DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

125. WRITE: Completing a Line Segment Plot the points $M(6, 8)$ and $A(2, 3)$ on a coordinate plane. If M is the midpoint of the line segment AB, find the coordinates of B. Write a brief description of the steps you took to find B and your reasons for taking them.

126. WRITE: Completing a Parallelogram Plot the points $P(0, 3)$, $Q(2, 2)$, and $R(5, 3)$ on a coordinate plane. Where should the point S be located so that the figure $PQRS$ is a parallelogram? Write a brief description of the steps you took and your reasons for taking them.

127. DISCOVER: Circle, Point, or Empty Set? Complete the squares in the general equation $x^2 + ax + y^2 + by + c = 0$, and simplify the result as much as possible. Under what conditions on the coefficients a, b, and c does this equation represent a circle? A single point? The empty set? In the case in which the equation does represent a circle, find its center and radius.

1.10 LINES

■ The Slope of a Line ■ Point-Slope Form of the Equation of a Line ■ Slope-Intercept Form of the Equation of a Line ■ Vertical and Horizontal Lines ■ General Equation of a Line ■ Parallel and Perpendicular Lines

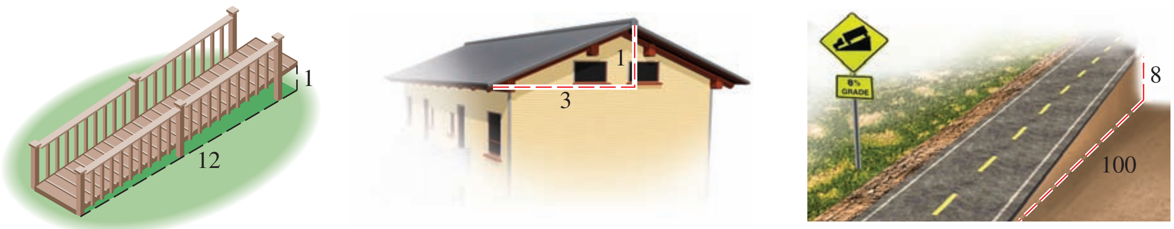
In this section we find equations for straight lines lying in a coordinate plane. The equations will depend on how the line is inclined, so we begin by discussing the concept of slope.

■ The Slope of a Line

We first need a way to measure the "steepness" of a line, or how quickly it rises (or falls) as we move from left to right. We define *run* to be the distance we move to the right and *rise* to be the corresponding distance that the line rises (or falls). The *slope* of a line is the ratio of rise to run:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Figure 1 shows situations in which slope is important. Carpenters use the term *pitch* for the slope of a roof or a staircase; the term *grade* is used for the slope of a road.



Slope of a ramp
Slope $= \frac{1}{12}$

Pitch of a roof
Slope $= \frac{1}{3}$

Grade of a road
Slope $= \frac{8}{100}$

FIGURE 1

If a line lies in a coordinate plane, then the **run** is the change in the x-coordinate and the **rise** is the corresponding change in the y-coordinate between any two points on the line (see Figure 2). This gives us the following definition of slope.

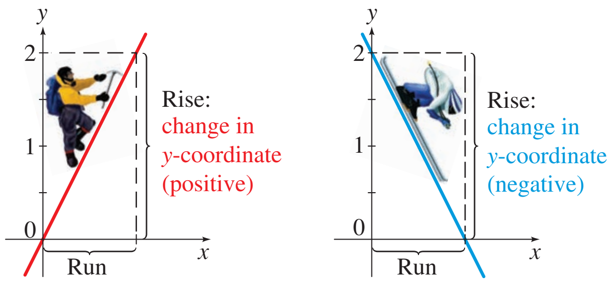


FIGURE 2

SLOPE OF A LINE

The **slope** m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

The slope is independent of which two points are chosen on the line. We can see that this is true from the similar triangles in Figure 3.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2' - y_1'}{x_2' - x_1'}$$

