The City College of New York **Department of Mathematics**

Final Examination **Math 19500**

Fall 2015

Your section: M195______ Your instructor's name ____________________________

Print your name: *ANSWER KEY*

Sign______________________________

Instructions

Show all work inside this booklet.
All electronic devices must be turned off and out of sight.
There are 4 parts, with 6 questions each. Answer 5 questions from each part, for a total of 20 questions.
Each question is worth 5 points.
In each part, cross out the question that you omit.
In exactly one box on each row of the chart below, **write the word OMIT** to show the problem you leave out. If you don't write the word OMIT, the first five questions will be graded. Otherwise do not write anything below.

#1	**#2**	**#3**	**#4**	**#5**	**#6**	**Part I Total**
#7	**#8**	**#9**	**#10**	**#11**	**#12**	**Part II Total**
#13	**#14**	**#15**	**#16**	**#17**	**#18**	**Part III Total**
#19	**#20**	**#21**	**#22**	**#23**	**#24**	**Part IV Total**

Exam Total:

1 Do five of the following six problems: 1–6 (5 points each)

1. (5 points) Prove the trigonometric identity $\frac{\sin 2x}{1+\cos 2x} = \tan x$.

$$\frac{2 \sin x \cos x}{1 + \cos^2 x - \sin^2 x} = \tan x$$

$$\frac{\cancel{2} \sin x \cancel{\cos x}}{\cancel{2} \cos^{\cancel{2}} x} = \tan x$$

✓

2. (5 points) Simplify $\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15}$.

$$\frac{\cancel{x+3}}{(2x+3)\cancel{(2x-3)}} \cdot \frac{\cancel{(2x-3)}(x+5)}{(x+4)\cancel{(x+3)}} = \boxed{\frac{x+5}{(2x+3)(x+4)}}$$

3. (5 points) Rewrite without parentheses or a radical sign $y^{\frac{3}{5}}(\sqrt{y} - \frac{1}{\sqrt{y}})$.

$y^{3/5}(y^{1/2} - y^{-1/2}) = \boxed{y^{11/10} - y^{1/10}}$

4. (5 points) Solve $\frac{5}{x+2} - \frac{6}{x} + 1 = 0$ for x.

LCD = $x(x+2)$

$\frac{5}{\cancel{x+2}} \cdot x\cancel{(x+2)} - \frac{6}{\cancel{x}} \cdot \cancel{x}(x+2) + 1 \cdot x(x+2) = 0$

$5x - 6(x+2) + x(x+2) = 0$

$5x - 6x - 12 + x^2 + 2x = 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\boxed{x = -4, 3}$

5. (5 points) Find all x that solve $x^3 > x^2$.

$x^3 - x^2 > 0$

$x^2(x-1) > 0$

$x = 0, 1$

		0		1	
x^2	+		+		+
$x-1$	−		−		+
$x^2(x-1)$	−		−		+

$\boxed{(1, \infty)}$

6. (5 points) Given $f(x) = 2x - 4x^2$. Find and simplify

(a) $f(3-a)$

(b) $f(\frac{a}{2})$

(a) $f(3-a) = 2(3-a) - 4(3-a)^2$

$= 6 - 2a - 4(9 - 6a + a^2)$

$= 6 - 2a - 36 + 24a - 4a^2$

$= \boxed{-4a^2 + 22a - 30}$

(b) $f(\frac{a}{2}) = 2(\frac{a}{2}) - 4(\frac{a}{2})^2$

$= \boxed{a - a^2}$

2 Do five of the following six problems: 7–12

7. (5 points) Show that the equation $x^2 + y^2 + \frac{1}{2}x - 2y + \frac{1}{16} = 0$ represents a circle, and find its center and radius.

$$\underbrace{x^2 + \frac{1}{2}x + \frac{1}{16}} + \underbrace{y^2 - 2y + 1} = -\frac{1}{16} + \frac{1}{16} + 1$$

$$\left(x + \frac{1}{4}\right)^2 + (y-1)^2 = 1^2$$

CENTER: $\left(-\frac{1}{4}, 1\right)$

RADIUS: 1

8. (5 points) Given $f(t) = t - 2t^2$. Find and simplify the average rate of change of f from $t = 2$ to $t = 2 + h$.

$$\frac{f(2+h) - f(2)}{2+h-2} = \frac{\left[2+h-2(2+h)^2\right] - \left[2 - 2(2)^2\right]}{h}$$

