Name: ANSWER KEY 12/21/2015

Math 195 Precalculus **Final Exam**

This exam has 4 parts, each containing 6 questions. **You must answer 5 questions and omit 1 question from each part**. Please indicate the questions you wish to omit by crossing them out with a large "X". Each question not omitted is worth 5 points, for a total of 100 points.

Please show all work and **box your final answers**. Calculators are not allowed, and cellphones should be put away.

Part I

1. Let $f(x) = 3x^2 - 2x + 1$. Find and simplify $\frac{f(x+h) - f(x)}{h}$.

$$f(x+h) = 3(x+h)^2 - 2(x+h) + 1$$

$$= 3(x^2 + 2xh + h^2) - 2x - 2h + 1$$

$$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 1$$

$$f(x) = 3x^2 - 2x + 1$$

$$f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{1} - (\cancel{3x^2} - \cancel{2x} + \cancel{1})$$

$$= h(6x + 3h - 2)$$

$$\frac{f(x+h) - f(x)}{h} = \boxed{6x - 2 + 3h}$$

2. Rewrite the following expression as a simplified sum

$$(x + x^{1/3})(x + x^{2/3}) + (2 - x)(x - x^{1/3})$$

$$xx + xx^{2/3} + x^{1/3}x + x^{1/3}x^{2/3} + 2x - 2x^{1/3} - xx + xx^{1/3}$$

$$\cancel{x^2} + x^{5/3} + x^{4/3} + x + 2x - 2x^{1/3} - \cancel{x^2} + x^{4/3}$$

$$\boxed{x^{5/3} + 2x^{4/3} + 3x - 2x^{1/3}}$$

3. Given the one-to-one function $f(x) = \dfrac{2}{3x - 2}$, find $f^{-1}(x)$.

$$y = \frac{2}{3x - 2}$$

$$y(3x - 2) = 2$$

$$3xy - 2y = 2$$

$$3xy = 2y + 2$$

$$x = \frac{2y + 2}{3y} \longrightarrow \boxed{f^{-1}(x) = \frac{2x + 2}{3x}}$$

4. Solve the following inequality and write the solution using interval notation.

$$5x^2 + 3x \geq 3x^2 + 2$$

$$2x^2 + 3x - 2 \geq 0$$

$$(2x-1)(x+2) = 0$$

$$x = \frac{1}{2} \qquad x = -2$$

		-2		$\frac{1}{2}$	
$(2x-1)$	$-$		$-$		$+$
$(x+2)$	$-$		$+$		$+$
$(2x-1)(x+2)$	$+$		$-$		$+$

$$\boxed{(-\infty, -2] \cup [\tfrac{1}{2}, \infty)}$$

5. Give an equation of the line passing through the point $(1, 7)$ that is perpendicular to the line passing through the points $(2, 5)$ and $(-1, 3)$.

