

## 5.5 INVERSE TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

■ The Inverse Sine Function ■ The Inverse Cosine Function ■ The Inverse Tangent Function ■ The Inverse Secant, Cosecant, and Cotangent Functions

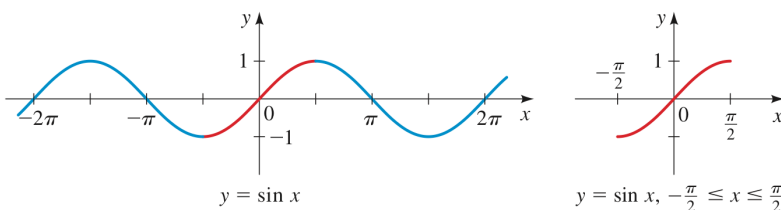


FIGURE 1 Graphs of the sine function and the restricted sine function

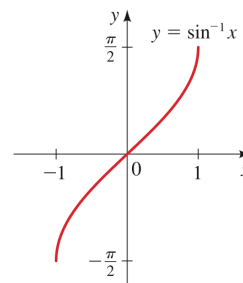


FIGURE 2 Graph of  $y = \sin^{-1} x$

### DEFINITION OF THE INVERSE SINE FUNCTION

The **inverse sine function** is the function  $\sin^{-1}$  with domain  $[-1, 1]$  and range  $[-\pi/2, \pi/2]$  defined by

$$\sin^{-1} x = y \iff \sin y = x$$

The inverse sine function is also called **arcsine**, denoted by **arcsin**.

### EXAMPLE 1 ■ Evaluating the Inverse Sine Function

Find each value.

(a)  $\sin^{-1} \frac{1}{2}$       (b)  $\sin^{-1} \left( -\frac{1}{2} \right)$       (c)  $\sin^{-1} \frac{3}{2}$

$$\sin(\sin^{-1} x) = x \quad \text{for} \quad -1 \leq x \leq 1$$

$$\sin^{-1}(\sin x) = x \quad \text{for} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

### EXAMPLE 3 ■ Evaluating Expressions with Inverse Sine

Find each value.

(a)  $\sin^{-1} \left( \sin \frac{\pi}{3} \right)$       (b)  $\sin^{-1} \left( \sin \frac{2\pi}{3} \right)$

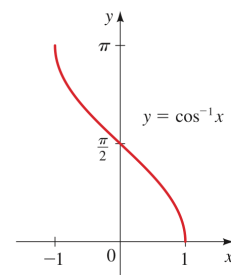
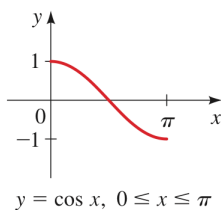
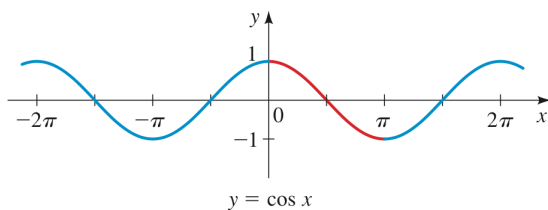


FIGURE 4 Graph of  $y = \cos^{-1}x$

### DEFINITION OF THE INVERSE COSINE FUNCTION

The **inverse cosine function** is the function  $\cos^{-1}$  with domain  $[-1, 1]$  and range  $[0, \pi]$  defined by

$$\cos^{-1}x = y \iff \cos y = x$$

The inverse cosine function is also called **arccosine**, denoted by **arccos**.

### EXAMPLE 4 ■ Evaluating the Inverse Cosine Function

Find each value.

(a)  $\cos^{-1} \frac{\sqrt{3}}{2}$       (b)  $\cos^{-1} 0$       (c)  $\cos^{-1} \left( -\frac{1}{2} \right)$

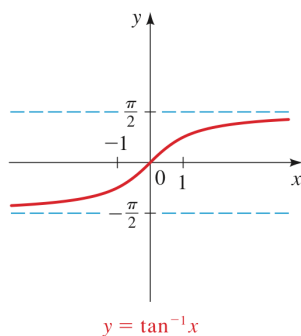
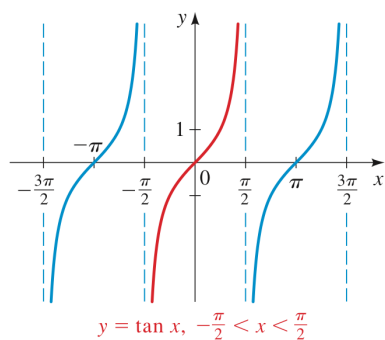
$$\cos(\cos^{-1}x) = x \quad \text{for } -1 \leq x \leq 1$$

$$\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi$$

### EXAMPLE 5 ■ Evaluating Expressions with Inverse Cosine

Find each value.

(a)  $\cos^{-1} \left( \cos \frac{2\pi}{3} \right)$       (b)  $\cos^{-1} \left( \cos \frac{5\pi}{3} \right)$



### DEFINITION OF THE INVERSE TANGENT FUNCTION

The **inverse tangent function** is the function  $\tan^{-1}$  with domain  $\mathbb{R}$  and range  $(-\pi/2, \pi/2)$  defined by

$$\tan^{-1} x = y \iff \tan y = x$$

The inverse tangent function is also called **arctangent**, denoted by **arctan**.

$$\begin{aligned} \tan(\tan^{-1} x) &= x \quad \text{for } x \in \mathbb{R} \\ \tan^{-1}(\tan x) &= x \quad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \end{aligned}$$

### EXAMPLE 6 ■ Evaluating the Inverse Tangent Function

Find each value.

(a)  $\tan^{-1} 1$       (b)  $\tan^{-1} \sqrt{3}$       (c)  $\tan^{-1}(20)$

39.  $\tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right)$

40.  $\tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)$

41.  $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

42.  $\sin^{-1}\left(\sin\left(\frac{11\pi}{4}\right)\right)$

43.  $\tan(\sin^{-1} \frac{1}{2})$

44.  $\cos(\sin^{-1} 0)$

45.  $\cos\left(\sin^{-1} \frac{\sqrt{3}}{2}\right)$

46.  $\tan\left(\sin^{-1} \frac{\sqrt{2}}{2}\right)$

47.  $\sin(\tan^{-1}(-1))$

48.  $\sin(\tan^{-1}(-\sqrt{3}))$