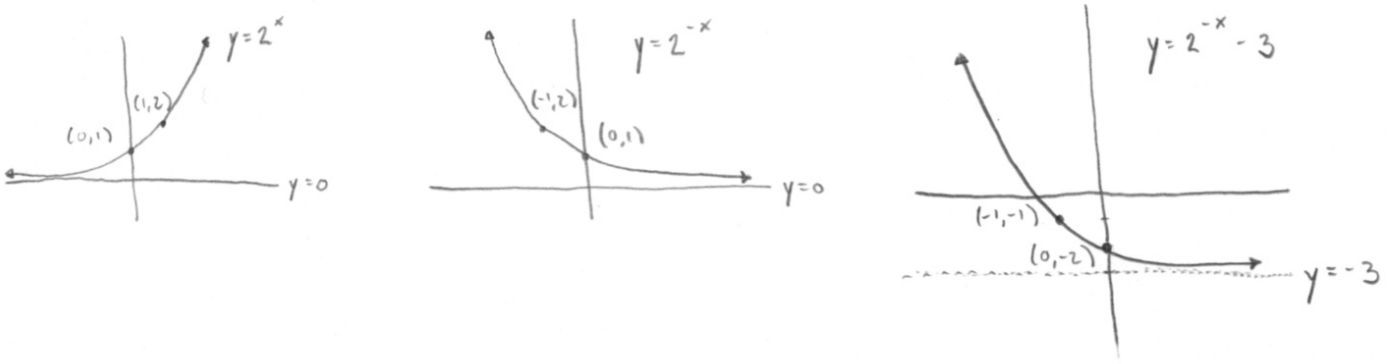
Name: * Answer Key *

Math 19500-FG Precalculus **5/8/2019**
Quiz 4

Please put away all papers and electronic devices. Show enough work that it is clear how you arrived at your answer. Correct answers with no work shown will not receive full credit. Box/circle your final answers. Good luck!

1. Let $f(x) = 2^{-x} - 3$.

   (a) (4 points) Use transformations to sketch the graph of f.



   (b) (4 points) State the domain and range of f and any asymptote(s) of its graph.

DOM $(f) = \mathbb{R}$ RAN $(f) = (-3, \infty)$

HORIZONTAL ASYMPTOTE : $y = -3$

2. Evaluate the following expressions.

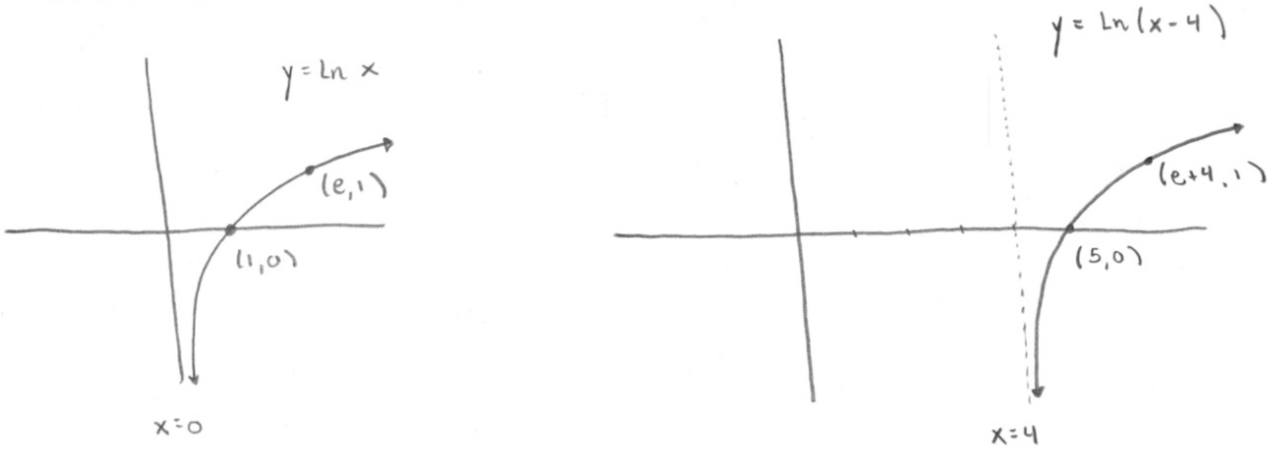
   (a) (2 points) $\log_8(64) = x \iff 8^x = 64 = 8^2 \quad \therefore x = \boxed{2}$

   (b) (2 points) $\log_2(1/8) = y \iff 2^y = \frac{1}{8} = 2^{-3} \quad \therefore y = \boxed{-3}$

   (c) (2 points) $\log_5(5^{13}) = z \iff 5^z = 5^{13} \quad \therefore z = \boxed{13}$

3. Let $f(x) = \ln(x-4)$.

(a) (4 points) Use transformations to sketch the graph of f.



