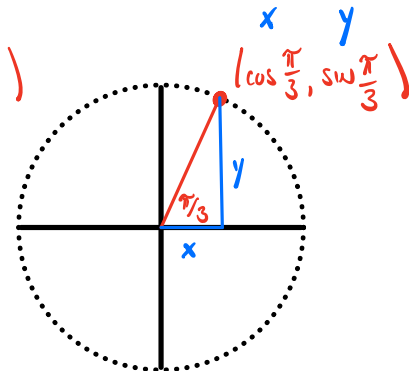
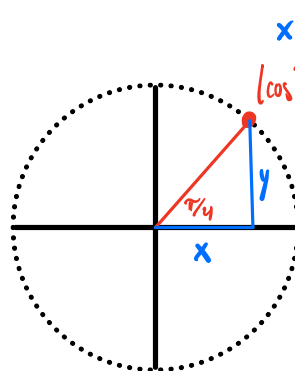
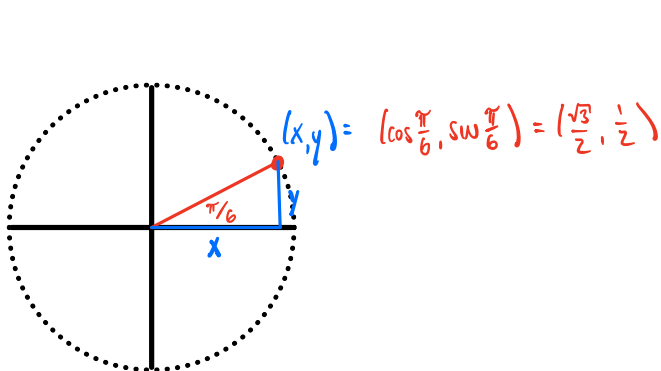


## § 5.3 TRIGONOMETRIC FUNCTIONS OF ANGLES

SO FAR, TWO IDEAS: ANGLES IN STANDARD POSITION (xy-PLANE) + TRIG RATIOS OF SIDES OF RIGHT TRIANGLES



$$\cos \frac{\pi}{6} = \frac{x}{1} \Rightarrow x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

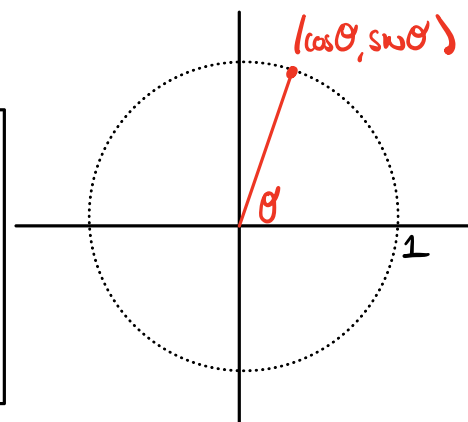
$$\sin \frac{\pi}{6} = \frac{y}{1} \Rightarrow y = \sin \frac{\pi}{6} = \frac{1}{2}$$

Put an angle in standard position.

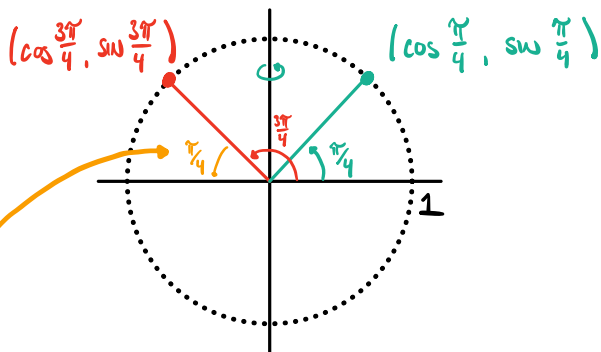
Consider the point where terminal side meets unit circle.

This point has coordinates  $x = \cos \theta$ ,  $y = \sin \theta$ .

This generalizes to ALL angles!



EX. FIND  $\cos \frac{3\pi}{4}$ ,  $\sin \frac{3\pi}{4}$ , ...



BY SYMMETRY,

$$\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

NEGATIVE

$$\sin \frac{3\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

POSITIVE

### REFERENCE ANGLE

Let  $\theta$  be an angle in standard position. The **reference angle**  $\bar{\theta}$  associated with  $\theta$  is the acute angle formed by the terminal side of  $\theta$  and the x-axis.

## EVALUATING TRIGONOMETRIC FUNCTIONS FOR ANY ANGLE

To find the values of the trigonometric functions for any angle  $\theta$ , we carry out the following steps.

1. Find the reference angle  $\bar{\theta}$  associated with the angle  $\theta$ .
2. Determine the sign of the trigonometric function of  $\theta$  by noting the quadrant in which  $\theta$  lies.
3. The value of the trigonometric function of  $\theta$  is the same, except possibly for sign, as the value of the trigonometric function of  $\bar{\theta}$ .

ex. Find  $\tan 390^\circ$ .