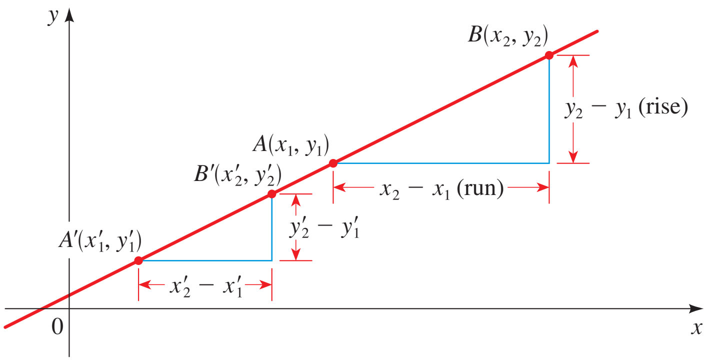
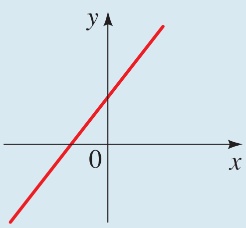


FIGURE 3

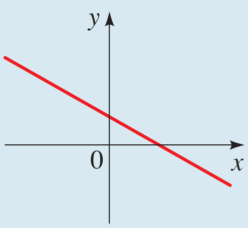
The figures in the box below show several lines labeled with their slopes. Notice that lines with positive slope slant upward to the right, whereas lines with negative slope slant downward to the right. The steepest lines are those for which the absolute value of the slope is the largest; a horizontal line has slope 0. The slope of a vertical line is undefined (it has a 0 denominator), so we say that a vertical line has no slope.

SLOPE OF A LINE

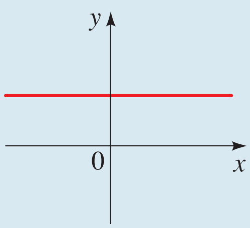
Positive Slope



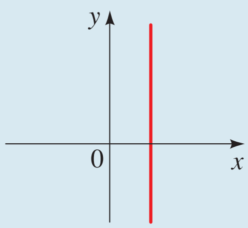
Negative Slope



Zero Slope



No Slope



EXAMPLE 1 ■ Finding the Slope of a Line Through Two Points

Find the slope of the line that passes through the points $P(2, 1)$ and $Q(8, 5)$.

SOLUTION Since any two different points determine a line, only one line passes through these two points. From the definition the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{8 - 2} = \frac{4}{6} = \frac{2}{3}$$

This says that for every 3 units we move to the right, the line rises 2 units. The line is drawn in Figure 4.

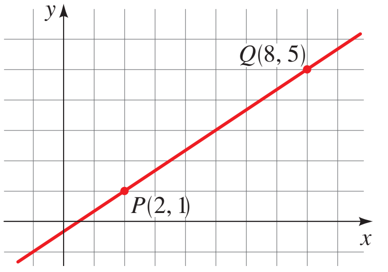


FIGURE 4

Now Try Exercise 9 ■

Point-Slope Form of the Equation of a Line

Now let's find the equation of the line that passes through a given point $P(x_1, y_1)$ and has slope m. A point $P(x, y)$ with $x \neq x_1$ lies on this line if and only if the slope of the line through P_1 and P is equal to m (see Figure 5), that is,

$$\frac{y - y_1}{x - x_1} = m$$

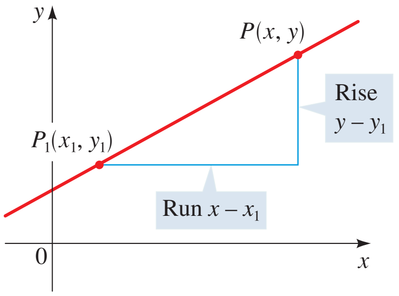


FIGURE 5

This equation can be rewritten in the form $y - y_1 = m(x - x_1)$; note that the equation is also satisfied when $x = x_1$ and $y = y_1$. Therefore it is an equation of the given line.

POINT-SLOPE FORM OF THE EQUATION OF A LINE

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1)$$

EXAMPLE 2 ■ Finding an Equation of a Line with Given Point and Slope

(a) Find an equation of the line through $(1, -3)$ with slope $-\frac{1}{2}$.

(b) Sketch the line.