$$\underbrace{(x_1, y_1) \quad (x_2, y_2)}$$

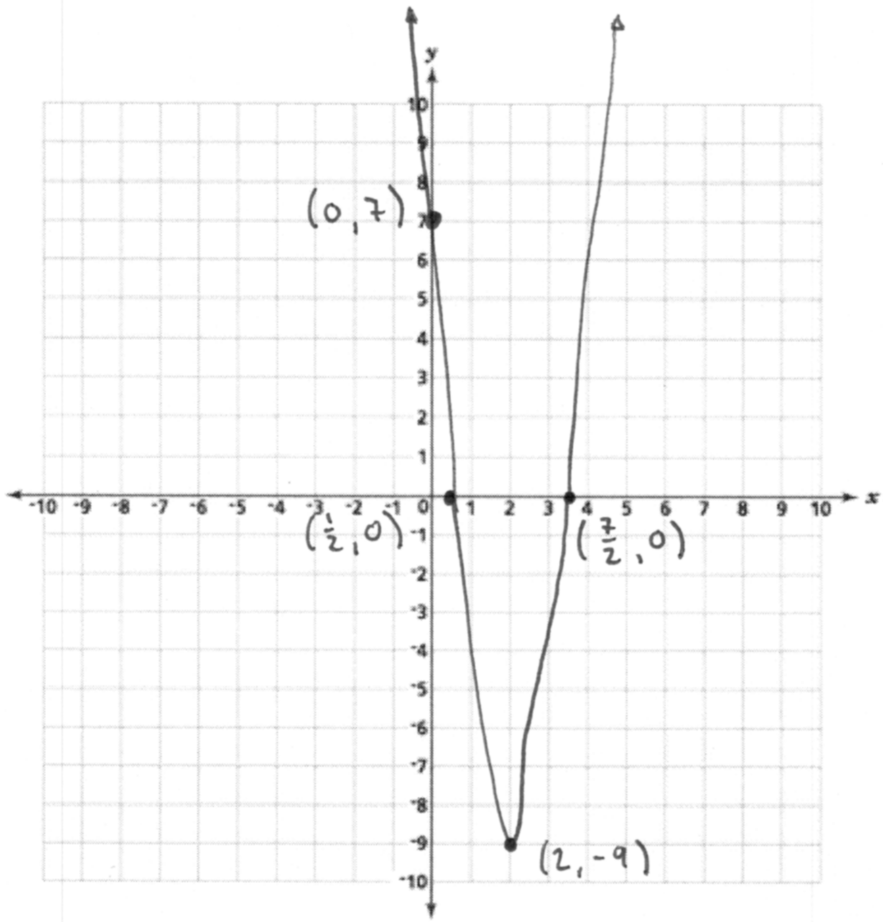
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{-1-2} = \frac{-2}{-3} = \frac{2}{3}$$

$$\perp m = -\frac{3}{2}$$

PNT-SLOPE : $y - 7 = -\frac{3}{2}(x-1)$

SLOPE-INT : $y = -\frac{3}{2}x + \frac{17}{2}$

6. In the xy-plane below, sketch the graph $y = 4x^2 - 16x + 7$. Include the coordinates of all intercepts and the vertex.



STANDARD FORM:

$y = 4(x^2 - 4x) + 7$

$= 4(x^2 - 4x + 4) + 7 - 16$

$= 4(x-2)^2 - 9$

$\therefore$ VERTEX $= (2, -9)$

Y-INT: 7

X-INT: $4x^2 - 16x + 7 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(4)(7)}}{2(4)}$$

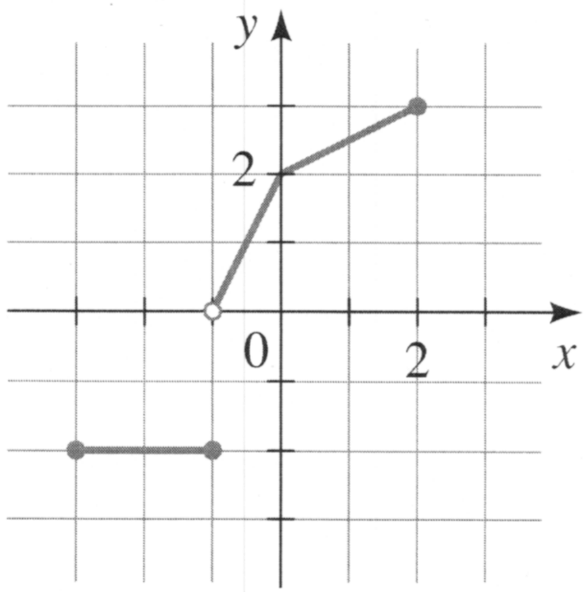
$$= \frac{16 \pm \sqrt{256 - 112}}{8} = \frac{16 \pm \sqrt{144}}{8}$$

$$= \frac{16 \pm 12}{8}$$

$$= 2 \pm \frac{3}{2} = \frac{1}{2}, \frac{7}{2}$$

Part II

7. Use interval notation to specify the domain and range of the function whose graph is given below.



DOMAIN: $[-3, 2]$

RANGE: $\{-2\} \cup (0, 3]$

8. Factor the following expression completely.

$$6(x-2)^{1/3} - 3x(x-2)^{-2/3}$$

GCF: $3(x-2)^{-2/3}$

$3(x-2)^{-2/3}\left[2(x-2) - x\right]$

$\boxed{3(x-2)^{-2/3}(x-4)}$

9. Let the functions f and g be defined as follows.

$$f(x) = \frac{1}{x^2+1}, \quad g(x) = \frac{1}{x} - 1$$

(a) Find $f(g(x))$.

