7.2 ADDITION AND SUBTRACTION FORMULAS

■ Addition and Subtraction Formulas ■ ~~Evaluating Expressions Involving Inverse Trigonometric Functions~~ ■ ~~Expressions of the form $A \sin x + B \cos x$~~

ADDITION AND SUBTRACTION FORMULAS

Formulas for sine:

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t \ *$$

Formulas for cosine:

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t \ *$$

* $\boxed{\begin{array}{l} \sin(-t) = -\sin t \\ \cos(-t) = \cos t \end{array}}$

EXAMPLE 1 ■ Using the Addition and Subtraction Formulas

Find the exact value of each expression.

(a) $\cos 75°$ **(b)** $\cos \dfrac{\pi}{12}$

ex. $\sin\left(\dfrac{7\pi}{12}\right)$

ex. $\sin\left(-\dfrac{5\pi}{12}\right)$

EXAMPLE 2 ■ Using the Addition Formula for Sine

Find the exact value of the expression $\sin 20° \cos 40° + \cos 20° \sin 40°$.

EXAMPLE 3 ■ Proving a Cofunction Identity

Prove the cofunction identity $\cos\left(\dfrac{\pi}{2} - u\right) = \sin u$.

3–14 ■ Values of Trigonometric Functions Use an Addition or Subtraction Formula to find the exact value of the expression, as demonstrated in Example 1.

3. $\sin 75°$ **4.** $\sin 15°$

5. $\cos 105°$ **6.** $\cos 195°$

7. $\tan 15°$ **8.** $\tan 165°$

9. $\sin \dfrac{19\pi}{12}$ **10.** $\cos \dfrac{17\pi}{12}$

11. $\tan\left(-\dfrac{\pi}{12}\right)$ **12.** $\sin\left(-\dfrac{5\pi}{12}\right)$

13. $\cos \dfrac{11\pi}{12}$ **14.** $\tan \dfrac{7\pi}{12}$

7.3 DOUBLE-ANGLE, HALF-ANGLE, AND PRODUCT-SUM FORMULAS

■ Double-Angle Formulas ■ Half-Angle Formulas ■ Evaluating Expressions Involving Inverse Trigonometric Functions ■ Product-Sum Formulas

DOUBLE-ANGLE FORMULAS

Formula for sine: $\sin 2x = 2 \sin x \cos x$

Formulas for cosine:

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

THESE FOLLOW IMMEDIATELY FROM ANGLE-SUM FORMULAS.

IF YOU KNOW THESE YOU ARE GOOD!

EXAMPLE 1 ■ Using the Double-Angle Formulas

If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$.

FORMULAS FOR LOWERING POWERS

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

HALF-ANGLE FORMULAS

$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}} \qquad \cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\cancel{\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}}$$

The choice of the $+$ or $-$ sign depends on the quadrant in which $u/2$ lies.

EXAMPLE 5 ■ Using a Half-Angle Form

3–10 ■ Double Angle Formulas Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information.

3. $\sin x = \frac{5}{13}$, x in Quadrant I
4. $\tan x = -\frac{4}{3}$, x in Quadrant II
5. $\cos x = \frac{4}{5}$, $\csc x < 0$
6. $\csc x = 4$, $\tan x < 0$
7. $\sin x = -\frac{3}{5}$, x in Quadrant III
8. $\sec x = 2$, x in Quadrant IV
9. $\tan x = -\frac{1}{3}$, $\cos x > 0$
10. $\cot x = \frac{2}{3}$, $\sin x > 0$

17–28 ■ Half Angle Formulas Use an appropriate Half-Angle Formula to find the exact value of the expression.

17. $\sin 15°$
18. $\tan 15°$
19. $\tan 22.5°$
20. $\sin 75°$
21. $\cos 165°$
22. $\cos 112.5°$
23. $\tan \frac{\pi}{8}$
24. $\cos \frac{3\pi}{8}$
25. $\cos \frac{\pi}{12}$
26. $\tan \frac{5\pi}{12}$
27. $\sin \frac{9\pi}{8}$
28. $\sin \frac{11\pi}{12}$