5.3 TRIGONOMETRIC GRAPHS

- Graphs of Sine and Cosine
- Graphs of Transformations of Sine and Cosine
- Using Graphing Devices to Graph Trigonometric Functions

t	$\sin t$	$\cos t$
$0 \to \frac{\pi}{2}$	$0 \to 1$	$1 \to 0$
$\frac{\pi}{2} \to \pi$	$1 \to 0$	$0 \to -1$
$\pi \to \frac{3\pi}{2}$	$0 \to -1$	$-1 \to 0$
$\frac{3\pi}{2} \to 2\pi$	$-1 \to 0$	$0 \to 1$

PERIODIC PROPERTIES OF SINE AND COSINE

The functions sine and cosine have period 2π:

$$\sin(t + 2\pi) = \sin t \qquad\qquad \cos(t + 2\pi) = \cos t$$

REAL # $t \longmapsto$ TERMINAL POINT $(\cos(t), \sin(t))$

https://www.geogebra.org/m/cNEtsbvC

https://bestanimations.com/gifs/Sin-Cos-PI-Shapes.html

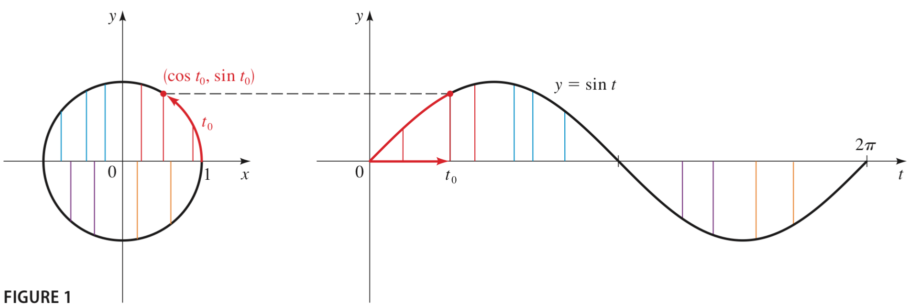

FIGURE 1

TABLE 2

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\sin t$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
$\cos t$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

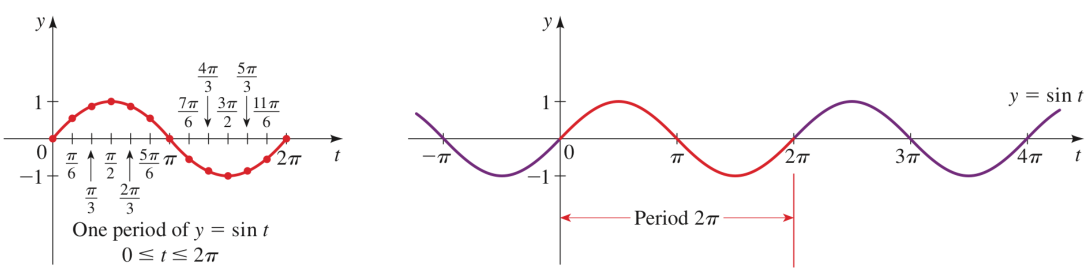

FIGURE 2 Graph of $\sin t$

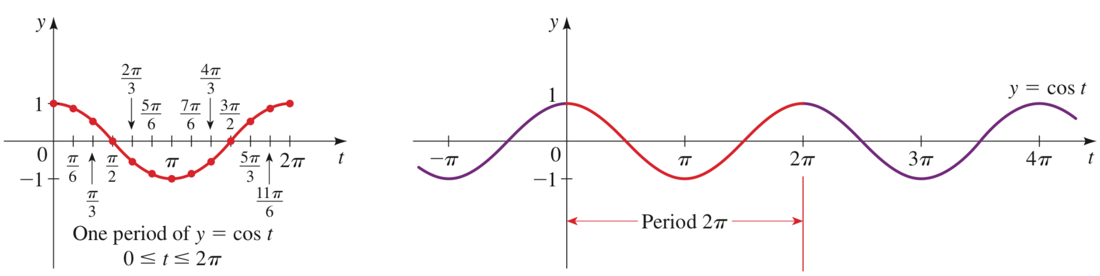

FIGURE 3 Graph of $\cos t$

SINE AND COSINE CURVES

The sine and cosine curves

$$y = a \sin kx \qquad \text{and} \qquad y = a \cos kx \qquad (k > 0)$$

have **amplitude** $|a|$ and **period** $2\pi/k$.

An appropriate interval on which to graph one complete period is $[0, 2\pi/k]$.

EXAMPLE 3 ■ Amplitude and Period

Find the amplitude and period of each function, and sketch its graph.

(a) $y = 4 \cos 3x$ **(b)** $y = -2 \sin \frac{1}{2}x$

25. $y = 10 \sin \frac{1}{2}x$

26. $y = 5 \cos \frac{1}{4}x$

27. $y = -\frac{1}{3} \cos \frac{1}{3}x$

28. $y = 4 \sin(-2x)$

29. $y = -2 \sin 2\pi x$

30. $y = -3 \sin \pi x$

SHIFTED SINE AND COSINE CURVES

The sine and cosine curves

$$y = a \sin k(x - b) \qquad \text{and} \qquad y = a \cos k(x - b) \qquad (k > 0)$$

have **amplitude** $|a|$, **period** $2\pi/k$, and **horizontal shift** b.

An appropriate interval on which to graph one complete period is $[b, b + (2\pi/k)]$.

EXAMPLE 4 ■ A Horizontally Shifted Sine Curve

Find the amplitude, period, and horizontal shift of $y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$, and graph one complete period.

EXAMPLE 5 ■ A Horizontally Shifted Cosine Curve

Find the amplitude, period, and horizontal shift of $y = \frac{3}{4} \cos\left(2x + \frac{2\pi}{3}\right)$, and graph one complete period.

33. $y = \cos\left(x - \frac{\pi}{2}\right)$

34. $y = 2 \sin\left(x - \frac{\pi}{3}\right)$

35. $y = -2 \sin\left(x - \frac{\pi}{6}\right)$

36. $y = 3 \cos\left(x + \frac{\pi}{4}\right)$

37. $y = -4 \sin 2\left(x + \frac{\pi}{2}\right)$

38. $y = \sin \frac{1}{2}\left(x + \frac{\pi}{4}\right)$

39. $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$

40. $y = 2 \sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$