7.2 ADDITION AND SUBTRACTION FORMULAS

■ Addition and Subtraction Formulas ■ Evaluating Expressions Involving Inverse

Trigonometric Functions \blacksquare Expressions of the form $A \sin x + B \cos x$

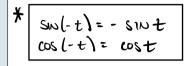
ADDITION AND SUBTRACTION FORMULAS

Formulas for sine: $\sin(s+t) = \sin s \cos t + \cos s \sin t$

$$\sin(s-t) = \sin s \cos t - \cos s \sin t +$$

Formulas for cosine: $\cos(s+t) = \cos s \cos t - \sin s \sin t$

$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$



EXAMPLE 1 Using the Addition and Subtraction Formulas

Find the exact value of each expression.

(b)
$$\cos \frac{\pi}{12}$$

$$ex.$$
 SIN $\left(-\frac{5\pi}{12}\right)$

EXAMPLE 2 Using the Addition Formula for Sine

Find the exact value of the expression $\sin 20^{\circ} \cos 40^{\circ} + \cos 20^{\circ} \sin 40^{\circ}$.

EXAMPLE 3 Proving a Cofunction Identity

Prove the cofunction identity $\cos\left(\frac{\pi}{2} - u\right) = \sin u$.

3–14 ■ Values of Trigonometric Functions Use an Addition or Subtraction Formula to find the exact value of the expression, as demonstrated in Example 1.

• 9.
$$\sin \frac{19\pi}{12}$$

10.
$$\cos \frac{17\pi}{12}$$

11.
$$\tan\left(-\frac{\pi}{12}\right)$$

12.
$$\sin\left(-\frac{5\pi}{12}\right)$$

13.
$$\cos \frac{11\pi}{12}$$

14.
$$\tan \frac{7\pi}{12}$$

7.3 DOUBLE-ANGLE, HALF-ANGLE, AND PRODUCT-SUM FORMULAS

■ Double-Angle Formulas ■ Half-Angle Formulas ■ Evaluating Expressions Involving Inverse Trigonometric Functions Product-Sum Formulas

DOUBLE-ANGLE FORMULAS

Formula for sine: $\sin 2x = 2 \sin x \cos x$

 $\cos 2x = \cos^2 x - \sin^2 x$ Formulas for cosine:

$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

THESE FOLLOW IMMEDIATELY FROM ANGLE-SUM FORMULAS.

IF YOU KNOW THESE YOU ARE GOOD

EXAMPLE 1 Using the Double-Angle Formulas

If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$.

FORMULAS FOR LOWERING POWERS

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

 $\lim_{N \to \infty} \frac{N}{N} = \lim_{N \to \infty} \frac{N}{N} = \lim_{N$

The choice of the + or - sign depends on the quadrant in which u/2 lies.

EXAMPLE 5 Using a Half-Angle Form

3–10 Double Angle Formulas Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information.

3.
$$\sin x = \frac{5}{13}$$
, x in Quadrant I

4.
$$\tan x = -\frac{4}{3}$$
, x in Quadrant II

5.
$$\cos x = \frac{4}{5}$$
, $\csc x < 0$

6.
$$\csc x = 4$$
, $\tan x < 0$

7.
$$\sin x = -\frac{3}{5}$$
, x in Quadrant III

8.
$$\sec x = 2$$
, x in Quadrant IV

9.
$$\tan x = -\frac{1}{3}$$
, $\cos x > 0$

10.
$$\cot x = \frac{2}{3}$$
, $\sin x > 0$

17–28 ■ Half Angle Formulas Use an appropriate Half-Angle Formula to find the exact value of the expression.

23.
$$\tan \frac{\pi}{2}$$

25.
$$\cos \frac{\pi}{12}$$

27.
$$\sin \frac{9\pi}{8}$$

18. tan 15°

24.
$$\cos \frac{3\pi}{8}$$

26.
$$\tan \frac{5\pi}{12}$$

28.
$$\sin \frac{11\pi}{12}$$