

Please show all work and **box your final answers**. Calculators are not allowed and cellphones should be put away. Good luck!

1. (8 points) Evaluate the following expressions.

(a) $\log_9(3)$ $\frac{1}{2}$

(b) $\log_{10}(0.01)$ -2

(c) $\ln(1)$ 0

(d) $\ln(e)$ 1

2. (8 points) Solve the logarithmic equation for x.

$$\log_{20}(2x-1) + \log_{20}(x+1) = 1$$

$$\log_{20}((2x-1)(x+1)) = 1$$

$$20 = (2x-1)(x+1) = 2x^2 + x - 1$$

$$0 = 2x^2 + x - 21$$

$$0 = (2x+7)(x-3)$$

$$2x+7=0$$

$$x-3=0$$

$$2x = -7$$

$x = 3$

$$x = -\frac{7}{2}$$

↑

NOTE THAT WHEN $x = -\frac{7}{2}$, $2x-1 = 2(-\frac{7}{2})-1 = -8$

SO $\log(2x-1)$ IS UNDEFINED WHEN $x = -\frac{7}{2}$

∴ REJECT THIS ANSWER

3. (8 points) Use the laws of logarithms to expand the logarithmic expression.

$$\ln\left(\frac{(xy)^{1/3}}{2(x^2+y^2)}\right)$$

$$= \ln\left((xy)^{1/3}\right) - \ln\left(2(x^2+y^2)\right)$$

$$= \frac{1}{3} \left[\ln(x) + \ln(y) \right] - \left[\ln(2) + \ln(x^2+y^2) \right]$$

$$= \boxed{\frac{1}{3} \ln x + \frac{1}{3} \ln y - \ln(2) - \ln(x^2+y^2)}$$

4. (8 points) A culture starts with 8,600 bacteria. After 1 hour the count is 10,000.

(a) Find a function $P(t)$ that models the number of bacteria after t hours.

$$P(t) = 8600 \left(\frac{10000}{8600}\right)^t$$

$$\boxed{P(t) = 8600 \left(\frac{50}{43}\right)^t}$$

$$\text{or } P(t) = 8600 e^{\ln\left(\frac{50}{43}\right)t}$$

- (b) After how many hours will the number of bacteria double?

$$P(t) = 8600 \left(\frac{50}{43}\right)^t = 2 \cdot 8600 \quad (\text{DOUBLE!})$$

$$\left(\frac{50}{43}\right)^t = 2$$

$$t \ln\left(\frac{50}{43}\right) = \ln(2)$$

$$\boxed{t = \frac{\ln(2)}{\ln(50) - \ln(43)}}$$

5. (8 points) In the xy -plane below, sketch the graph $y = 1 + \log_2(x + 4)$. Include any horizontal/vertical asymptotes.