SOLUTION

(a) Using the point-slope form with $m = -\frac{1}{2}$, $x_1 = 1$, and $y_1 = -3$, we obtain an equation of the line as

$$y + 3 = -\tfrac{1}{2}(x - 1) \qquad \text{Slope } m = -\tfrac{1}{2}, \text{ point } (1, -3)$$

$$2y + 6 = -x + 1 \qquad \text{Multiply by 2}$$

$$x + 2y + 5 = 0 \qquad \text{Rearrange}$$

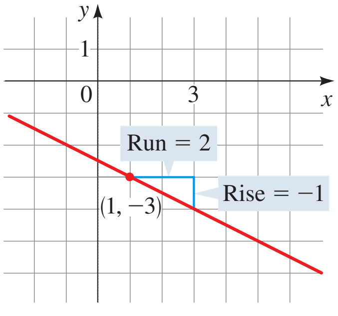


FIGURE 6

(b) The fact that the slope is $-\frac{1}{2}$ tells us that when we move to the right 2 units, the line drops 1 unit. This enables us to sketch the line in Figure 6.

Now Try Exercise 25 ■

EXAMPLE 3 ■ Finding an Equation of a Line Through Two Given Points

Find an equation of the line through the points $(-1, 2)$ and $(3, -4)$.

SOLUTION The slope of the line is

$$m = \frac{-4 - 2}{3 - (-1)} = -\frac{6}{4} = -\frac{3}{2}$$

We can use *either* point, $(-1, 2)$ *or* $(3, -4)$, in the point-slope equation. We will end up with the same final answer.

Using the point-slope form with $x_1 = -1$ and $y_1 = 2$, we obtain

$$y - 2 = -\tfrac{3}{2}(x + 1) \qquad \text{Slope } m = -\tfrac{3}{2}, \text{ point } (-1, 2)$$

$$2y - 4 = -3x - 3 \qquad \text{Multiply by 2}$$

$$3x + 2y - 1 = 0 \qquad \text{Rearrange}$$

Now Try Exercise 29 ■

■ Slope-Intercept Form of the Equation of a Line

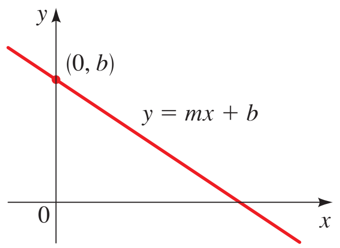


FIGURE 7

Suppose a nonvertical line has slope m and y-intercept b (see Figure 7). This means that the line intersects the y-axis at the point $(0, b)$, so the point-slope form of the equation of the line, with $x = 0$ and $y = b$, becomes

$$y - b = m(x - 0)$$

This simplifies to $y = mx + b$, which is called the **slope-intercept form** of the equation of a line.

SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$

EXAMPLE 4 ■ Lines in Slope-Intercept Form

(a) Find an equation of the line with slope 3 and y-intercept -2.

(b) Find the slope and y-intercept of the line $3y - 2x = 1$.

SOLUTION

(a) Since $m = 3$ and $b = -2$, from the slope-intercept form of the equation of a line we get

$$y = 3x - 2$$

(b) We first write the equation in the form $y = mx + b$.

$$3y - 2x = 1$$

$$3y = 2x + 1 \qquad \text{Add } 2x$$

$$y = \tfrac{2}{3}x + \tfrac{1}{3} \qquad \text{Divide by 3}$$

Slope | y-intercept

$y = \tfrac{2}{3}x + \tfrac{1}{3}$

From the slope-intercept form of the equation of a line, we see that the slope is $m = \frac{2}{3}$ and the y-intercept is $b = \frac{1}{3}$.

Now Try Exercises 23 and 61 ■

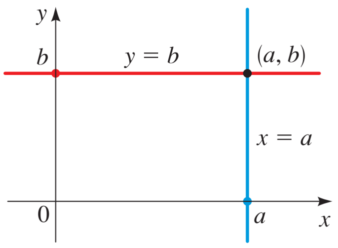


FIGURE 8

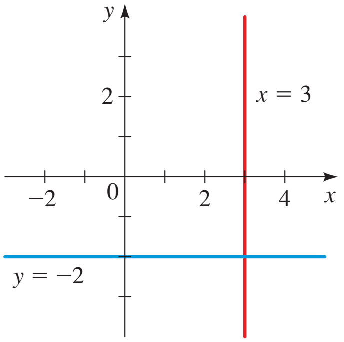


FIGURE 9

Vertical and Horizontal Lines

If a line is horizontal, its slope is $m = 0$, so its equation is $y = b$, where b is the y-intercept (see Figure 8). A vertical line does not have a slope, but we can write its equation as $x = a$, where a is the x-intercept, because the x-coordinate of every point on the line is a.

