

## §5.2 TRIGONOMETRIC FUNCTIONS OF REAL NUMBERS

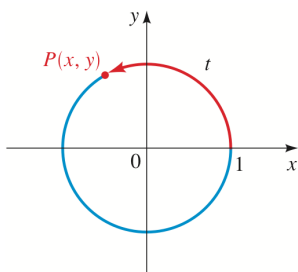


FIGURE 1

### DEFINITION OF THE TRIGONOMETRIC FUNCTIONS

Let  $t$  be any real number and let  $P(x, y)$  be the terminal point on the unit circle determined by  $t$ . We define

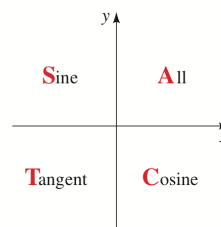
$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x} \quad (x \neq 0)$$

$$\csc t = \frac{1}{y} \quad (y \neq 0) \qquad \sec t = \frac{1}{x} \quad (x \neq 0) \qquad \cot t = \frac{x}{y} \quad (y \neq 0)$$

### DOMAINS OF THE TRIGONOMETRIC FUNCTIONS

Function	Domain
sin, cos	All real numbers
tan, sec	All real numbers other than $\frac{\pi}{2} + n\pi$ for any integer $n$
cot, csc	All real numbers other than $n\pi$ for any integer, $n$

The following mnemonic device will help you remember which trigonometric functions are positive in each quadrant: All of them, Sine, Tangent, or Cosine.



You can remember this as "All Students Take Calculus."

### SIGNS OF THE TRIGONOMETRIC FUNCTIONS

Quadrant	Positive Functions	Negative Functions
I	all	none
II	sin, csc	cos, sec, tan, cot
III	tan, cot	sin, csc, cos, sec
IV	cos, sec	sin, csc, tan, cot

### EVALUATING TRIGONOMETRIC FUNCTIONS FOR ANY REAL NUMBER

To find the values of the trigonometric functions for any real number  $t$ , we carry out the following steps.

- Find the reference number.** Find the reference number  $\bar{t}$  associated with  $t$ .
- Find the sign.** Determine the sign of the trigonometric function of  $t$  by noting the quadrant in which the terminal point lies.
- Find the value.** The value of the trigonometric function of  $t$  is the same, except possibly for sign, as the value of the trigonometric function of  $\bar{t}$ .

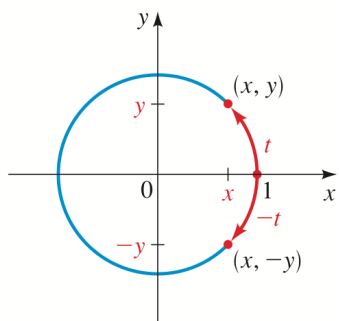
5–22 ■ Evaluating Trigonometric Functions Find the exact value of the trigonometric function at the given real number.

5. (a)  $\sin \frac{7\pi}{6}$       (b)  $\cos \frac{17\pi}{6}$       (c)  $\tan \frac{7\pi}{6}$
6. (a)  $\sin \frac{5\pi}{3}$       (b)  $\cos \frac{11\pi}{3}$       (c)  $\tan \frac{5\pi}{3}$
7. (a)  $\sin \frac{11\pi}{4}$       (b)  $\sin \left(-\frac{\pi}{4}\right)$       (c)  $\sin \frac{5\pi}{4}$
8. (a)  $\cos \frac{19\pi}{6}$       (b)  $\cos \left(-\frac{7\pi}{6}\right)$       (c)  $\cos \left(-\frac{\pi}{6}\right)$

17. (a)  $\csc \frac{7\pi}{6}$       (b)  $\sec \left(-\frac{\pi}{6}\right)$       (c)  $\cot \left(-\frac{5\pi}{6}\right)$
18. (a)  $\sec \frac{3\pi}{4}$       (b)  $\cos \left(-\frac{2\pi}{3}\right)$       (c)  $\tan \left(-\frac{7\pi}{6}\right)$
19. (a)  $\sin \frac{4\pi}{3}$       (b)  $\sec \frac{11\pi}{6}$       (c)  $\cot \left(-\frac{\pi}{3}\right)$

We can easily remember the sines and cosines of the basic angles by writing them in the form  $\sqrt{\square}/2$ :

$t$	$\sin t$	$\cos t$
0	$\sqrt{0}/2$	$\sqrt{4}/2$
$\pi/6$	$\sqrt{1}/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	$\sqrt{1}/2$
$\pi/2$	$\sqrt{4}/2$	$\sqrt{0}/2$



### EVEN-ODD PROPERTIES

Sine, cosecant, tangent, and cotangent are odd functions; cosine and secant are even functions.

$$\sin(-t) = -\sin t$$

$$\cos(-t) = \cos t$$

$$\tan(-t) = -\tan t$$

$$\csc(-t) = -\csc t$$

$$\sec(-t) = \sec t$$

$$\cot(-t) = -\cot t$$

### EXAMPLE 4 ■ Even and Odd Trigonometric Functions

Use the even-odd properties of the trigonometric functions to determine each value.

(a)  $\sin\left(-\frac{\pi}{6}\right)$       (b)  $\cos\left(-\frac{\pi}{4}\right)$

### FUNDAMENTAL IDENTITIES

#### Reciprocal Identities

$$\csc t = \frac{1}{\sin t} \quad \sec t = \frac{1}{\cos t} \quad \cot t = \frac{1}{\tan t} \quad \tan t = \frac{\sin t}{\cos t} \quad \cot t = \frac{\cos t}{\sin t}$$

#### Pythagorean Identities

$$\sin^2 t + \cos^2 t = 1 \quad \tan^2 t + 1 = \sec^2 t \quad 1 + \cot^2 t = \csc^2 t$$

### EXAMPLE 5 ■ Finding All Trigonometric Functions from the Value of One

If  $\cos t = \frac{3}{5}$  and  $t$  is in Quadrant IV, find the values of all the trigonometric functions at  $t$ .

### EXAMPLE 6 ■ Writing One Trigonometric Function in Terms of Another

Write  $\tan t$  in terms of  $\cos t$ , where  $t$  is in Quadrant III.