### EXAMPLE 3 ■ Using the Reference Angle to Evaluate Trigonometric Functions

Find (a)  $\sin 240^\circ$  and (b)  $\cot 495^\circ$ .

### EXAMPLE 4 ■ Using the Reference Angle to Evaluate Trigonometric Functions

Find (a)  $\sin \frac{16\pi}{3}$  and (b)  $\sec\left(-\frac{\pi}{4}\right)$ .

## FUNDAMENTAL IDENTITIES

### Reciprocal Identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

### EXAMPLE 5 ■ Expressing One Trigonometric Function in Terms of Another

- Express  $\sin \theta$  in terms of  $\cos \theta$ .
- Express  $\tan \theta$  in terms of  $\sin \theta$ , where  $\theta$  is in Quadrant II.

### EXAMPLE 6 ■ Evaluating a Trigonometric Function

If  $\tan \theta = \frac{2}{3}$  and  $\theta$  is in Quadrant III, find  $\cos \theta$ .

### EXAMPLE 7 ■ Evaluating Trigonometric Functions

If  $\sec \theta = 2$  and  $\theta$  is in Quadrant IV, find the other five trigonometric functions of  $\theta$ .

**13–36 ■ Values of Trigonometric Functions** Find the exact value of the trigonometric function.

- |   |  |  |
|---|--|--|
| 13. $\cos 150^\circ$                    | 14. $\sin 240^\circ$                   | 15. $\tan 330^\circ$                   |
| 16. $\sin(-30^\circ)$                   | 17. $\cot(-120^\circ)$                 | 18. $\csc 300^\circ$                   |
| 19. $\csc(-630^\circ)$                  | 20. $\cot 210^\circ$                   | 21. $\cos 570^\circ$                   |
| 22. $\sec 120^\circ$                    | 23. $\tan 750^\circ$                   | 24. $\cos 660^\circ$                   |
| 25. $\sin \frac{3\pi}{2}$               | 26. $\cos \frac{4\pi}{3}$              | 27. $\tan\left(-\frac{4\pi}{3}\right)$ |
| 28. $\cos\left(-\frac{11\pi}{6}\right)$ | 29. $\csc\left(-\frac{5\pi}{6}\right)$ | 30. $\sec \frac{7\pi}{6}$              |
| 31. $\sec \frac{17\pi}{3}$              | 32. $\csc \frac{5\pi}{4}$              | 33. $\cot\left(-\frac{\pi}{4}\right)$  |
| 34. $\cos \frac{7\pi}{4}$               | 35. $\tan \frac{5\pi}{2}$              | 36. $\sin \frac{11\pi}{6}$             |

**37–40 ■ Quadrant in Which an Angle Lies** Find the quadrant in which  $\theta$  lies from the information given.

- $\sin \theta < 0$  and  $\cos \theta < 0$
- $\tan \theta < 0$  and  $\sin \theta < 0$
- $\sec \theta > 0$  and  $\tan \theta < 0$
- $\csc \theta > 0$  and  $\cos \theta < 0$

**41–46 ■ Expressing One Trigonometric Function in Terms of Another** Write the first trigonometric function in terms of the second for  $\theta$  in the given quadrant.

- $\tan \theta$ ,  $\cos \theta$ ;  $\theta$  in Quadrant III
- $\cot \theta$ ,  $\sin \theta$ ;  $\theta$  in Quadrant II
- $\cos \theta$ ,  $\sin \theta$ ;  $\theta$  in Quadrant IV
- $\sec \theta$ ,  $\sin \theta$ ;  $\theta$  in Quadrant I
- $\sec \theta$ ,  $\tan \theta$ ;  $\theta$  in Quadrant II
- $\csc \theta$ ,  $\cot \theta$ ;  $\theta$  in Quadrant III

**47–54 ■ Values of Trigonometric Functions** Find the values of the trigonometric functions of  $\theta$  from the information given.

- $\sin \theta = -\frac{4}{5}$ ,  $\theta$  in Quadrant IV
- $\tan \theta = \frac{4}{3}$ ,  $\theta$  in Quadrant III
- $\cos \theta = \frac{7}{12}$ ,  $\sin \theta < 0$
- $\cot \theta = -\frac{8}{9}$ ,  $\cos \theta > 0$
- $\csc \theta = 2$ ,  $\theta$  in Quadrant I
- $\cot \theta = \frac{1}{4}$ ,  $\sin \theta < 0$
- $\cos \theta = -\frac{2}{7}$ ,  $\tan \theta < 0$
- $\tan \theta = -4$ ,  $\sin \theta > 0$