$$f(g(x)) = \frac{1}{g(x)^2 + 1} = \boxed{\frac{1}{\left(\frac{1}{x} - 1\right)^2 + 1}}$$

(b) Find $g(f(-4))$.

$$f(-4) = \frac{1}{(-4)^2+1} = \frac{1}{17}$$

$$g(f(-4)) = g\left(\frac{1}{17}\right) = \frac{1}{\frac{1}{17}} - 1 = 17 - 1 = \boxed{16}$$

10. (a) Convert $\frac{5\pi}{12}$ radians to degrees. $\frac{5\pi}{12} \cdot \frac{180}{\pi} = \boxed{75^\circ}$

(b) Use the addition formula for cosine to find $\cos \frac{5\pi}{12}$.
Note that you may work in degrees, using your answer to part (a), if you prefer.

$$\cos(45^\circ + 30^\circ) = \cos(45^\circ)\cos(30^\circ) - \sin(45^\circ)\sin(30^\circ)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

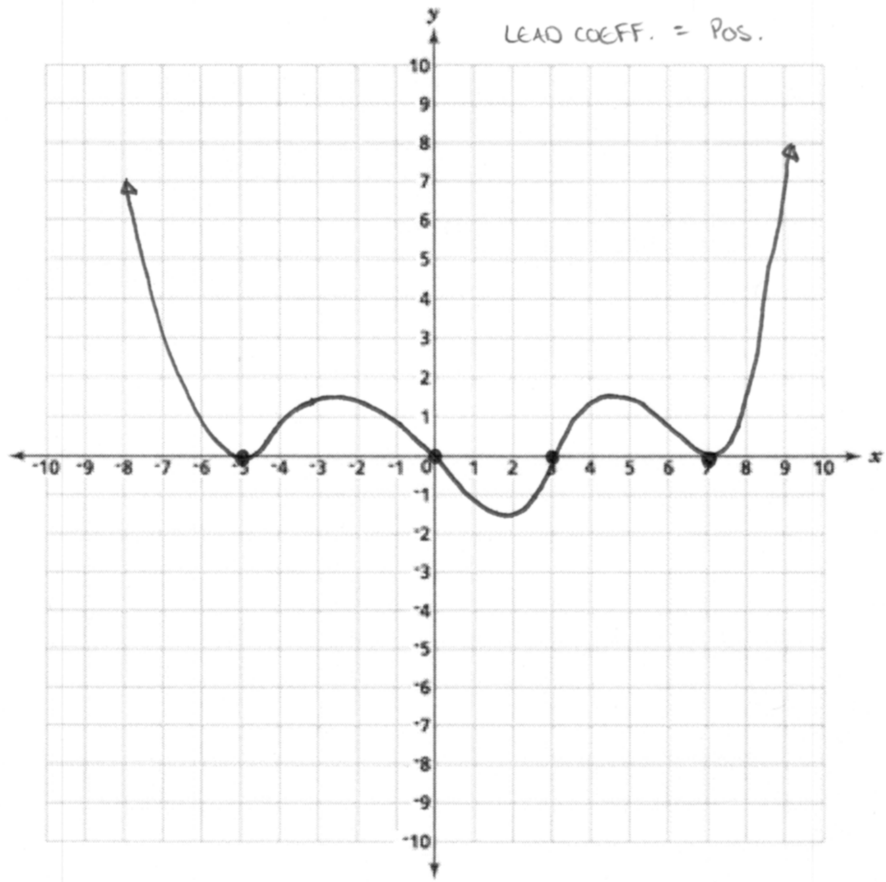
$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4} \quad \text{or} \quad \frac{\sqrt{3} - 1}{2\sqrt{2}}}$$

CROSS BOUNCE CROSS BOUNCE

11. In the xy-plane below, sketch the graph $y = x(x+5)^4(x-3)^3(x-7)^2$. Be sure to clearly show the *end behavior* of the graph, as well as where the graph *crosses* the x-axis and where the graph *bounces off* the x-axis.

