§4.3 Logarithmic Functions

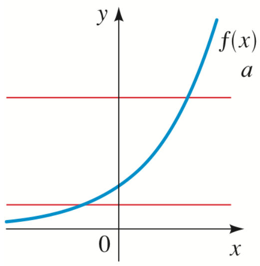


FIGURE 1 $f(x) = a^x$ is one-to-one.

$f(x) = a^x = \text{Exp}_a\, x$ = Exponential function base a

Has inverse $f^{-1}(x) = \log_a x$ = Logarithmic function base a

Def: $\log_a x = y \iff a^y = x$

$\underbrace{}$ The power we raise a to to make x.

ex. Use the definition to evaluate

(a) $\log_3 9 \longrightarrow$ write $\log_3 9 = p \iff 3^p = 9$

(b) $\log_{10} 10{,}000$

(c) $\log_2\left(\frac{1}{8}\right)$

(d) $\log_{13}\left(13^5\right)$

PROPERTIES OF LOGARITHMS

Property	Reason
1. $\log_a 1 = 0$	We must raise a to the power 0 to get 1.
2. $\log_a a = 1$	We must raise a to the power 1 to get a.
3. $\log_a a^x = x$	We must raise a to the power x to get a^x.
4. $a^{\log_a x} = x$	$\log_a x$ is the power to which a must be raised to get x.

Cancellation Properties of Inverse Functions

$f(x) = a^x$, $f^{-1}(x) = \log_a x$

3. $f^{-1}(f(x)) = x$

4. $f(f^{-1}(x)) = x$

Know these graphs!!

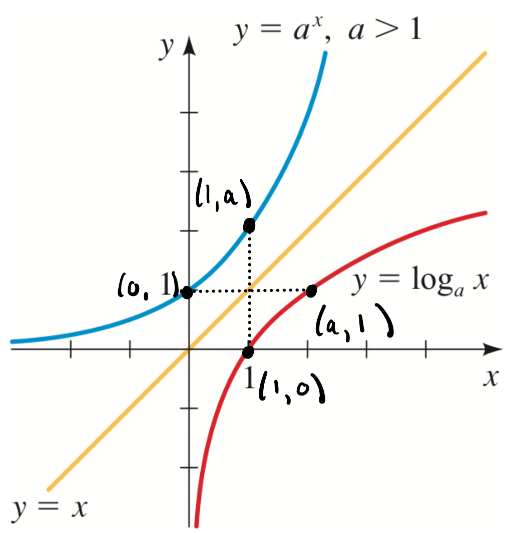
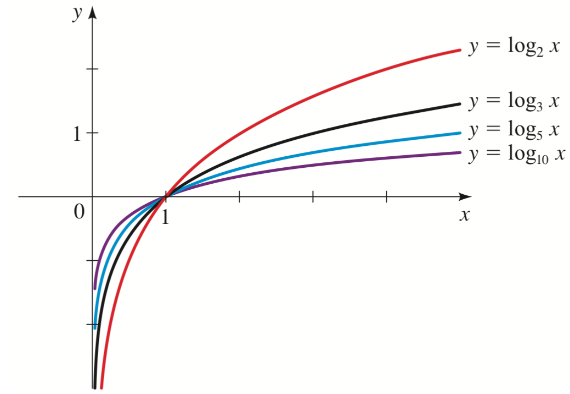


FIGURE 2 Graph of the logarithmic function $f(x) = \log_a x$



EXAMPLE 5 ■ Reflecting Graphs of Logarithmic Functions

Sketch the graph of each function. State the domain, range, and asymptote.

(a) $g(x) = -\log_2 x$ **(b)** $h(x) = \log_2(-x)$

EXAMPLE 6 ■ Shifting Graphs of Logarithmic Functions

Sketch the graph of each function. State the domain, range, and asymptote.

(a) $g(x) = 2 + \log_5 x$ **(b)** $h(x) = \log_{10}(x - 3)$

THE NATURAL LOGARITHM

DEF: $\boxed{\ln x = \log_e x, \quad e = 2.718281828\ldots}$

THE INVERSE OF THE NATURAL EXPONENTIAL e^x

ex. EVALUATE

(a) $\ln(e^x)$ (d) $e^{\ln x}$

(b) $\ln(e^3)$ (e) $e^{\ln 5}$

(c) $\ln\left(\frac{1}{\sqrt{e}}\right)$

PROPERTIES OF NATURAL LOGARITHMS

Property	Reason
1. $\ln 1 = 0$	We must raise e to the power 0 to get 1.
2. $\ln e = 1$	We must raise e to the power 1 to get e.
3. $\ln e^x = x$	We must raise e to the power x to get e^x.
4. $e^{\ln x} = x$	$\ln x$ is the power to which e must be raised to get x.

EXAMPLE 10 ■ Finding the Domain of a Logarithmic Function

Find the domain of the function $f(x) = \ln(4 - x^2)$.

FIND THE DOMAIN:

77. $h(x) = \ln x + \ln(2 - x)$

78. $h(x) = \sqrt{x - 2} - \log_5(10 - x)$