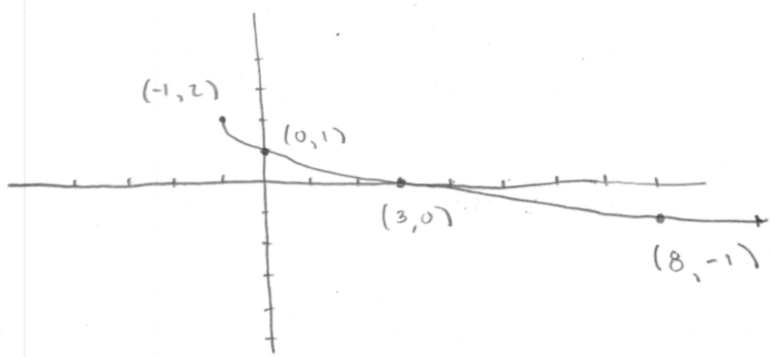
$$= \frac{1}{h}\left(\cancel{2} + h - \cancel{8} - 8h - 2h^2 - \cancel{2} + \cancel{8}\right)$$

$$= \frac{1}{h}\left(-2h^2 - 7h\right) = \boxed{-2h - 7}$$

9. (5 points) Sketch the graph of the function $y = 2 - \sqrt{x+1}$. Label at least three points on your graph including any intercepts. Begin with $y = \sqrt{x}$ and indicate the steps needed to transform its graph to the graph of $y = 2 - \sqrt{x+1}$.

$$y = \sqrt{x} \xrightarrow[\text{X-AXIS}]{\text{REFLECT}} y = -\sqrt{x} \xrightarrow[\text{LEFT 1}]{\text{SHIFT}} y = -\sqrt{x+1}$$

$$\xrightarrow[\text{UP 2}]{\text{SHIFT}} y = 2 - \sqrt{x+1}$$



10. (5 points) Given $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{2}{x}$. Evaluate and simplify $f(g(7)) - g(f(7))$.

$$g(7) = \frac{2}{7} \Rightarrow f(g(7)) = f\left(\frac{2}{7}\right) = \frac{\frac{2}{7}}{\frac{2}{7}+1} = \frac{2}{2+7} = \frac{2}{9}$$

$$f(7) = \frac{7}{7+1} = \frac{7}{8} \Rightarrow g(f(7)) = \frac{2}{\frac{7}{8}} = \frac{16}{7}$$

$$\therefore f(g(7)) - g(f(7)) = \frac{2}{9} - \frac{16}{7}$$

$$= \frac{2(7)}{63} - \frac{16(9)}{63} = \boxed{-\frac{130}{63}}$$

11. (5 points) Let

$$f(x) = \begin{cases} 1-2x & \text{if } x \le 1, \\ 2x & \text{if } x > 1 \end{cases}$$

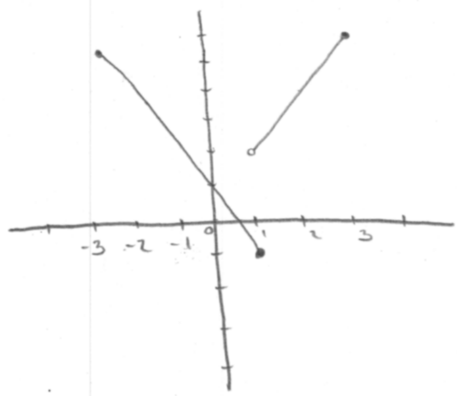
(a) Evaluate $f(-2)$ and $f(1)$.

(b) Sketch the graph of $f(x)$ from $x=-3$ to $x=3$.

(a) $f(-2) = 1-2(-2) = 1+4 = 5$

$f(1) = 1-2(1) = 1-2 = -1$

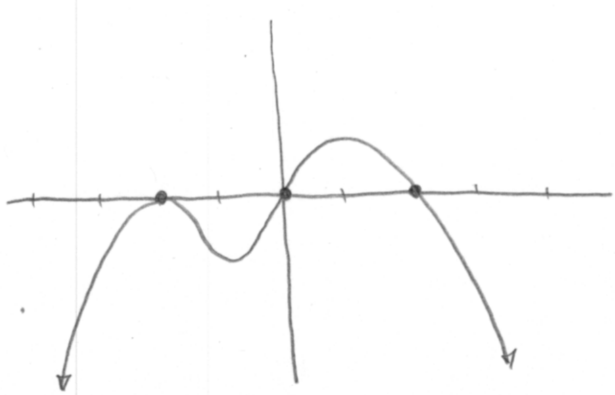
(b)



12. (5 points) Sketch the graph of $p(x) = -x(x+2)^2(x-2)^3$. Label all intercepts and indicate the end behavior.

