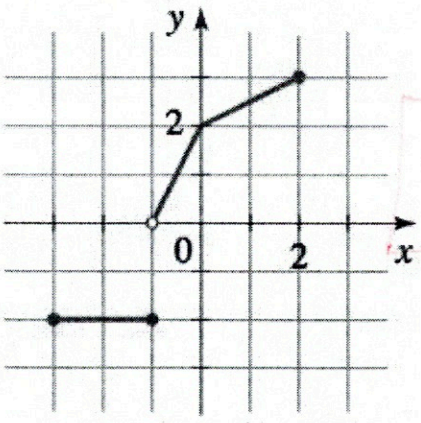
Name: * ANSWER KEY * 10/28/2015

Math 195 Precalculus Quiz 2

Please show all work and **box your final answers**. Calculators are not allowed and cellphones should be put away. Good luck!

1. Determine if the following graph is the graph of a function. If it is not, explain why. If it is, explain why and find its domain and range.



IT IS A FUNCTION BECAUSE IT PASSES THE VERTICAL LINE TEST

DOMAIN : $[-3, 2]$

RANGE : $\{-2\} \cup (0, 3]$

2. (8 points) Given $f(x) = x^2 - 3x + 2$, rewrite $\dfrac{f(x+h) - f(x)}{h}$ as a simplified polynomial.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 2 - x^2 + 3x - 2}{h}$$

CANCEL

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

$$= \frac{2xh + h^2 - 3h}{h} = \frac{(2x + h - 3)h}{h}$$

$$= \boxed{2x + h - 3}$$

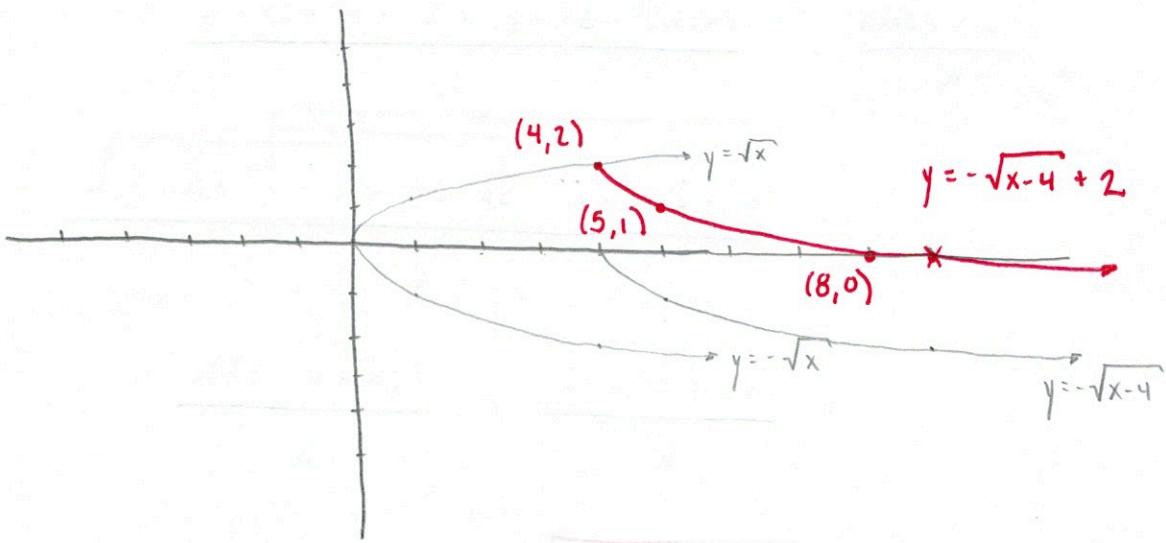
3. Find the average rate of change of the function $f(x) = \dfrac{2}{x+1}$ over the interval $[0, h]$.

$$\frac{f(h)-f(0)}{h-0} = \frac{1}{h}\left(\frac{2}{h+1} - \frac{2}{0+1}\right)$$

$$= \frac{1}{h}\left(\frac{2-2(h+1)}{h+1}\right)$$

$$= \frac{1}{h}\cdot\frac{-2h}{h+1} = \boxed{\frac{-2}{h+1}}$$

4. (8 points) Sketch the graph of $y = -\sqrt{x-4}+2$.



5. (8 points) Consider the following functions.

$$f(x) = \frac{1}{x-a} + a, \qquad g(x) = x^2 - 2x, \qquad h(x) = 4x + 1$$

(a) Find $(f \circ f)(x)$, i.e. $f(f(x))$.

$$f(f(x)) = \frac{1}{f(x) - a} + a = \frac{1}{\frac{1}{x-a} + a - a} + a$$

$$= \frac{1}{\frac{1}{x-a}} + a = x - a + a = \boxed{x}$$

(b) Find $(g \circ h)(2)$, i.e. $g(h(2))$.

$$h(2) = 4(2) + 1 = 9$$

So $g(h(2)) = g(9) = (9)^2 - 2(9) = 81 - 18 = \boxed{63}$

6. (8 points) Let f be the one-to-one function defined by $f(x) = \dfrac{3x+2}{2x-1}$.

(a) Find $f^{-1}(x)$.

$y = \dfrac{3x+2}{2x-1}$. SOLVE FOR x.

$2xy - y = 3x + 2$ (MULTIPLY BOTH SIDES BY $2x-1$)

$2xy - 3x = y + 2$

$(2y-3)x = y + 2$

$x = \dfrac{y+2}{2y-3}$ SWITCH x & y $\longrightarrow$ $\boxed{f^{-1}(x) = \dfrac{x+2}{2x-3}}$

(b) Find the range of f. (Hint: The range of f is equal to the domain of f^{-1}.)

DOMAIN OF f^{-1} IS ALL REAL # EXCEPT WHEN DENOMINATOR = 0.

THAT IS : $\mathbb{R}$ TAKE AWAY $2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

i.e. $\boxed{(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)}$

7. **This is a bonus question. You will be rewarded for a correct answer, and not penalized for an incorrect answer.**

(a) (4 points) Suppose you start with the graph of the equation $y = \sqrt{x^3+1}$ and then apply the following transformations in the following order.

1. Move the graph 3 units to the left.
2. Reflect the graph through the y-axis.
3. Move the graph 2 units down.

What is the equation of the resulting graph? **Do not sketch the graph.**

1. $y = \sqrt{(x+3)^3+1}$
2. $y = \sqrt{(-x+3)^3+1}$
3. $y = \sqrt{(-x+3)^3+1} - 2$

(b) (4 points) Now suppose you start with the same graph of the same equation $y = \sqrt{x^3+1}$ and you apply the same transformations but in a different order.

1. Reflect the graph through the y-axis.
2. Move the graph 3 units to the left.
3. Move the graph 2 units down.

Now what is the equation of the resulting graph? **Do not sketch the graph.**

1. $y = \sqrt{-x^3+1}$ OR $y = \sqrt{(-x)^3+1}$
2. $y = \sqrt{-(x+3)^3+1}$ OR $y = \sqrt{(-(x+3))^3+1}$
3. $y = \sqrt{-(x+3)^3+1} - 2$

⋮

- NOTE THAT THE EQUATIONS IN PART (a) & (b) ARE DIFFERENT!