VERTICAL AND HORIZONTAL LINES

- An equation of the vertical line through (a, b) is $x = a$.
- An equation of the horizontal line through (a, b) is $y = b$.

EXAMPLE 5 ■ Vertical and Horizontal Lines

(a) An equation for the vertical line through $(3, 5)$ is $x = 3$.

(b) The graph of the equation $x = 3$ is a vertical line with x-intercept 3.

(c) An equation for the horizontal line through $(8, -2)$ is $y = -2$.

(d) The graph of the equation $y = -2$ is a horizontal line with y-intercept -2.

The lines are graphed in Figure 9.

Now Try Exercises 35, 37, 63, and 65 ■

General Equation of a Line

A **linear equation** in the variables x and y is an equation of the form

$$Ax + By + C = 0$$

where A, B, and C are constants and A and B are not both 0. An equation of a line is a linear equation:

- A nonvertical line has the equation $y = mx + b$ or $-mx + y - b = 0$, which is a linear equation with $A = -m$, $B = 1$, and $C = -b$.
- A vertical line has the equation $x = a$ or $x - a = 0$, which is a linear equation with $A = 1$, $B = 0$, and $C = -a$.

  Conversely, the graph of a linear equation is a line.
- If $B \neq 0$, the equation becomes

$$y = -\frac{A}{B}x - \frac{C}{B} \qquad \text{Divide by } B$$

  and this is the slope-intercept form of the equation of a line (with $m = -A/B$ and $b = -C/B$).
- If $B = 0$, the equation becomes

$$Ax + C = 0 \qquad \text{Set } B = 0$$

  or $x = -C/A$, which represents a vertical line.

We have proved the following.

GENERAL EQUATION OF A LINE

The graph of every **linear equation**

$$Ax + By + C = 0 \qquad (A, B \text{ not both zero})$$

is a line. Conversely, every line is the graph of a linear equation.

EXAMPLE 6 ■ Graphing a Linear Equation

Sketch the graph of the equation $2x - 3y - 12 = 0$.

SOLUTION 1 Since the equation is linear, its graph is a line. To draw the graph, it is enough to find any two points on the line. The intercepts are the easiest points to find.

x-intercept: Substitute $y = 0$, to get $2x - 12 = 0$, so $x = 6$

y-intercept: Substitute $x = 0$, to get $-3y - 12 = 0$, so $y = -4$

With these points we can sketch the graph in Figure 10.

SOLUTION 2 We write the equation in slope-intercept form.

$$\begin{aligned} 2x - 3y - 12 &= 0 & \\ 2x - 3y &= 12 & \text{Add 12} \\ -3y &= -2x + 12 & \text{Subtract } 2x \\ y &= \tfrac{2}{3}x - 4 & \text{Divide by } -3 \end{aligned}$$

This equation is in the form $y = mx + b$, so the slope is $m = \frac{2}{3}$ and the y-intercept is $b = -4$. To sketch the graph, we plot the y-intercept and then move 3 units to the right and 2 units up as shown in Figure 11.

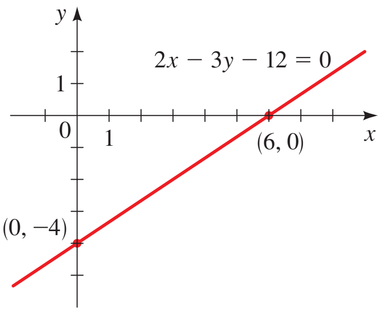


FIGURE 10

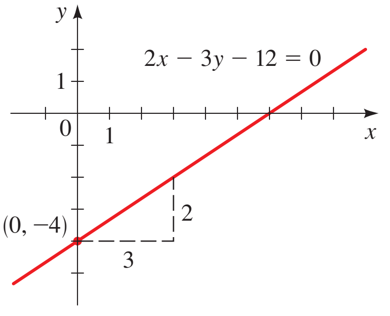


FIGURE 11

Now Try Exercise 67 ■

■ Parallel and Perpendicular Lines

Since slope measures the steepness of a line, it seems reasonable that parallel lines should have the same slope. In fact, we can prove this.