DEGREE = 6 (EVEN)
LEAD COEFF = -1 (NEG)
} END BEHAVIOR

ZEROS: $x=0$ MULTIPLICITY 1 (CROSS)
$x=-2$ MULTIPLICITY 2 (BOUNCE)
$x=2$ MULTIPLICITY 3 (CROSS)



3 Do five of the following six problems: 13–18

13. (5 points) A bacteria culture starts with 900 bacteria. After one hour the count is 1000.
    (a) Assuming that bacteria population grows exponentially, find a function that models the number of bacteria $n(t)$ after t hours. (You may leave e, log, or ln in your answer).
    (b) After how many hours will the number of bacteria double? (You may leave e, log, or ln in your answer).

(a) $\boxed{n(t) = 900\left(\frac{10}{9}\right)^t}$

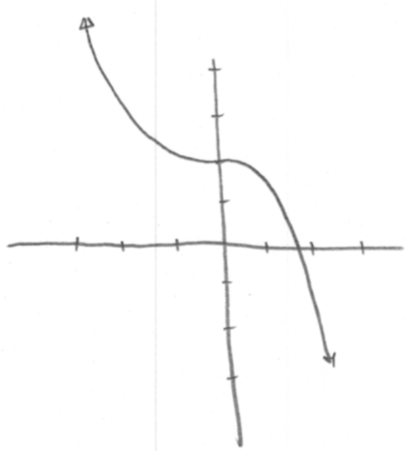
(b) $n(t) = 900\left(\frac{10}{9}\right)^t = 1800$

$\left(\frac{10}{9}\right)^t = 2$

$t \operatorname{Ln}\left(\frac{10}{9}\right) = \boxed{\dfrac{\operatorname{Ln}(2)}{\operatorname{Ln}\left(\frac{10}{9}\right)}}$

14. (5 points) Given the function $f(x) = 2 - x^3$.
    (a) Sketch the graph of f.
    (b) Find the formula for the inverse function $f^{-1}(x)$.

(a)



(b) $y = 2 - x^3$

$y - 2 = -x^3$

$x^3 = 2 - y$

$x = (2 - y)^{1/3}$

$\boxed{f^{-1}(x) = (x - y)^{1/3}}$

15. (5 points) Evaluate $\log_3(\frac{1}{\sqrt{27}})$.

$= \log_3(1) - \log_3\sqrt{27}$

$= 0 - \frac{1}{2}\log_3(27)$

$= \boxed{-\frac{3}{2}}$

16. (5 points) Solve for t in each of the following parts. You may leave e, log, or ln in your answer.

(a) $5e^{2t} - 20 = 0$

(b) $3 - \log(3 - x) = 1$.

(a) $5e^{2t} = 20$

$e^{2t} = 4$

$2t = \ln(4)$

$t = \frac{1}{2}\ln(4) = \ln(4^{1/2}) = \boxed{\ln(2)}$

(b) $2 = \log(3-x)$

$10^2 = 3 - x$

$\boxed{x = -97}$

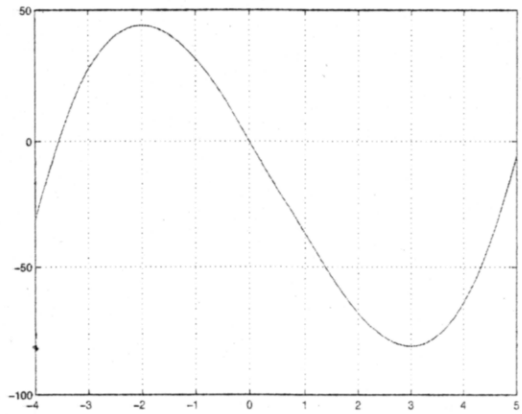


Figure 1:

17. (5 points) For the graph of the function in Figure 1, determine the interval(s) on which the function is increasing.

$\boxed{[-4,-2] \cup [3,5]}$

18. Find the exact value of $\tan(-\frac{20\pi}{3})$.

$$\tan\left(-\frac{20\pi}{3}\right) = \tan\left(-\frac{20\pi}{3} + 8\pi\right)$$

$$= \tan\left(\frac{4\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) \quad \text{(REFERENCE ANGLE)}$$

$$= \boxed{\sqrt{3}}$$

4 Do five of the following six problems: 19–24

19. (5 points) Perform the subtraction $\frac{7}{x+6} - \frac{1}{x^2+8x+12}$ and simplify.