(b) (4 points) State the domain and range of f and any asymptote(s) of its graph.

$\text{DOM}(f) = \{x \in \mathbb{R} \mid x-4 > 0\}$ $\qquad$ $\text{RAN}(f) = \mathbb{R}$

$= (4, \infty)$

VERTICAL ASYMPTOTE: $x = 4$

4. (4 points) Use the laws of logarithms to simply the expression $\ln\left(\left(\frac{\sqrt[3]{a}}{b^2c}\right)^5\right)$.

$= 5 \ln\left(\frac{\sqrt[3]{a}}{b^2c}\right)$

$= 5\left[\ln(a^{1/3}) - \ln(b^2c)\right]$

$= 5\left[\frac{1}{3}\ln a - \left(\ln(b^2) + \ln c\right)\right]$

$= \boxed{5\left(\frac{1}{3}\ln a - 2\ln b - \ln c\right)}$

5. (4 points) Solve: $e^{3-5x} = 16$.

$$\ln\left(e^{3-5x}\right) = \ln(16)$$

$$3 - 5x = \ln(16)$$

$$\boxed{x = \frac{3 - \ln(16)}{5}}$$

6. Suppose a colony of bacteeria grows exponentially. It is observed that there are 235 bacteria in the colony at 12pm, and there are 425 bacteria in the colony at 3pm.

   (a) (4 points) Find a function $P(t)$ for the population of the colony t hours past 12pm.

$$P(t) = P_0 e^{rt} \qquad P(0) = P_0 = 235$$

$$P(t) = 235\, e^{rt} \qquad P(3) = 235\, e^{3r} = 425$$

$$e^{3r} = \frac{425}{235} \rightarrow r = \frac{1}{3}\ln\left(\frac{425}{235}\right)$$

$$\therefore \boxed{P(t) = 235\, e^{\frac{1}{3}\ln\left(\frac{425}{235}\right)t} = 235\left(\frac{425}{235}\right)^{t/3}}$$

   (b) (4 points) How long after 12pm does the population reach 1000?

$$P(t) = 235\left(\frac{425}{235}\right)^{t/3} = 1000 \qquad \text{SOLVE FOR } t.$$

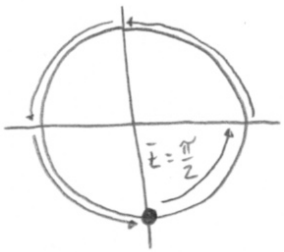
$$\left(\frac{425}{235}\right)^{t/3} = \frac{1000}{235}$$

$$\frac{t}{3}\ln\left(\frac{425}{235}\right) = \ln\left(\frac{1000}{235}\right)$$

$$\boxed{t = \frac{3\ln\left(\frac{1000}{235}\right)}{\ln\left(\frac{425}{235}\right)}}$$

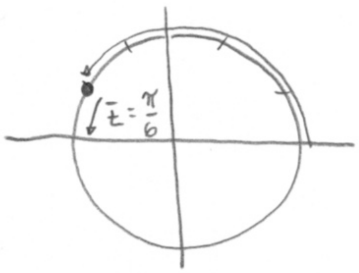
7. Find the terminal point $P(x, y)$ on the unit circle determined by each of the following real numbers t.

(a) (2 points) $t = 3\pi/2$



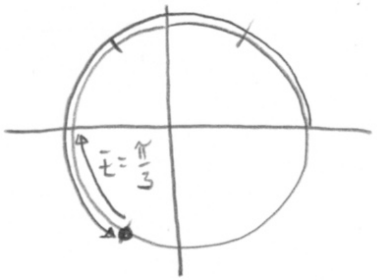
$(0, -1)$

(b) (2 points) $t = 5\pi/6$



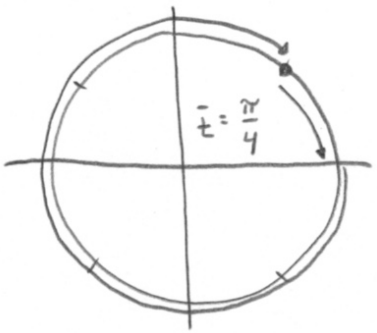
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(c) (2 points) $t = 4\pi/3$



$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(d) (2 points) $t = -7\pi/4$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$