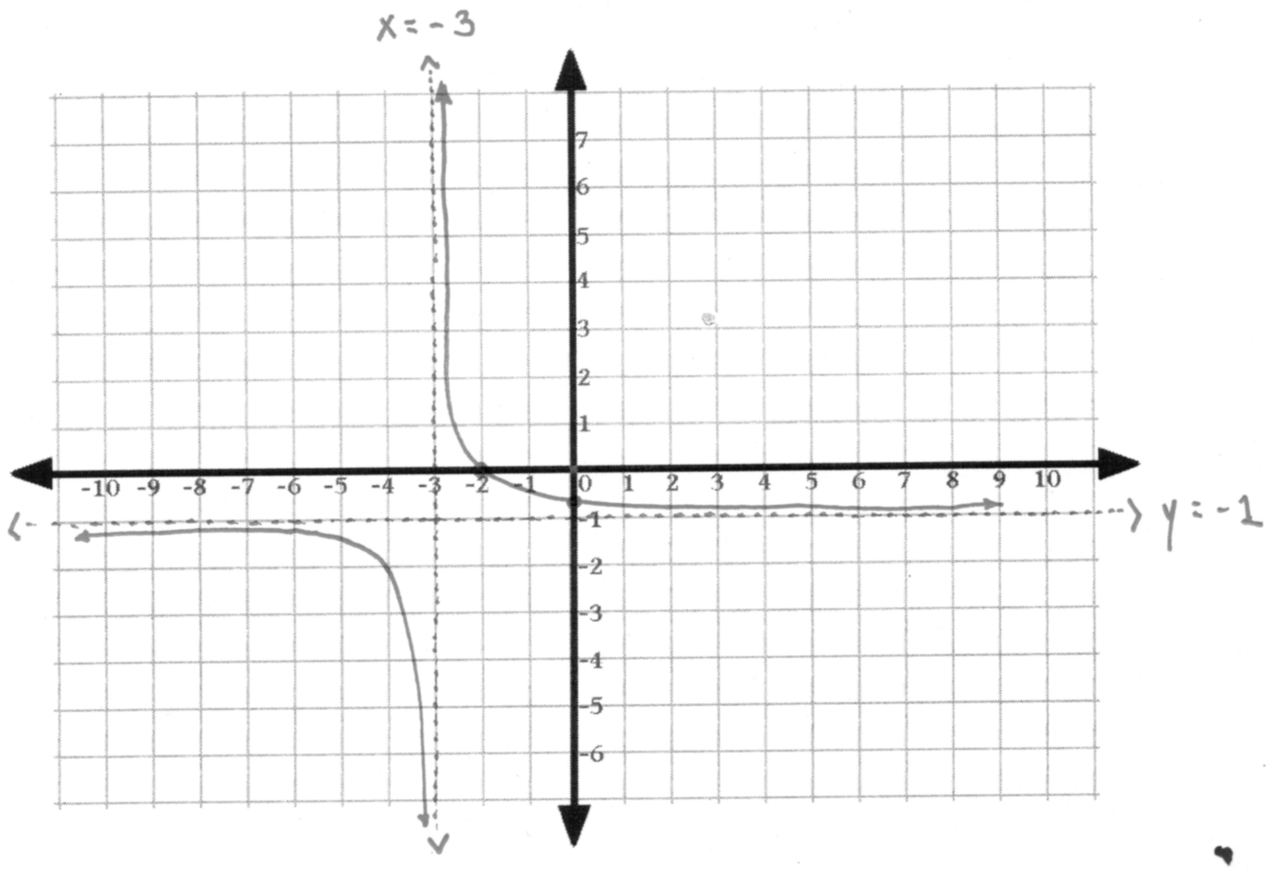
$$\boxed{\begin{array}{l} Q(x) = x^4 - 4x^2 \\ R(x) = 1 \end{array}}$$

$$\frac{x^7 - 4x^5 - x^4 + 4x^2 + 1}{x^3 - 1} = x^4 - 4x^2 + \frac{1}{x^3 - 1}$$

6. (8 points) Sketch the following graph.

$$y = \frac{1}{x+3} - 1$$

Label all x-intercepts and y-intercepts and give equations for any horizontal asymptotes and vertical asymptotes.



X-INT : $0 = \frac{1}{x+3} - 1$

$1 = \frac{1}{x+3}$

$x + 3 = 1$

$x = -2$

y-INT : $y = \frac{1}{0+3} - 1$

$y = \frac{1}{3} - 1 = -\frac{2}{3}$

bonusbonusbonus

7. (5 bonus points) Let f and g be defined as is question 2. That is,

$$f(x) = x^2 - 3x + 1 \qquad g(x) = \sqrt{x} + 1$$

Use interval notation to describe the domain of $(g \circ f)(x)$.

THE DOMAIN OF $g(f(x)) = \sqrt{x^2 - 3x + 1} + 1$

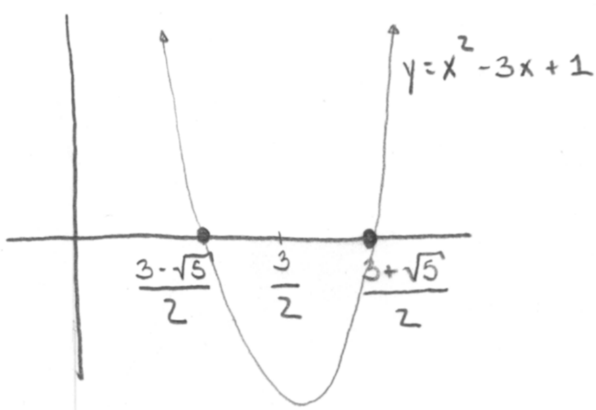
IS THE SET OF ALL REAL NUMBERS SUCH THAT

THE RADICAND IS NOT NEGATIVE. THAT IS,

$$x^2 - 3x + 1 \geq 0.$$

WE USE QUADRATIC FORMULA TO FIND WHERE IT EQUALS 0:

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$



$x^2 - 3x + 1$ IS POSITIVE

ON THE SAME INTERVALS ON WHICH

ITS GRAPH IS ABOVE THE X-AXIS.

THEREFORE WE SEE THAT $x^2 - 3x + 1 \geq 0$ WHEN EITHER

$$x \leq \frac{3-\sqrt{5}}{2} \quad \text{or} \quad x \geq \frac{3+\sqrt{5}}{2}$$

i.e. $\boxed{\left(-\infty, \frac{3-\sqrt{5}}{2}\right] \cup \left[\frac{3+\sqrt{5}}{2}, \infty\right)}$