PARALLEL LINES

Two nonvertical lines are parallel if and only if they have the same slope.

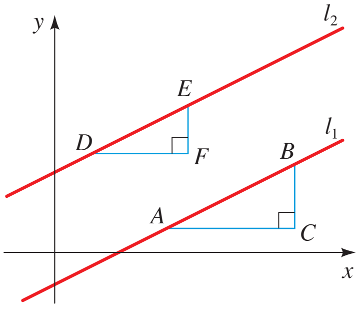


FIGURE 12

Proof Let the lines l_1 and l_2 in Figure 12 have slopes m_1 and m_2. If the lines are parallel, then the right triangles ABC and DEF are similar, so

$$m_1 = \frac{d(B, C)}{d(A, C)} = \frac{d(E, F)}{d(D, F)} = m_2$$

Conversely, if the slopes are equal, then the triangles will be similar, so $\angle BAC = \angle EDF$ and the lines are parallel. ■

EXAMPLE 7 ■ Finding an Equation of a Line Parallel to a Given Line

Find an equation of the line through the point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.

SOLUTION First we write the equation of the given line in slope-intercept form.

$$4x + 6y + 5 = 0$$

$$6y = -4x - 5 \qquad \text{Subtract } 4x + 5$$

$$y = -\tfrac{2}{3}x - \tfrac{5}{6} \qquad \text{Divide by 6}$$

So the line has slope $m = -\frac{2}{3}$. Since the required line is parallel to the given line, it also has slope $m = -\frac{2}{3}$. From the point-slope form of the equation of a line we get

$$y - 2 = -\tfrac{2}{3}(x - 5) \qquad \text{Slope } m = -\tfrac{2}{3}, \text{ point } (5, 2)$$

$$3y - 6 = -2x + 10 \qquad \text{Multiply by 3}$$

$$2x + 3y - 16 = 0 \qquad \text{Rearrange}$$

Thus an equation of the required line is $2x + 3y - 16 = 0$.

Now Try Exercise 43 ■

The condition for perpendicular lines is not as obvious as that for parallel lines.

PERPENDICULAR LINES

Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$, that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

Also, a horizontal line (slope 0) is perpendicular to a vertical line (no slope).

Proof In Figure 13 we show two lines intersecting at the origin. (If the lines intersect at some other point, we consider lines parallel to these that intersect at the origin. These lines have the same slopes as the original lines.)

If the lines l_1 and l_2 have slopes m_1 and m_2, then their equations are $y = m_1 x$ and $y = m_2 x$. Notice that $A(1, m_1)$ lies on l_1 and $B(1, m_2)$ lies on l_2. By the Pythagorean Theorem and its converse (see Appendix A) $OA \perp OB$ if and only if

$$[d(O, A)]^2 + [d(O, B)]^2 = [d(A, B)]^2$$

By the Distance Formula this becomes

$$(1^2 + m_1^2) + (1^2 + m_2^2) = (1 - 1)^2 + (m_2 - m_1)^2$$

$$2 + m_1^2 + m_2^2 = m_2^2 - 2m_1 m_2 + m_1^2$$

$$2 = -2m_1 m_2$$

$$m_1 m_2 = -1$$

■

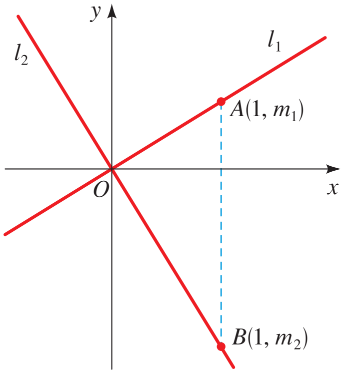


FIGURE 13

EXAMPLE 8 ■ Perpendicular Lines

Show that the points $P(3, 3)$, $Q(8, 17)$, and $R(11, 5)$ are the vertices of a right triangle.

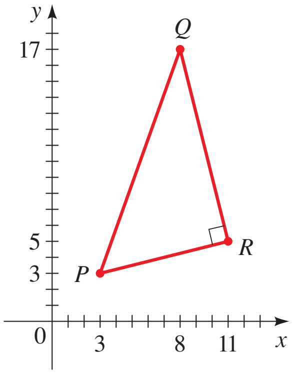


FIGURE 14

SOLUTION The slopes of the lines containing PR and QR are, respectively,

$$m_1 = \frac{5-3}{11-3} = \frac{1}{4} \qquad \text{and} \qquad m_2 = \frac{5-17}{11-8} = -4$$

Since $m_1 m_2 = -1$, these lines are perpendicular, so PQR is a right triangle. It is sketched in Figure 14.

