

§4.4 LAWS OF LOGARITHMS

ANYTHING SAID USING EXPONENTS CAN ALSO BE SAID USING LOGARITHMS

EXPONENTS	LOGARITHMS
$a^x = y$ $2^4 = 16$ $a^{x+y} = a^x a^y$	$\log_2 16 = 4$ $\log_a y = x$ $\log_a (a^x a^y) = x + y$ $= \log_a (a^x) + \log_a (a^y)$ i.e. $\log_a (AB) = \log_a A + \log_a B$
$a^{x-y} = \frac{a^x}{a^y}$	$\log_a \left(\frac{a^x}{a^y} \right) = x - y$ $= \log_a (a^x) - \log_a (a^y)$ i.e. $\log_a \left(\frac{A}{B} \right) = \log_a A - \log_a B$
$a^{xy} = (a^x)^y$ \Downarrow	$\log_a \left((a^x)^y \right) = xy$ $= y \log_a (a^x)$ i.e. $\log_a (A^B) = B \log_a A$

LAWS OF LOGARITHMS

Let a be a positive number, with $a \neq 1$. Let A , B , and C be any real numbers with $A > 0$ and $B > 0$.

Law	Description
1. $\log_a(AB) = \log_a A + \log_a B$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
2. $\log_a \left(\frac{A}{B} \right) = \log_a A - \log_a B$	The logarithm of a quotient of numbers is the difference of the logarithms of the numbers.
3. $\log_a(A^C) = C \log_a A$	The logarithm of a power of a number is the exponent times the logarithm of the number.

EXAMPLE 1 ■ Using the Laws of Logarithms to Evaluate Expressions

Evaluate each expression.

(a) $\log_4 2 + \log_4 32$

(b) $\log_2 80 - \log_2 5$

(c) $-\frac{1}{3} \log 8$

EXAMPLE 2 ■ Expanding Logarithmic Expressions

Use the Laws of Logarithms to expand each expression.

(a) $\log_2(6x)$ (b) $\log_5(x^3y^6)$ (c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

EXAMPLE 3 ■ Combining Logarithmic Expressions

Use the Laws of Logarithms to combine each expression into a single logarithm.

(a) $3 \log x + \frac{1}{2} \log(x + 1)$

(b) $3 \ln s + \frac{1}{2} \ln t - 4 \ln(t^2 + 1)$

Note: $\log x = \log_{10} x$
(in our textbook)

WARNING!

$\log_a(x + y) \neq \log_a x + \log_a y$

$\frac{\log 6}{\log 2} \neq \log\left(\frac{6}{2}\right)$ and $(\log_2 x)^3 \neq 3 \log_2 x$