

Please show all work and **box your final answers**. Calculators are not allowed and cellphones should be put away. Good luck!

1. Consider the quadratic function  $q(x) = 9x^2 - 45x + 13$ .

(a) (4 points) Rewrite  $q(x)$  in standard form, that is  $q(x) = a(x - h)^2 + k$ .

$$\begin{aligned} q(x) &= 9(x^2 - 5x) + 13 \\ &= 9\left(x^2 - 5x + \frac{25}{4}\right) + 13 - \underbrace{(9)\left(\frac{25}{4}\right)}_{-56\frac{1}{4}} \\ &= \boxed{9\left(x - \frac{5}{2}\right)^2 - \frac{173}{4}} \end{aligned}$$

$-43\frac{1}{4}$

NOTE:  $13 - (9)\left(\frac{25}{4}\right)$

$$\begin{aligned} &= \frac{52}{4} - \frac{225}{4} \\ &= \frac{-173}{4} = -43\frac{1}{4} \end{aligned}$$

(b) (4 points) Find the maximum/minimum value of  $q$  and state whether it is a maximum or minimum value. Then give the range of  $q$ .

MINIMUM VALUE :  $-\frac{173}{4} = -43\frac{1}{4}$

RANGE :  $\left[-\frac{173}{4}, \infty\right)$

(c) (4 points) Does the graph of  $y = q(x)$  intersect the  $x$ -axis? If so, give the  $x$ -intercepts in simplified form.

$$9\left(x - \frac{5}{2}\right)^2 - \frac{173}{4} = 0$$

$$9\left(x - \frac{5}{2}\right)^2 = \frac{173}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{173}{36}$$

$$x - \frac{5}{2} = \pm \frac{\sqrt{173}}{6}$$

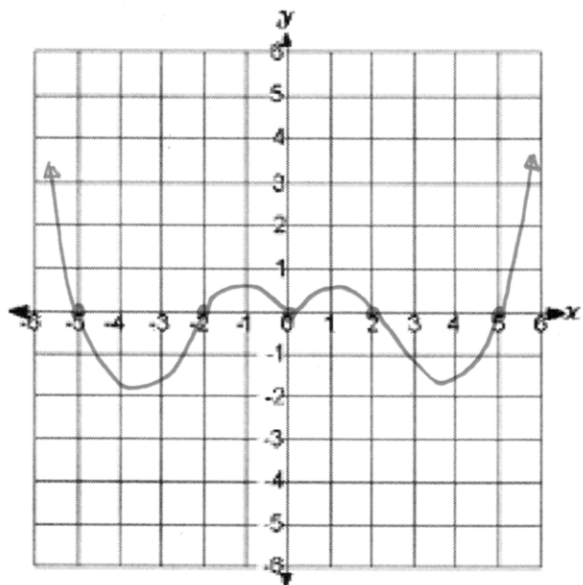
$$x = \frac{5}{2} \pm \frac{\sqrt{173}}{6}$$

OR  $\frac{15 \pm \sqrt{173}}{6}$

OR  $\frac{45 \pm \sqrt{2025 - 468}}{18}$

$$= \frac{45 \pm \sqrt{1557}}{18}$$

2. (8 points) In the coordinate plane below, sketch the graph of  $y = x^8 - 29x^6 + 100x^4$ . Do not worry about the scale of the y-axis.



① FIND ZEROS:

$$x^8 - 29x^6 + 100x^4 = 0$$

$$x^4(x^4 - 29x^2 + 100) = 0$$

$$x^4(x^2 - 25)(x^2 - 4) = 0$$

$$x^4(x+5)(x-5)(x+2)(x-2) = 0$$

$$x = 0, -5, 5, -2, 2$$

$\underbrace{\hspace{10em}}_{\text{MULTIPLICITY 1}}$   
 MULT. 4

② END BEHAVIOR: EVEN DEGREE, LEAD COEFFICIENT POSITIVE

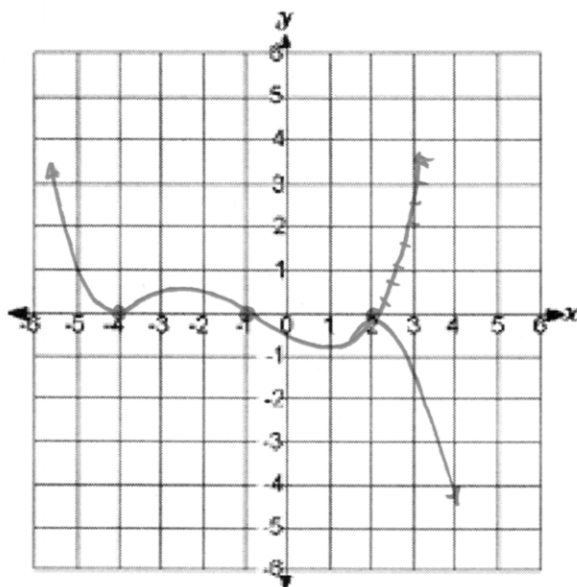


③ GRAPH TOUCHES X-AXIS AT  $-5, -2, 0, 2, 5$ .

IT CROSSES THE X-AXIS AT  $-5, -2, 2, 5$  (ODD MULTIPLICITIES)

AND BOUNCES OFF THE X-AXIS AT  $0$ . (EVEN MULTIPLICITY)

3. (8 points) In the coordinate plane below, sketch the graph of  $y = -(x+4)^8(x+1)^5(x-2)^6$ . Do not worry about the scale of the  $y$ -axis.



① ZEROS:  $x = -4, -1, 2$

② END BEHAVIOR:  $8 + 5 + 6 = 19^{\text{th}}$  DEGREE POLYNOMIAL

ODD DEGREE

LEAD COEFFICIENT  $-1$  (NEGATIVE)



③  $x = -4$  IS ZERO WITH MULTIPLICITY 8 (EVEN)  $\Rightarrow$  BOUNCE

$x = -1$  IS ZERO WITH MULTIPLICITY 5 (ODD)  $\Rightarrow$  CROSS

$x = 2$  IS ZERO WITH MULTIPLICITY 6 (EVEN)  $\Rightarrow$  BOUNCE

4. In the coordinate plane below, sketch and label the graphs of the following equations. If a graph has an asymptote, sketch and label that as well.

(a) (4 points)  $y = 2^x$

(b) (4 points)  $y = \left(\frac{1}{2}\right)^x$

(c) (4 points)  $y = 2^{x+4} - 2$