Now Try Exercise 81 ■

EXAMPLE 9 ■ Finding an Equation of a Line Perpendicular to a Given Line

Find an equation of the line that is perpendicular to the line $4x + 6y + 5 = 0$ and passes through the origin.

SOLUTION In Example 7 we found that the slope of the line $4x + 6y + 5 = 0$ is $-\frac{2}{3}$. Thus the slope of a perpendicular line is the negative reciprocal, that is, $\frac{3}{2}$. Since the required line passes through $(0, 0)$, the point-slope form gives

$$y - 0 = \tfrac{3}{2}(x - 0) \qquad \text{Slope } m = \tfrac{3}{2}, \text{ point } (0, 0)$$

$$y = \tfrac{3}{2}x \qquad \text{Simplify}$$

Now Try Exercise 47 ■

EXAMPLE 10 ■ Graphing a Family of Lines

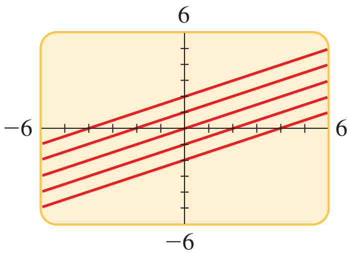


FIGURE 15 $y = 0.5x + b$

Use a graphing calculator to graph the family of lines

$$y = 0.5x + b$$

for $b = -2, -1, 0, 1, 2$. What property do the lines share?

SOLUTION We use a graphing calculator to graph the lines in the viewing rectangle $[-6, 6]$ by $[-6, 6]$. The graphs are shown in Figure 15. The lines all have the same slope, so they are parallel.

Now Try Exercise 53 ■

EXAMPLE 11 ■ Application: Interpreting Slope

A swimming pool is being filled with a hose. The water depth y (in feet) in the pool t hours after the hose is turned on is given by

$$y = 1.5t + 2$$

(a) Find the slope and y-intercept of the graph of this equation.

(b) What do the slope and y-intercept represent?

SOLUTION

(a) This is the equation of a line with slope 1.5 and y-intercept 2.

(b) The slope represents an increase of 1.5 ft. in water depth for every hour. The y-intercept indicates that the water depth was 2 ft. at the time the hose was turned on.

Now Try Exercise 87 ■

1.10 EXERCISES

CONCEPTS

1. We find the "steepness," or slope, of a line passing through two points by dividing the difference in the ____-coordinates of these points by the difference in the ____-coordinates. So the line passing through the points (0, 1) and (2, 5) has slope ________.

2. A line has the equation $y = 3x + 2$.

(a) This line has slope ________.

(b) Any line parallel to this line has slope ________.

(c) Any line perpendicular to this line has slope ________.

3. The point-slope form of the equation of the line with slope 3 passing through the point $(1, 2)$ is ________.

4. For the linear equation $2x + 3y - 12 = 0$, the x-intercept is ________ and the y-intercept is ________. The equation in slope-intercept form is $y =$ ________________. The slope of the graph of this equation is ________.

5. The slope of a horizontal line is ________. The equation of the horizontal line passing through (2, 3) is ________.

6. The slope of a vertical line is ________. The equation of the vertical line passing through (2, 3) is ________.

7. *Yes or No*? If *No*, give a reason.

(a) Is the graph of $y = -3$ a horizontal line?

(b) Is the graph of $x = -3$ a vertical line?

(c) Does a line perpendicular to a horizontal line have slope 0?

(d) Does a line perpendicular to a vertical line have slope 0?

8. Sketch a graph of the lines $y = -3$ and $x = -3$. Are the lines perpendicular?

SKILLS

9–16 ■ Slope Find the slope of the line through P and Q.

