§7.2 ADDITION & SUBTRACTION FORMULAS

#9-12, 13, 25-28, 31, 32, 35, 36, 42, 43

9. $\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{16\pi}{12} + \frac{3\pi}{12}\right)^{*} = \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{4\pi}{3}\cos\frac{\pi}{4} + \cos\frac{4\pi}{3}\sin\frac{\pi}{4}$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2}\left(\sqrt{3}+1\right)}{4}}$

* NOTE: THIS IS NOT THE ONLY WAY:

$$\frac{19\pi}{12} = \underbrace{\frac{16\pi}{12} + \frac{3\pi}{12}}_{\frac{4\pi}{3} + \frac{\pi}{4}} = \underbrace{\frac{15\pi}{12} + \frac{4\pi}{12}}_{\frac{5\pi}{4} + \frac{\pi}{3}} = \underbrace{\frac{10\pi}{12} + \frac{9\pi}{12}}_{\frac{5\pi}{6} + \frac{3\pi}{4}}$$

$$= \underbrace{\frac{21\pi}{12} - \frac{2\pi}{12}}_{\frac{7\pi}{4} - \frac{\pi}{6}} = \underbrace{\frac{22\pi}{12} - \frac{3\pi}{12}}_{\frac{11\pi}{6} - \frac{\pi}{4}} = \underbrace{\frac{28\pi}{12} - \frac{9\pi}{12}}_{\frac{7\pi}{3} - \frac{3\pi}{4}} \quad \text{ETC.}$$

10. $\cos\frac{17\pi}{12} = \cos\left(\frac{14\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{7\pi}{6}\cos\frac{\pi}{4} - \sin\frac{7\pi}{6}\sin\frac{\pi}{4}$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}\left(1-\sqrt{3}\right)}{4}}$

12. $\sin\left(-\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{2\pi}{3} - \cos\frac{\pi}{4}\sin\frac{2\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{-\sqrt{2}\left(1+\sqrt{3}\right)}{4}}$

13. $\cos\frac{11\pi}{12} = \cos\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{2}\left(\sqrt{3}+1\right)}{4}}$

25. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \underbrace{\cos\frac{\pi}{2}}_{0} - \cos x \underbrace{\sin\frac{\pi}{2}}_{1} = -\cos x$ ✓

26. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \underbrace{\cos\frac{\pi}{2}}_{0} + \sin x \underbrace{\sin\frac{\pi}{2}}_{1} = \sin x$ ✓

27. $\sin\left(x - \pi\right) = \sin x \underbrace{\cos\pi}_{-1} - \cos x \underbrace{\sin\pi}_{0} = -\sin x$ ✓

28. $\cos\left(x - \pi\right) = \cos x \underbrace{\cos\pi}_{-1} + \sin x \underbrace{\sin\pi}_{0} = -\cos x$ ✓

31. $\sin\left(\frac{\pi}{2} - x\right) = \underbrace{\sin\frac{\pi}{2}}_{1}\cos x - \underbrace{\cos\frac{\pi}{2}}_{0}\sin x = \cos x$

$\sin\left(\frac{\pi}{2} + x\right) = \underbrace{\sin\frac{\pi}{2}}_{1}\cos x + \underbrace{\cos\frac{\pi}{2}}_{0}\sin x = \cos x$

SAME!

32. $\cos\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{6}\right)$

$= \cos x \underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}} - \sin x \underbrace{\sin\frac{\pi}{3}}_{\frac{\sqrt{3}}{2}} + \sin x \underbrace{\cos\frac{\pi}{6}}_{\frac{\sqrt{3}}{2}} - \cos x \underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}}$

$= \left(\frac{1}{2} - \frac{1}{2}\right)\cos x + \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\sin x = 0$

35. $\sin(x+y) - \sin(x-y)$

$= \cancel{\sin x \cos y} + \cos x \sin y - \left(\cancel{\sin x \cos y} - \cos x \sin y\right) = 2\cos x \sin y$

36. $\cos(x+y) + \cos(x-y)$

$= \cos x \cos y - \cancel{\sin x \sin y} + \cos x \cos y + \cancel{\sin x \sin y} = 2\cos x \cos y$

42. $$\frac{\sin(x+y)-\sin(x-y)}{\cos(x+y)+\cos(x-y)}$$

$$= \frac{\cancel{\sin x\cos y} + \cos x\sin y - (\cancel{\sin x\cos y} - \cos x\sin y)}{\cos x\cos y - \cancel{\sin x\sin y} + \cos x\cos y + \cancel{\sin x\sin y}}$$

$$= \frac{\cancel{2}\,\cancel{\cos x}\,\sin y}{\cancel{2}\,\cancel{\cos x}\,\cos y} = \frac{\sin y}{\cos y} = \tan y \quad \checkmark$$

43. $$\cos(x+y)\cos(x-y)$$

$$= [\cos x\cos y - \sin x\sin y][\cos x\cos y + \sin x\sin y]$$

$$= (\cos x\cos y)^2 - (\sin x\sin y)^2$$

$$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$$

$$= \cos^2 x(1-\sin^2 y) - \sin^2 x\sin^2 y = \cos^2 x - \sin^2 y\,(\underbrace{\cos^2 x + \sin^2 x}_{1})$$

$$= \cos^2 x - \sin^2 y \quad \checkmark$$