Math 190

Tuesday, July 12, 2022 10:46 AM

3.1 Quadratics

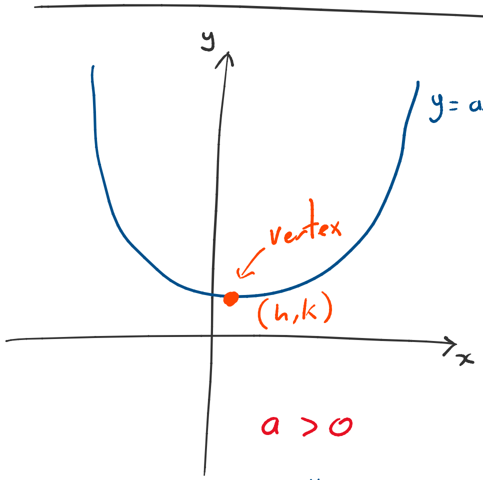
In General a quadratic function is a function of the form:

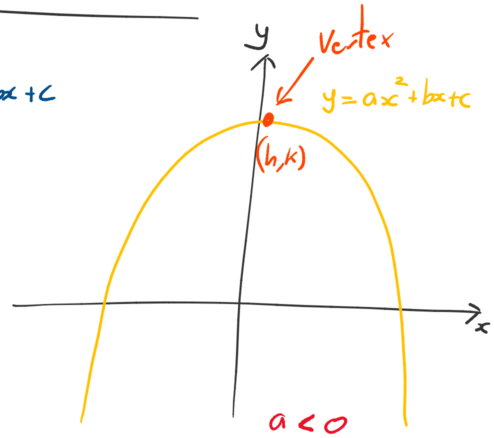
$$\boxed{f(x) = ax^2 + bx + c}$$ General Form

↓

$$\boxed{f(x) = a(x-h)^2 + k}$$ Standard form

Graph of a Quadratic function (Parabola)





$a > 0$ $a < 0$

"Smile ☺" "Frown ☹"

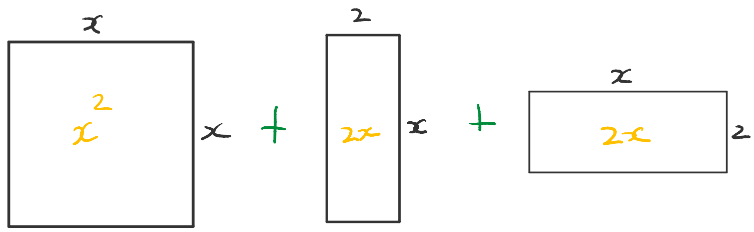
Minimum = K Maximum = K

Completing the Square

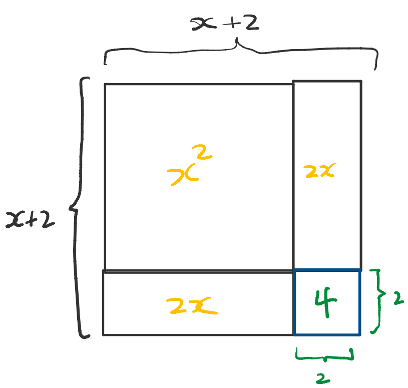
Example

Express $x^2 + 4x$ in the form $a(x-h)^2 + k$ by completing the square.

$$x^2 + 4x = x^2 + 2x + 2x$$



$=$



$$= \boxed{(x+2)^2 - 4}$$

Check that $(x+2)^2 = x^2 + 4x + 4$

Steps to Complete the Square

Given $ax^2 + bx + c$

Step 1: Factor out the coefficient of x^2

Step 1: Factor out the coefficient of x^2 from each term.

$$ax^2+bx+c = a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$$

Step 2: (a) Half the new coefficient of x and form a perfect square.

(b) Then subtract $(\text{new coefficient of } x)^2$ from outside the perfect square.

$$a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] = a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2\right]$$

Step 3: Simplify

Recall: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{(2a)^2}\right]$$

Recall: $(ab)^n = a^n b^n$

$$= a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right]$$

$$a \times \frac{b^2}{4a^2} = \frac{\cancel{a}\, b^2}{4a \times \cancel{a}} = \frac{b^2}{4a}$$

$$= a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\Leftrightarrow a(x-h)^2 + k$$

where $\boxed{h = -\frac{b}{2a}}$ and $\boxed{k = c - \frac{b^2}{4a}}$

__Example 1__ (Case 1 : $a = 1$)

Let $f(x) = x^2 + 8x + 13$

(a) Express f in standard form : $a(x-h)^2 + k$

(b) Find the vertex of Graph $y = f(x)$

(c) Find the x and y intercepts of $y = f(x)$

(d) Sketch graph of $y = f(x)$

(e) State the minimum / maximum value.

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Solution

(a) $f(x) = x^2 + 8x + 13$

$= (x+4)^2 + 13 - 4^2$

$= (x+4)^2 + 13 - 16$

$= \boxed{(x+4)^2 - 3}$

$a(x-h)^2 + k$

(b) Vertex : $(h, k) = (-4, -3)$

(c) $\boxed{y\text{-intercept (set } x=0)}$

$x^2 + 8x + 13$

when $x = 0$: $y = 0^2 + 8(0) + 13 = 13$

when $x = 0$: $y = 0 + 8(0) + 13 = 13$

So y intercept will be at the point $\boxed{(0, 13)}$

Note: for a quadratic $ax^2 + bx + c$, the y-intercept will always be $(0, c)$.

