§3.1 Quadratics

Recall Def. of Polynomial:

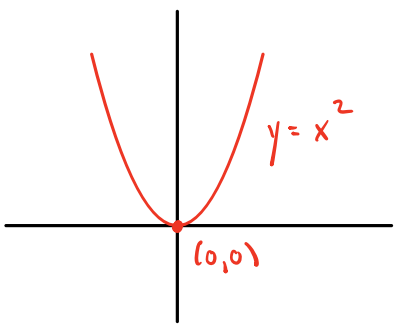
$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

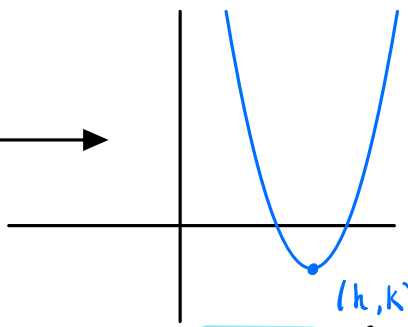
- $a_n x^n$: lead term (variable raised to highest exponent; also called the degree of the polynomial)
- a_n: lead coefficient (cannot be 0)
- $a_n \neq 0$
- $a_{n-1}, a_{n-2}, \dots, a_1$: other coefficients (any of these may be 0)
- a_0: constant term (may be 0)

Quadratic functions are degree 2 polynomial functions

$$a_2 x^2 + a_1 x + a_0 \xrightarrow{\text{by convention, we rename the coefficients}} f(x) = ax^2 + bx + c$$

Standard form of quadratic functions: $f(x) = a(x-h)^2 + k$





Vertex of parabola

- Horizontal shift
- Vertical shift
- Vertical stretch/shrink

$f(x) = ax^2 + bx + c$ — convert by completing the square → $f(x) = a(x-h)^2 + k$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - \frac{b^2}{4a}$$

Adding $a\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a}$ — subtracting same thing!

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = a\left(x - \left(\frac{-b}{2a}\right)\right) + \left(c - \frac{b^2}{4a}\right)$$

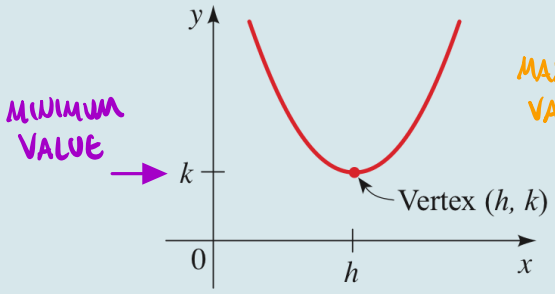
(h = $\frac{-b}{2a}$, k = $c - \frac{b^2}{4a}$)

STANDARD FORM OF A QUADRATIC FUNCTION

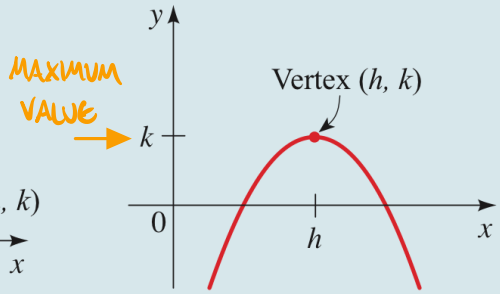
A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k); the parabola opens upward if $a > 0$ or downward if $a < 0$.



$f(x) = a(x - h)^2 + k,\ a > 0$



$f(x) = a(x - h)^2 + k,\ a < 0$

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ex. Let $f(x) = x^2 + 8x + 13$ ($a = 1$: EASIER)

(a) EXPRESS f IN STANDARD FORM ⟶ $(x+4)^2 - 3$
(b) FIND VERTEX OF GRAPH $y = f(x)$
(c) FIND x- & y-INTERCEPTS OF $y = f(x)$
(d) SKETCH $y = f(x)$
(e) MINIMUM / MAXIMUM VALUE ?

ex. Let $f(x) = -3x^2 - 30x - 73$ ($a \neq 1$: TRICKY)

(a) EXPRESS f IN STANDARD FORM ⟶ $-3(x-5)^2 + 2$
(b) FIND VERTEX OF GRAPH $y = f(x)$
(c) FIND x- & y-INTERCEPTS OF $y = f(x)$
(d) SKETCH $y = f(x)$
(e) MINIMUM / MAXIMUM VALUE ?

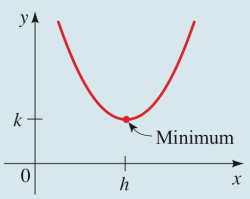
SHORTCUTS TO MAX/MIN VALUES:

MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

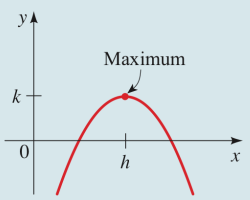
Let f be a quadratic function with standard form $f(x) = a(x - h)^2 + k$. The maximum or minimum value of f occurs at $x = h$.

If $a > 0$, then the **minimum value** of f is $f(h) = k$.

If $a < 0$, then the **maximum value** of f is $f(h) = k$.



$f(x) = a(x - h)^2 + k,\ a > 0$



$f(x) = a(x - h)^2 + k,\ a < 0$

MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a}$$ ← INPUT THAT PRODUCES MAX/MIN OUTPUT

If $a > 0$, then the **minimum value** is $f\left(-\frac{b}{2a}\right)$.

If $a < 0$, then the **maximum value** is $f\left(-\frac{b}{2a}\right)$.

Additional Practice:

25–34 ■ Maximum and Minimum Values A quadratic function f is given. **(a)** Express f in standard form. **(b)** Sketch a graph of f. **(c)** Find the maximum or minimum value of f.

25. $f(x) = x^2 + 2x - 1$

26. $f(x) = x^2 - 8x + 8$

27. $f(x) = 3x^2 - 6x + 1$

28. $f(x) = 5x^2 + 30x + 4$

29. $f(x) = -x^2 - 3x + 3$

30. $f(x) = 1 - 6x - x^2$

31. $g(x) = 3x^2 - 12x + 13$

32. $g(x) = 2x^2 + 8x + 11$

33. $h(x) = 1 - x - x^2$

34. $h(x) = 3 - 4x - 4x^2$

35–44 ■ Formula for Maximum and Minimum Values Find the maximum or minimum value of the function.

35. $f(x) = 2x^2 + 4x - 1$

36. $f(x) = 3 - 4x - x^2$

37. $f(t) = -3 + 80t - 20t^2$

38. $f(x) = 6x^2 - 24x - 100$

39. $f(s) = s^2 - 1.2s + 16$

40. $g(x) = 100x^2 - 1500x$

41. $h(x) = \frac{1}{2}x^2 + 2x - 6$

42. $f(x) = -\frac{x^2}{3} + 2x + 7$

43. $f(x) = 3 - x - \frac{1}{2}x^2$

44. $g(x) = 2x(x - 4) + 7$