DEGREE = 1 + 4 + 3 + 2 = 10

LEAD COEFF. = POS.



12. Suppose the population of a particular colony of ants is found to be 650 on January 1, 2010, and 900 on January 1, 2011. Assume the population of the colony grows exponentially.

(a) Find a function $P(t)$ that gives the population of the ant colony t years after January 1, 2010.

$$P(t) = P_0 e^{rt} = 650 e^{rt}$$

$$P(1) = 650 e^{r} = 900$$

$$e^{r} = \frac{900}{650} = \frac{18}{13}$$

$$r = \ln\left(\frac{18}{13}\right)$$

$$P(t) = 650\, e^{\ln\left(\frac{18}{13}\right) t}$$

$$\boxed{P(t) = 650\left(\frac{18}{13}\right)^{t} \quad \text{or} \quad P(t) = 650\left(\frac{900}{650}\right)^{t}}$$

(b) How many years after January 1, 2010 will the population of the ant colony reach 3000?

$$P(t) = 650\left(\frac{18}{13}\right)^{t} = 3000$$

$$\left(\frac{18}{13}\right)^{t} = \frac{3000}{650} = \frac{60}{13}$$

$$t \ln\left(\frac{18}{13}\right) = \ln\left(\frac{60}{13}\right)$$

$$\boxed{t = \frac{\ln\left(\frac{60}{13}\right)}{\ln\left(\frac{18}{13}\right)} \quad \text{or} \quad \log_{\frac{18}{13}}\left(\frac{60}{13}\right)}$$

Part III

13. Solve the following logarithmic equation.

$$\log_5(x+1) - \log_5(x-1) = 2$$

$$\log_5\left(\frac{x+1}{x-1}\right) = 2$$

$$\frac{x+1}{x-1} = 5^2 = 25$$

$$x+1 = 25(x-1)$$

$$x+1 = 25x - 25$$

$$26 = 24x$$

$$\boxed{x = \frac{13}{12}}$$

14. Find *all* solutions to the following trigonometric equation.

$$4\sin^2\theta - 3 = 0$$

$$4\sin^2\theta = 3$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\sqrt{\frac{3}{4}}$$

$$\sin\theta = \frac{\sqrt{3}}{2} \qquad \sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3} + n2\pi \qquad \theta = \frac{4\pi}{3} + n2\pi$$

$$\frac{2\pi}{3} + n2\pi \qquad \frac{5\pi}{3} + n2\pi$$

WHERE $n = 0, \pm 1, \pm 2, \ldots$

15. Add the following rational expressions and simplify the sum.

$$\frac{3}{x^2+3x}+\frac{1}{x^2+7x+12}-\frac{3}{x^2+4x}$$

$$x(x+3) \quad (x+3)(x+4) \quad x(x+4)$$

$$LCD = x(x+3)(x+4)$$

$$\frac{3(x+4) + x - 3(x+3)}{x(x+3)(x+4)} = \frac{\cancel{3x}+12+x-\cancel{3x}-9}{x(x+3)(x+4)}$$