$\boxed{x\text{-intercepts (set } y = 0)}$

$$y = x^2 + 8x + 13 = 0$$

$$\Rightarrow x^2 + 8x + 13 = 0$$

$$(x+4)^2 - 3 = 0$$

$$(x+4)^2 = 3$$

$$x + 4 = \pm\sqrt{3}$$

$$x = \pm\sqrt{3} - 4$$

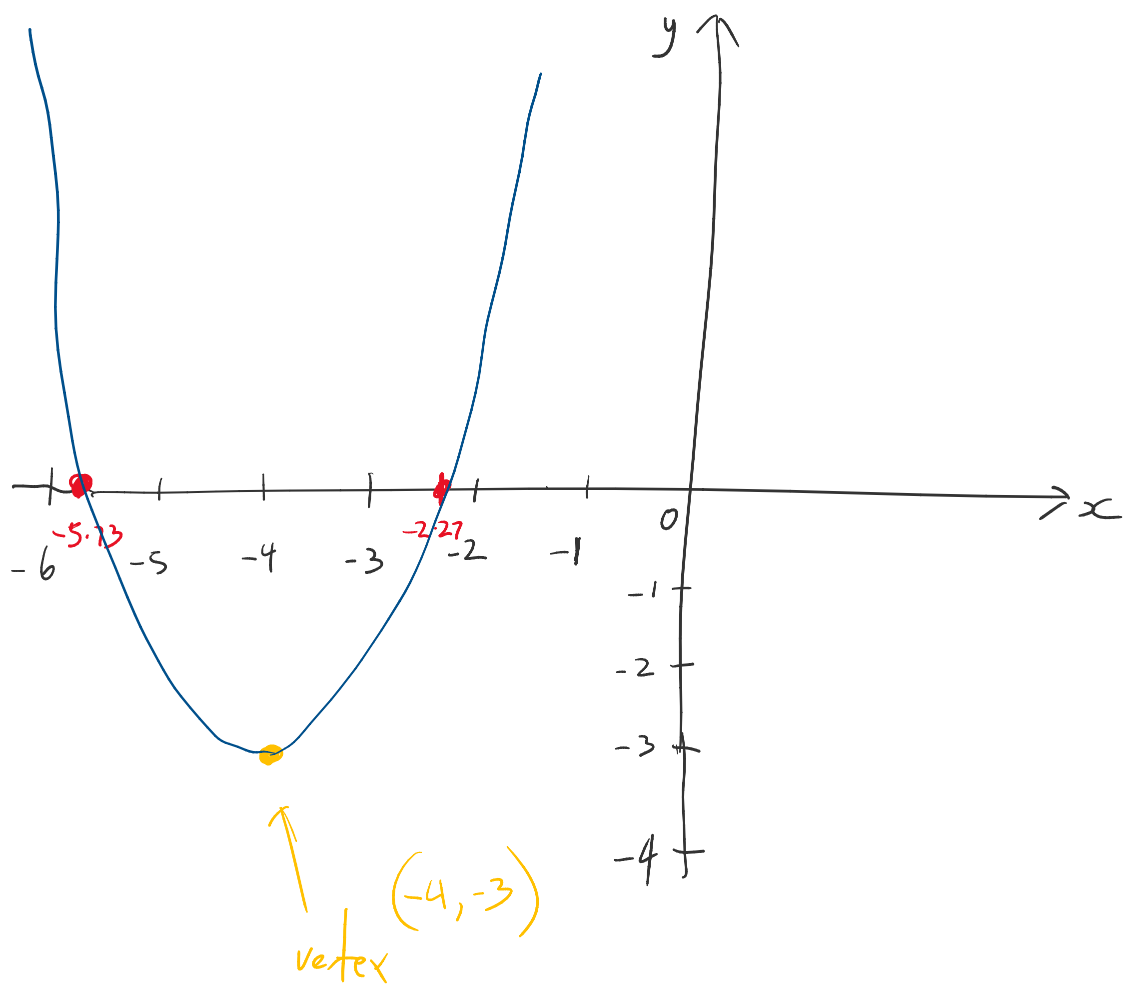
$$x = \sqrt{3} - 4 \quad \text{OR} \quad x = -\sqrt{3} - 4$$

$$\Rightarrow \boxed{x = -2.27} \quad \text{OR} \quad \boxed{x = -5.73}$$

$$\boxed{x\text{-intercepts}: \; (-2.27, 0), \; (-5.73, 0)}$$

(d) Graph of $y = x^2 + 8x + 13$

$$\Rightarrow y = (x+4)^2 - 3$$



(e) Minimum value is $\boxed{y = -3}$

Example 2 (Case 2 : $a \neq 1$)

Let $f(x) = -3x^2 - 30x - 73$

(a) Express f in standard form

(b) Find the vertex.

Solution

(a) $f(x) = -3x^2 - 30x - 73$

$$= -3\left[x^2 + 10x + \frac{73}{3}\right]$$

$$= -3\left[(x+5)^2 + \frac{73}{3} - (5^2)\right]$$

$$= -3\left[(x+5)^2 + \frac{73}{3} - 25\right]$$

$$= -3(x+5)^2 - 73 + 75$$

$$= \boxed{-3(x+5)^2 + 2}$$ Standard form

(b) Vertex $= (-5, 2)$