9. $P(-1, 2)$, $Q(0, 0)$

10. $P(0, 0)$, $Q(3, -1)$

11. $P(2, -2)$, $Q(7, -1)$

12. $P(-5, 1)$, $Q(3, -2)$

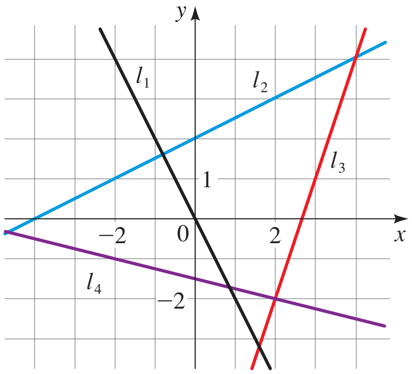
13. $P(5, 4)$, $Q(0, 4)$

14. $P(4, 3)$, $Q(1, -1)$

15. $P(10, -2)$, $Q(6, -5)$

16. $P(3, -2)$, $Q(6, -2)$

17. Slope Find the slopes of the lines l_1, l_2, l_3, and l_4 in the figure below.



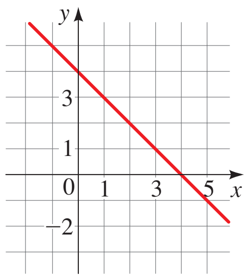
18. Slope

(a) Sketch lines through $(0, 0)$ with slopes $1, 0, \frac{1}{2}, 2$, and -1.

(b) Sketch lines through $(0, 0)$ with slopes $\frac{1}{3}, \frac{1}{2}, -\frac{1}{3}$, and 3.

19–22 ■ Equations of Lines Find an equation for the line whose graph is sketched.

19.



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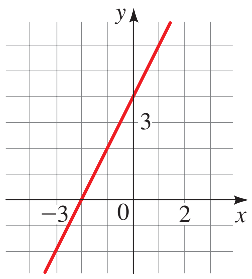

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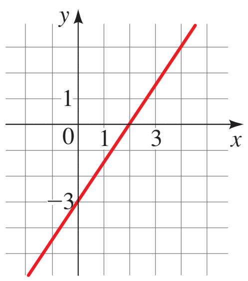

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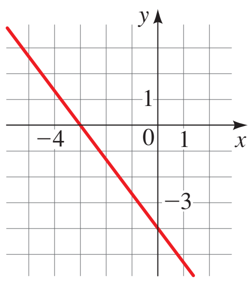


23–50 ■ Finding Equations of Lines Find an equation of the line that satisfies the given conditions.

23. Slope 3; y-intercept -2

24. Slope $\frac{2}{5}$; y-intercept 4

25. Through $(2, 3)$; slope 5

26. Through $(-2, 4)$; slope -1

27. Through $(1, 7)$; slope $\frac{2}{3}$

28. Through $(-3, -5)$; slope $-\frac{7}{2}$

29. Through $(2, 1)$ and $(1, 6)$

30. Through $(-1, -2)$ and $(4, 3)$

31. Through $(-2, 5)$ and $(-1, -3)$

32. Through $(1, 7)$ and $(4, 7)$

33. x-intercept 1; y-intercept -3

34. x-intercept -8; y-intercept 6

35. Through $(1, 3)$; slope 0

36. Through $(-1, 4)$; slope undefined

37. Through $(2, -1)$; slope undefined

38. Through $(5, 1)$; slope 0

39. Through $(1, 2)$; parallel to the line $y = 3x - 5$

40. Through $(-3, 2)$; perpendicular to the line $y = -\frac{1}{2}x + 7$

41. Through $(4, 5)$; parallel to the x-axis

42. Through $(4, 5)$; parallel to the y-axis

43. Through $(1, -6)$; parallel to the line $x + 2y = 6$

44. y-intercept 6; parallel to the line $2x + 3y + 4 = 0$

45. Through $(-1, 2)$; parallel to the line $x = 5$

46. Through $(2, 6)$; perpendicular to the line $y = 1$

47. Through $(-1, -2)$; perpendicular to the line $2x + 5y + 8 = 0$

48. Through $\left(\frac{1}{2}, -\frac{2}{3}\right)$; perpendicular to the line $4x - 8y = 1$

49. Through $(1, 7)$; parallel to the line passing through $(2, 5)$ and $(-2, 1)$

50. Through $(-2, -11)$; perpendicular to the line passing through $(1, 1)$ and $(5, -1)$

51. Finding Equations of Lines and Graphing

(a) Sketch the line with slope $\frac{3}{2}$ that passes through the point $(-2, 1)$.

