

## 7.1 TRIGONOMETRIC IDENTITIES

■ Simplifying Trigonometric Expressions ■ Proving Trigonometric Identities

So FAR:

### FUNDAMENTAL TRIGONOMETRIC IDENTITIES

#### Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

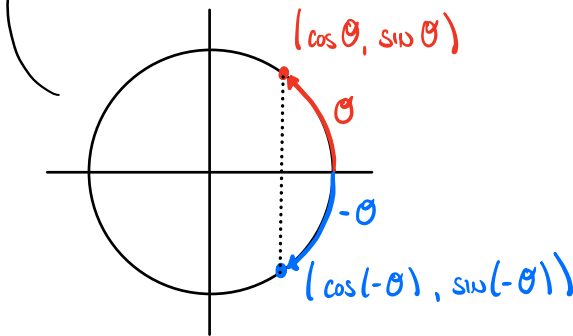
$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

#### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

#### Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$



$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta} = -\tan \theta$$

### EXAMPLE 1 ■ Simplifying a Trigonometric Expression

Simplify the expression  $\cos t + \tan t \sin t$ .

### EXAMPLE 2 ■ Simplifying by Combining Fractions

Simplify the expression  $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$ .

### GUIDELINES FOR PROVING TRIGONOMETRIC IDENTITIES

- Start with one side.** Pick one side of the equation, and write it down. Your goal is to transform it into the other side. It's usually easier to start with the more complicated side.
- Use known identities.** Use algebra and the identities you know to change the side you started with. Bring fractional expressions to a common denominator, factor, and use the fundamental identities to simplify expressions.
- Convert to sines and cosines.** If you are stuck, you may find it helpful to rewrite all functions in terms of sines and cosines.

**EXAMPLE 3 ■ Proving an Identity by Rewriting in Terms of Sine and Cosine**

Consider the equation  $\cos \theta (\sec \theta - \cos \theta) = \sin^2 \theta$ .

(a) Verify algebraically that the equation is an identity.

**EXAMPLE 4 ■ Proving an Identity by Combining Fractions**

Verify the identity

$$2 \tan x \sec x = \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x}$$

**EXAMPLE 5 ■ Proving an Identity by Introducing Something Extra**

Verify the identity  $\frac{\cos u}{1 - \sin u} = \sec u + \tan u$ .

**EXAMPLE 6 ■ Proving an Identity by Working with Both Sides Separately**

Verify the identity  $\frac{1 + \cos \theta}{\cos \theta} = \frac{\tan^2 \theta}{\sec \theta - 1}$ .

**EXAMPLE 7 ■ Trigonometric Substitution**

Substitute  $\sin \theta$  for  $x$  in the expression  $\sqrt{1 - x^2}$ , and simplify. Assume that  $0 \leq \theta \leq \pi/2$ .

**13–28 ■ Simplifying Trigonometric Expressions** Simplify the trigonometric expression.

13.  $\frac{\sin x \sec x}{\tan x}$

14.  $\frac{\cos x \sec x}{\cot x}$

15.  $\frac{\sin t + \tan t}{\tan t}$

16.  $\frac{1 + \cot A}{\csc A}$

17.  $\cos^3 x + \sin^2 x \cos x$

18.  $\sin^4 \alpha - \cos^4 \alpha + \cos^2 \alpha$

19.  $\frac{\sec^2 x - 1}{\sec^2 x}$

20.  $\frac{\sec x - \cos x}{\tan x}$

21.  $\frac{1 + \cos y}{1 + \sec y}$

22.  $\frac{1 + \sin y}{1 + \csc y}$

23.  $\frac{1 + \sin u}{\cos u} + \frac{\cos u}{1 + \sin u}$

24.  $\frac{\sin t}{1 - \cos t} - \csc t$

25.  $\frac{\cos x}{\sec x + \tan x}$

26.  $\frac{\cot A - 1}{1 + \tan(-A)}$

27.  $\frac{1}{1 - \sin \alpha} + \frac{1}{1 + \sin \alpha}$

28.  $\frac{2 + \tan^2 x}{\sec^2 x} - 1$

**31–88 ■ Proving Identities** Verify the identity.

31.  $\frac{\sin \theta}{\tan \theta} = \cos \theta$

32.  $\frac{\tan x}{\sec x} = \sin x$

33.  $\frac{\cos u \sec u}{\tan u} = \cot u$

34.  $\frac{\cot x \sec x}{\csc x} = 1$

35.  $\frac{\tan y}{\csc y} = \frac{1}{\cos y} - \frac{1}{\sec y}$

36.  $\frac{\cos^2 v}{\sin v} = \csc v - \sin v$

37.  $\cos(-x) - \sin(-x) = \cos x + \sin x$

38.  $\cot(-\alpha) \cos(-\alpha) + \sin(-\alpha) = -\csc \alpha$

39.  $\tan \theta + \cot \theta = \sec \theta \csc \theta$

40.  $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

41.  $(1 - \cos \beta)(1 + \cos \beta) = \frac{1}{\csc^2 \beta}$

42.  $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$