$$= \frac{\cancel{(x+3)}}{x\cancel{(x+3)}(x+4)} = \boxed{\frac{1}{x(x+4)}}$$

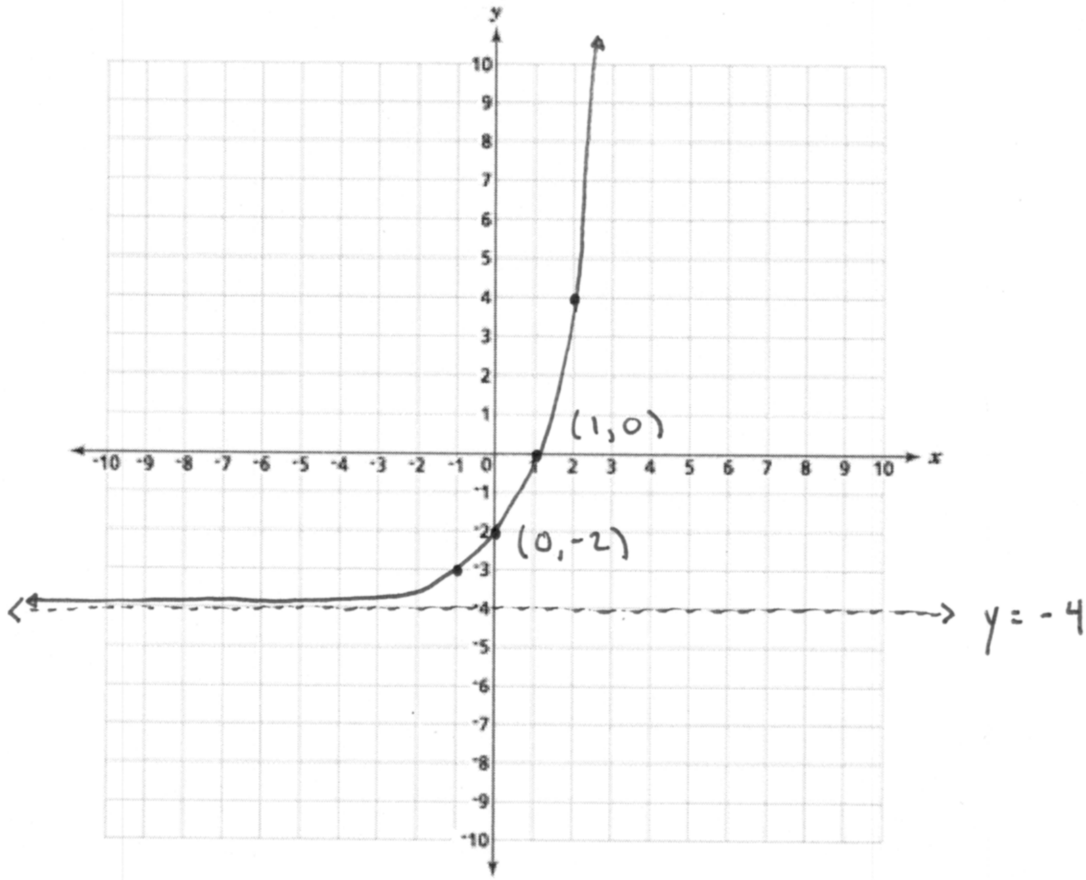
16. Rewrite the following expression as a simplified sum.

$$(x-5)^2-(x+3)(2x-1)+2x^2$$

$$x^2 - 10x + 25 - (\cancel{2x^2} + 5x - 3) + \cancel{2x^2}$$

$$\boxed{x^2 - 15x + 28}$$

17. In the xy-plane below, sketch the graph $y = 2^{x+1} - 4$ and label all intercepts. Also sketch and give an equation for any asymptotes.



18. Simplify the following rational expression.

$$\frac{x^3 - x^2 - 4x + 4}{x^2 - 3x + 2}$$

FACTOR NUMERATOR WITH "FACTOR BY GROUPING":

$$[x^3 - x^2] + [-4x + 4]$$

$$x^2(x-1) - 4(x-1) = (x^2-4)(x-1)$$

$$= (x+2)(x-2)(x-1)$$

DENOMINATOR: $x^2 - 3x + 2 = (x-1)(x-2)$

$$\therefore \frac{(x+2)\cancel{(x-2)}\cancel{(x-1)}}{\cancel{(x-1)}\cancel{(x-2)}} = \boxed{x+2}$$

