§1.7 Inequalities

Linear (variable raised only to power 1, never in denominator)

e.g. Solve: $4x + 6 \leq 3x + 9$

* Is 0 a solution? 1? 10?

$x \leq 3$ or $-3 \leq -x$

same ✓ $3 \geq x$

Same rules as equations but with 2 twists!

* If $A < B$ & C is negative then $CA > CB$
* If $0 < A < B$ (both positive) then $\frac{1}{A} > \frac{1}{B}$
* ... and one new rule

  If $A < B$ and $C < D$

  then $A + C < B + D$

ex. $\frac{2}{3} - \frac{1}{2}x \geq \frac{1}{6} + x$

ex. Simultaneous ineq.:

$-3 \leq 3x + 7 \leq \frac{1}{2}$

ex. $-\frac{1}{2} \leq \frac{4 - 3x}{5} \leq \frac{1}{4}$

Non Linear

Positive or Negative? (compare to 0)

$\frac{(0)(13)(-7)}{(-11)(102)(-29)}$ 🤔

Observe: If the # of negative factors in a product/quotient is

- Even then the product/quotient is positive
- Odd then the product/quotient is negative

ex. $x^2 + 40 < 13x$

ex. $x^2 \geq 9$

ex. $5x^2 + 3x \geq 3x^2 + 2$

ex. $16x \leq x^3$

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ex. $x^3(x+3)^2(x+1) > 0$

ex. $x^2(x^2-1) \geq 0$

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ex. $\dfrac{4-x}{x+4} < 0$

ex. $\dfrac{x}{2} \geq \dfrac{5}{x+1} + 4$

ex. $-2 < \dfrac{x+1}{x-3}$

ex. $\dfrac{x+2}{x+3} < \dfrac{x-1}{x-2}$

WHY CAN'T WE CROSS-MULTIPLY?

HOW WOULD WE SOLVE THESE IF THEY WERE EQUATIONS?

WHY CAN'T WE JUST GET RID OF DENOMINATORS BY MULTIPLYING EVERYTHING BY LCD?

🤔

GUIDELINES FOR SOLVING NONLINEAR INEQUALITIES

1. **Move All Terms to One Side.** If necessary, rewrite the inequality so that all nonzero terms appear on one side of the inequality sign. If the nonzero side of the inequality involves quotients, bring them to a common denominator.
2. **Factor.** Factor the nonzero side of the inequality.
3. **Find the Intervals.** Determine the values for which each factor is zero. These numbers will divide the real line into intervals. List the intervals that are determined by these numbers.
4. **Make a Table or Diagram.** Use **test values** to make a table or diagram of the signs of each factor on each interval. In the last row of the table determine the sign of the product (or quotient) of these factors.
5. **Solve.** Use the sign table to find the intervals on which the inequality is satisfied. Check whether the **endpoints** of these intervals satisfy the inequality. (This may happen if the inequality involves $\leq$ or $\geq$.)