$(x+6)(x+2)$

LCD = $(x+6)(x+2)$

$$\frac{7(x+2)}{(x+6)(x+2)} - \frac{1}{(x+6)(x+2)} = \frac{7x+14-1}{(x+6)(x+2)}$$

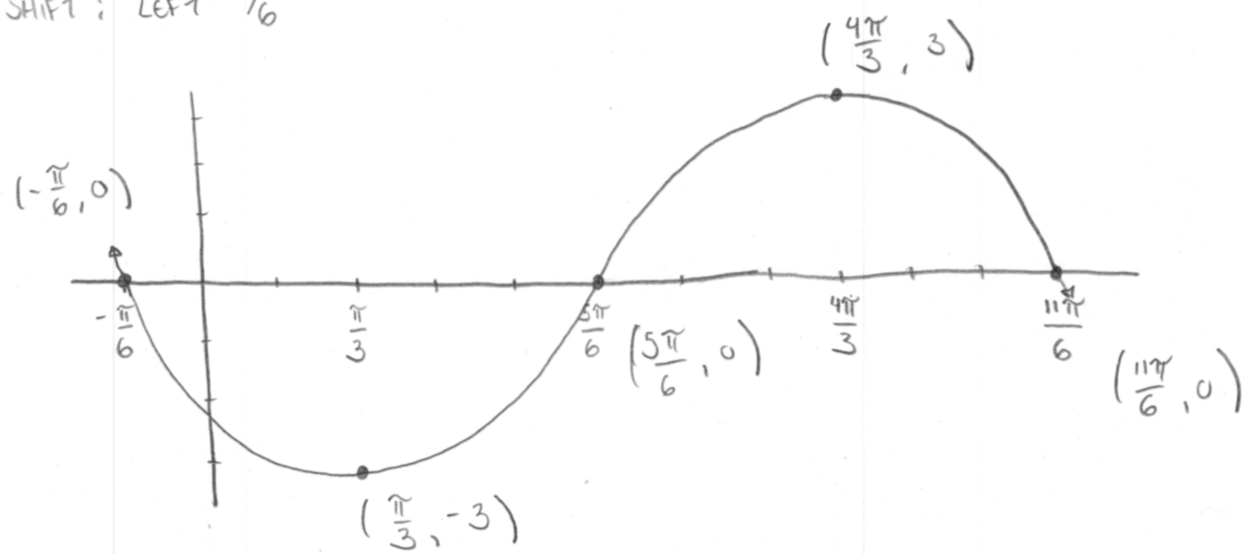
$$= \boxed{\frac{7x+13}{(x+6)(x+2)}}$$

20. (5 points) For $y = -3\sin(x + \frac{\pi}{6})$ find the amplitude, period, phase shift and then graph. Label the coordinates of three points on your graph: one maximum point, one minimum point and one intercept.

AMPLITUDE: 3

PERIOD: 2π

PHASE SHIFT: LEFT $\pi/6$



21. (5 points) Find $\cos 105°$.

$$\cos(60° + 45°) = \cos(60°)\cos(45°) - \sin(60°)\sin(45°)$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

22. Find all solutions θ to $6\cos^2\theta - 3 = 0$ for θ in the interval $-\pi \leq \theta \leq \pi$.

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{2}}{2} \qquad \cos\theta = -\frac{\sqrt{2}}{2}$$

$$\theta = -\frac{\pi}{4}, \frac{\pi}{4} \qquad \theta = -\frac{3\pi}{4}, \frac{3\pi}{4}$$

$$\boxed{\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}}$$

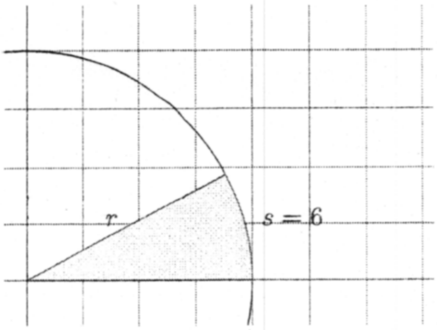


Figure 2:

23. (5 points) Find the radius r of the circle in Figure 2 with shaded angle of measure $\frac{1}{2}$ radians that is subtended by the arc s of length 6.

$s = r\theta$

$6 = \frac{r}{2}$

$\boxed{r = 12}$

24. (5 points) Evaluate each of the following
    (a) $\sin^{-1}(\frac{\sqrt{2}}{2})$
    (b) $\arcsin(\sin(\frac{11\pi}{6}))$

(a) $\frac{\pi}{4}$

(b) $\sin^{-1}\left(\sin\left(\frac{11\pi}{6}\right)\right) = \sin^{-1}\left(\sin\left(\frac{11\pi}{6} - 2\pi\right)\right)$

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= \boxed{\frac{-\pi}{6}}$