Part IV

19. (a) Find $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

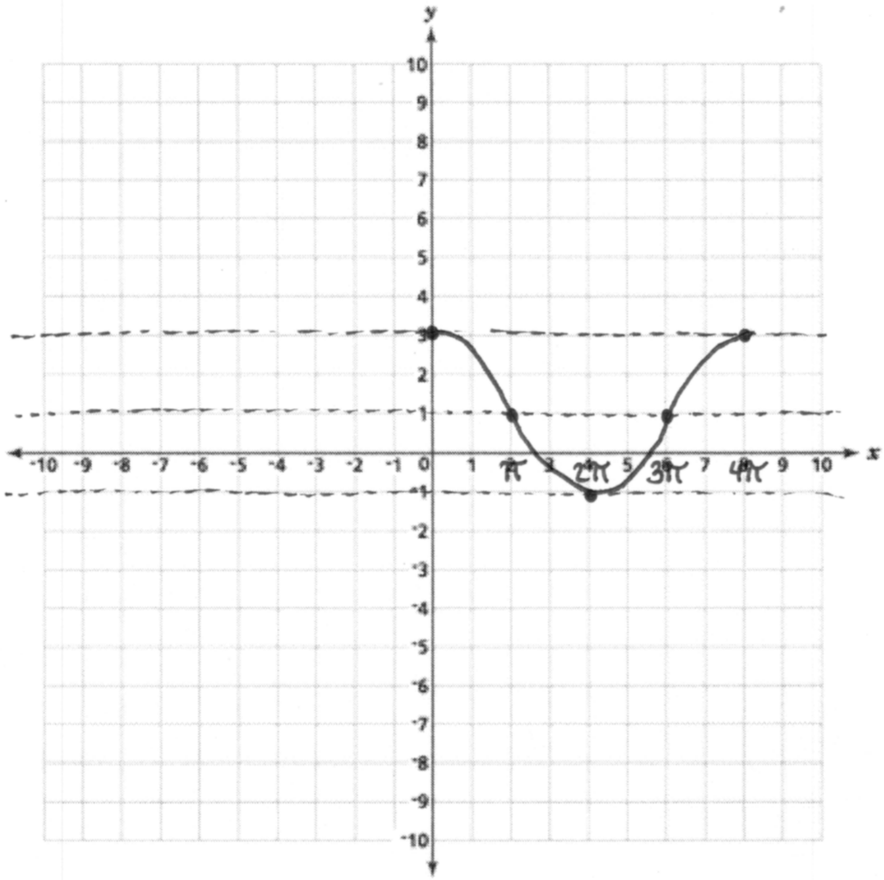
$\boxed{-\frac{\pi}{3}}$

(b) Verify the trigonometric identity $\underbrace{(1 - \cos^2\theta)}_{\sin^2\theta}(1 + \cot^2\theta) = 1$.

$$\boxed{\sin^2\theta\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right) = \sin^2\theta + \cos^2\theta = 1}$$

✓

20. In the xy-plane below, sketch one period of the graph $y = 2\cos\left(\dfrac{x}{2}\right) + 1$. Include the coordinates of all intercepts, local maximums, and local minimums.



AMPLITUDE = 2

PERIOD = $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$

21. Use interval notation to describe the solution set of the following inequality.

$$\left|\frac{1}{4}x - 2\right| \geq 3$$

$\frac{1}{4}x - 2 \geq 3$ or $\frac{1}{4}x - 2 \leq -3$

$\frac{1}{4}x \geq 5$ $\frac{1}{4}x \leq -1$

$x \geq 20$ $x \leq -4$

$$\boxed{(-\infty, -4] \cup [20, \infty)}$$

22. Solve the following exponential equation.

$$1 + e^{4x+1} = 20$$

$$e^{4x+1} = 19$$

$$4x + 1 = \ln(19)$$

$$4x = \ln(19) - 1$$

$$\boxed{x = \frac{\ln(19) - 1}{4}}$$

23. Use interval notation to describe the domain of the domain of the following function.

$$f(x) = \frac{x^2 - 25}{\sqrt{x^2 - 4}}$$

$x^2 - 4 > 0$

$x^2 > 4$

$|x| > 2$

$\boxed{(-\infty, -2) \cup (2, \infty)}$

24. Find the center and radius of the circle described by the following equation.

$$x^2 + y^2 + 4x + 12 = 6y$$

$x^2 + 4x + 4 + y^2 - 6y \boxed{+9} = -12 + 4 \boxed{+9}$

$(x+2)^2 + (y-3)^2 = 1$

CENTER: $(-2, 3)$

RADIUS: 1