

## § 6.2 TRIGONOMETRY OF RIGHT TRIANGLES

CONSIDER A RIGHT TRIANGLE (ONE ANGLE IS  $90^\circ = \frac{\pi}{2}$  RAD).

LET ONE OF THE ACUTE ANGLES BE  $\theta$  (THE OTHER MUST BE  $(90 - \theta)^\circ = \frac{\pi}{2} - \theta$  RAD.)

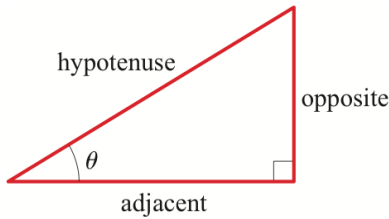


FIGURE 1

### THE TRIGONOMETRIC RATIOS

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

### EXAMPLE 1 ■ Finding Trigonometric Ratios

Find the six trigonometric ratios of the angle  $\theta$  in Figure 3.

**SOLUTION** By the definition of trigonometric ratios, we get

$$\sin \theta = \frac{2}{3} \quad \cos \theta = \frac{\sqrt{5}}{3} \quad \tan \theta = \frac{2}{\sqrt{5}}$$

$$\csc \theta = \frac{3}{2} \quad \sec \theta = \frac{3}{\sqrt{5}} \quad \cot \theta = \frac{\sqrt{5}}{2}$$

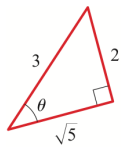


FIGURE 3

**Now Try Exercise 3**

### EXAMPLE 2 ■ Finding Trigonometric Ratios

If  $\cos \alpha = \frac{3}{4}$ , sketch a right triangle with acute angle  $\alpha$ , and find the other five trigonometric ratios of  $\alpha$ .

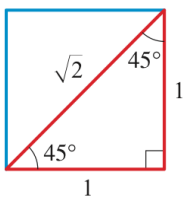
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

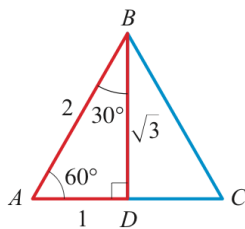
$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

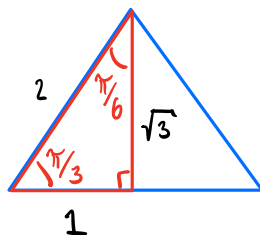
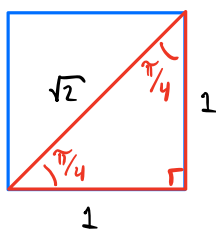
## SPECIAL TRIANGLES & TRIG TABLE



SQUARE



EQUILATERAL TRIANGLE



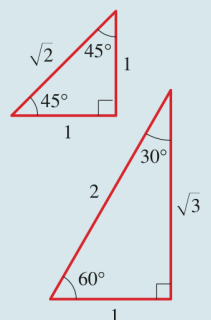
BOTH HAVE SHORTEST SIDE LENGTH 1

$$\left( \text{PYTH THM: } a^2 + b^2 = c^2 \Rightarrow \sin^2 \theta + \cos^2 \theta = 1 \right)$$

### SPECIAL VALUES OF THE TRIGONOMETRIC FUNCTIONS

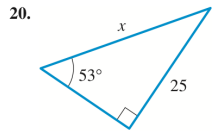
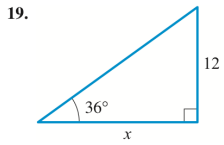
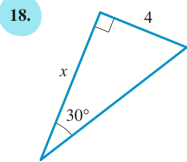
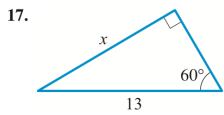
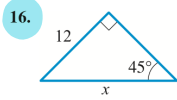
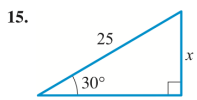
The following values of the trigonometric functions are obtained from the special triangles.

| $\theta$ in degrees | $\theta$ in radians | $\sin \theta$        | $\cos \theta$        | $\tan \theta$        | $\csc \theta$         | $\sec \theta$         | $\cot \theta$        |
|---------------------|---------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|----------------------|
| 0                   | 0                   | 0                    | 1                    | 0                    | —                     | 1                     | —                    |
| $30^\circ$          | $\frac{\pi}{6}$     | $\frac{1}{2}$        | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | 2                     | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$           |
| $45^\circ$          | $\frac{\pi}{4}$     | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1                    | $\sqrt{2}$            | $\sqrt{2}$            | 1                    |
| $60^\circ$          | $\frac{\pi}{3}$     | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$        | $\sqrt{3}$           | $\frac{2\sqrt{3}}{3}$ | 2                     | $\frac{\sqrt{3}}{3}$ |
| $90^\circ$          | $\frac{\pi}{2}$     | 1                    | 0                    | —                    | 1                     | —                     | 0                    |

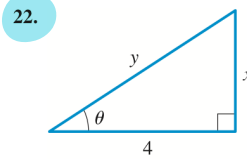
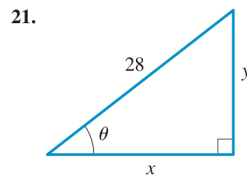


REUNDANT

**15–20 ■ Finding an Unknown Side** Find the side labeled  $x$ . In Exercises 17 and 18 state your answer rounded to five decimal places.



**21–22 ■ Trigonometric Ratios** Express  $x$  and  $y$  in terms of trigonometric ratios of  $\theta$ .



**23–28 ■ Trigonometric Ratios** Sketch a triangle that has acute angle  $\theta$ , and find the other five trigonometric ratios of  $\theta$ .

23.  $\tan \theta = \frac{5}{6}$

24.  $\cos \theta = \frac{12}{13}$

25.  $\cot \theta = 1$

26.  $\tan \theta = \sqrt{3}$

27.  $\csc \theta = \frac{11}{6}$

28.  $\cot \theta = \frac{5}{3}$

**EXAMPLE 5 ■ Finding the Height of a Tree**

A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is  $25.7^\circ$ .

**56. Distance at Sea** From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is  $23^\circ$ . How far is the ship from the base of the lighthouse?