# §4.4 Laws of Logarithms

Anything said using exponents can also be said using logarithms

| Exponents | Logarithms |
|---|---|
| $a^x = y$ | $\log_a y = x$ |
| $2^4 = 16$ | $\log_2 16 = 4$ |
| $a^{x+y} = a^x a^y$ | $\log_a(a^x a^y) = x + y = \log_a(a^x) + \log_a(a^y)$ |
| i.e. $\log_a(AB) = \log_a A + \log_a B$ | |
| $a^{x-y} = \frac{a^x}{a^y}$ | $\log_a\left(\frac{a^x}{a^y}\right) = x - y = \log_a(a^x) - \log_a(a^y)$ |
| i.e. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$ | |
| $a^{xy} = (a^x)^y$ | $\log_a((a^x)^y) = xy = y\log_a(a^x)$ |
| i.e. $\log_a(A^B) = B\log_a A$ | |

## LAWS OF LOGARITHMS

Let $a$ be a positive number, with $a \neq 1$. Let $A$, $B$, and $C$ be any real numbers with $A > 0$ and $B > 0$.

| Law | Description |
|---|---|
| 1. $\log_a(AB) = \log_a A + \log_a B$ | The logarithm of a product of numbers is the sum of the logarithms of the numbers. |
| 2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$ | The logarithm of a quotient of numbers is the difference of the logarithms of the numbers. |
| 3. $\log_a(A^C) = C\log_a A$ | The logarithm of a power of a number is the exponent times the logarithm of the number. |

## EXAMPLE 1 ■ Using the Laws of Logarithms to Evaluate Expressions

Evaluate each expression.

**(a)** $\log_4 2 + \log_4 32$

**(b)** $\log_2 80 - \log_2 5$

**(c)** $-\frac{1}{3}\log 8$

## EXAMPLE 2 ■ Expanding Logarithmic Expressions

Use the Laws of Logarithms to expand each expression.

**(a)** $\log_2(6x)$ **(b)** $\log_5(x^3y^6)$ **(c)** $\ln\left(\dfrac{ab}{\sqrt[3]{c}}\right)$

## EXAMPLE 3 ■ Combining Logarithmic Expressions

Use the Laws of Logarithms to combine each expression into a single logarithm.

**(a)** $3\log x + \frac{1}{2}\log(x+1)$

**(b)** $3\ln s + \frac{1}{2}\ln t - 4\ln(t^2+1)$

Note: $\log x = \log_{10} x$ (in our textbook)

WARNING!

$$\log_a(x+y) \neq \log_a x + \log_a y$$

$$\frac{\log 6}{\log 2} \neq \log\left(\frac{6}{2}\right) \quad \text{and} \quad (\log_2 x)^3 \neq 3\log_2 x$$