Name: *ANSWER KEY* 2/20/2019

Math 19500-FG Precalculus Quiz 1

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed and cellphones should be put away. Good luck!

1. Multiply the following expressions.

   (a) (4 points) $x^{3/2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$

   $$x^{3/2}\left(x^{1/2}-x^{-1/2}\right) = x^{3/2+1/2} - x^{3/2-1/2}$$

   $$= \boxed{x^2 - x}$$

   (b) (4 points) $(\sqrt{h^2+1}+1)(\sqrt{h^2+1}-1)$

   $$(a+b)(a-b) = a^2 - b^2$$

   $$\longrightarrow (\sqrt{h^2+1})^2 - (1)^2 = h^2+1-1 = \boxed{h^2}$$

2. Factor the following expressions completely.

   (a) (4 points) $9x^2-36x-45$

   $$9(x^2-4x-5) = \boxed{9(x-5)(x+1)}$$

   GREATEST COMMON FACTOR (GCF)

   (b) (4 points) $3x^3-x^2-12x+4$ (FACTOR BY GROUPING)

   $$= x^2(3x-1) - 4(3x-1)$$

   $$= (3x-1)(x^2-4) = \boxed{(3x-1)(x+2)(x-2)}$$

3. Rewrite the following expressions as one simplified fraction.

(a) (4 points) $\dfrac{\left(\dfrac{2x^2-3x-2}{x^2-1}\right)}{\left(\dfrac{2x^2+5x+2}{x^2+x-2}\right)}$

$$= \frac{(2x+1)(x-2)}{(x+1)(x-1)} \div \frac{(2x+1)(x+2)}{(x+2)(x-1)}$$

$$= \frac{\cancel{(2x+1)}(x-2)}{(x+1)\cancel{(x-1)}} \cdot \frac{\cancel{(x+2)}\cancel{(x-1)}}{\cancel{(2x+1)}\cancel{(x+2)}} = \boxed{\frac{x-2}{x+1}}$$

(b) (4 points) $\dfrac{1}{x+1} - \dfrac{2}{x^2+2x+1} + \dfrac{3}{x^2-1}$

$$\frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{(x+1)(x-1)} \qquad \text{LCD} = (x+1)^2(x-1)$$

$$= \frac{1}{x+1} \cdot \frac{(x+1)(x-1)}{(x+1)(x-1)} - \frac{2}{(x+1)^2} \cdot \frac{x-1}{x-1} + \frac{3}{(x+1)(x-1)} \cdot \frac{x+1}{x+1}$$

$$= \frac{x^2-1-2x+2+3x+3}{(x+1)^2(x-1)} = \boxed{\frac{x^2+x+4}{(x+1)^2(x-1)}}$$

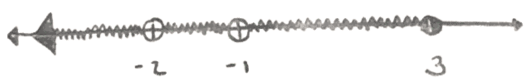
4. (4 points) Use interval notation to describe the domain of the following expression.

$$\frac{\sqrt{3-x}}{x^2+3x+2}$$

$3 - x \geq 0$ AND $x^2 + 3x + 2 \neq 0$

$x \leq 3$ $(x+2)(x+1) \neq 0$

$x \neq -2, \; x \neq -1$



$$\boxed{(-\infty, -2) \cup (-2, -1) \cup (-1, 3]}$$

5. Solve the following equations.

(a) (4 points) $\dfrac{4}{x-1} + \dfrac{2}{x+1} = \dfrac{35}{x^2-1}$ MULTIPLY EVERYTHING BY LCD $= (x+1)(x-1)$

(ASSUMING $x \neq \pm 1$)

$4(x+1) + 2(x-1) = 35$

$4x + 4 + 2x - 2 = 35$

$6x = 33, \quad x = \dfrac{33}{6} = \boxed{\dfrac{11}{2}}$

(b) (4 points) $\dfrac{10}{x} - \dfrac{12}{x-3} = -4$ MULTIPLY EVERYTHING BY LCD $= x(x-3)$

(ASSUMING $x \neq 0, 3$)

$10(x-3) - 12x = -4x(x-3)$

$10x - 30 - 12x = -4x^2 + 12x$

$4x^2 - 14x - 30 = 0$

$2(2x^2 - 7x - 15) = 0$

$2(2x+3)(x-5) = 0$

$2x + 3 = 0 \qquad x - 5 = 0$

$$\boxed{x = -\frac{3}{2} \qquad x = 5}$$

6. Use interval notation to describe the solution set of the following inequalities.

(a) (4 points) $(x+3)^2(x-2)(x+5) \ge 0$

INCLUDED

ZEROS: $x = -3,\ 2,\ -5$

	$x<-5$	$-5<x<-3$	$-3<x<2$	$x>2$
$(x+3)^2$	+	+	+	+
$(x-2)$	−	−	−	+
$(x+5)$	−	+	+	+
$(x+3)^2(x-2)(x+5)$	+	−	−	+

$$\boxed{(-\infty,-5] \cup \{-3\} \cup [2,\infty)}$$

(b) (4 points) $1+\dfrac{2}{x+1} \le \dfrac{2}{x}$ (NOTE THAT $x \neq 0,\ x \neq -1$)

$$1 \cdot \frac{x(x+1)}{x(x+1)} + \frac{2}{x+1}\cdot\frac{x}{x} - \frac{2}{x}\cdot\frac{x+1}{x+1} \le 0$$

$$\frac{x^2+x+2x-2x-2}{x(x+1)} = \frac{x^2+x-2}{x(x+1)} = \frac{(x+2)(x-1)}{x(x+1)} \le 0$$

ZEROS: $x = -2,\ 1,\ 0,\ -1$

0 UNDEFINED UND. 0

	$x<-2$	$-2<x<-1$	$-1<x<0$	$0<x<1$	$x>1$
$(x+2)$	−	+	+	+	+
$(x-1)$	−	−	−	−	+
x	−	−	−	+	+
$(x+1)$	−	−	+	+	+
$\frac{(x+2)(x-1)}{x(x+1)}$	+	−	+	−	+

$$\boxed{[-2,-1) \cup (0,1]}$$

(c) (4 points) $|8x+3| > 12$

$8x+3 > 12$ or $8x+3 < -12$

$8x > 9 \qquad 8x < -15$

$x > \frac{9}{8} \qquad x < -\frac{15}{8}$

$$\boxed{\left(-\infty,-\frac{15}{8}\right) \cup \left(\frac{9}{8},\infty\right)}$$