(b) Find an equation for this line.

52. Finding Equations of Lines and Graphing

(a) Sketch the line with slope -2 that passes through the point $(4, -1)$.

(b) Find an equation for this line.

53–56 ■ Families of Lines Use a graphing device to graph the given family of lines in the same viewing rectangle. What do the lines have in common?

53. $y = -2x + b$ for $b = 0, \pm 1, \pm 3, \pm 6$

54. $y = mx - 3$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$

55. $y = m(x - 3)$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$

56. $y = 2 + m(x + 3)$ for $m = 0, \pm 0.5, \pm 1, \pm 2, \pm 6$

57–66 ■ Using Slopes and y-Intercepts to Graph Lines Find the slope and y-intercept of the line, and draw its graph.

57. $y = 3 - x$

58. $y = \frac{2}{3}x - 2$

59. $-2x + y = 7$

60. $2x - 5y = 0$

61. $4x + 5y = 10$

62. $3x - 4y = 12$

63. $y = 4$

64. $x = -5$

65. $x = 3$

66. $y = -2$

67–72 ■ Using x- and y-Intercepts to Graph Lines Find the x- and y-intercepts of the line, and draw its graph.

67. $5x + 2y - 10 = 0$

68. $6x - 7y - 42 = 0$

69. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$

70. $\frac{1}{3}x - \frac{1}{5}y - 2 = 0$

71. $y = 6x + 4$

72. $y = -4x - 10$

73–78 ■ Parallel and Perpendicular Lines The equations of two lines are given. Determine whether the lines are parallel, perpendicular, or neither.

73. $y = 2x + 3$; $2y - 4x - 5 = 0$

74. $y = \frac{1}{2}x + 4$; $2x + 4y = 1$

75. $-3x + 4y = 4$; $4x + 3y = 5$

76. $2x - 3y = 10$; $3y - 2x - 7 = 0$

77. $7x - 3y = 2$; $9y + 21x = 1$

78. $6y - 2x = 5$; $2y + 6x = 1$

SKILLS Plus

79–82 ■ Using Slopes Verify the given geometric property.

79. Use slopes to show that $A(1, 1)$, $B(7, 4)$, $C(5, 10)$, and $D(-1, 7)$ are vertices of a parallelogram.

80. Use slopes to show that $A(-3, -1)$, $B(3, 3)$, and $C(-9, 8)$ are vertices of a right triangle.

81. Use slopes to show that $A(1, 1)$, $B(11, 3)$, $C(10, 8)$, and $D(0, 6)$ are vertices of a rectangle.

82. Use slopes to determine whether the given points are collinear (lie on a line).

(a) $(1, 1), (3, 9), (6, 21)$

(b) $(-1, 3), (1, 7), (4, 15)$

83. Perpendicular Bisector Find an equation of the perpendicular bisector of the line segment joining the points $A(1, 4)$ and $B(7, -2)$.

84. Area of a Triangle Find the area of the triangle formed by the coordinate axes and the line

$$2y + 3x - 6 = 0$$

85. Two-Intercept Form

(a) Show that if the x- and y-intercepts of a line are nonzero numbers a and b, then the equation of the line can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

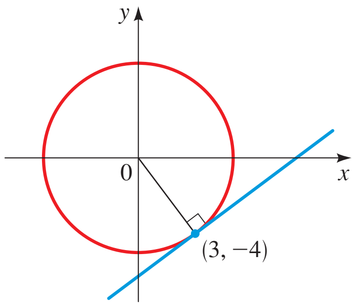
This is called the **two-intercept form** of the equation of a line.

(b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.

86. Tangent Line to a Circle

(a) Find an equation for the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$. (See the figure.)

(b) At what other point on the circle will a tangent line be parallel to the tangent line in part (a)?



APPLICATIONS

87. Global Warming Some scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature can be modeled by

$$T = 0.02t + 15.0$$

where T is temperature in °C and t is years since 1950.

(a) What do the slope and T-intercept represent?

(b) Use the equation to predict the average global surface temperature in 2050.

88. Drug Dosages If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation

$$c = 0.0417D(a + 1)$$

Suppose the dosage for an adult is 200 mg.

(a) Find the slope. What does it represent?

(b) What is the dosage for a newborn?

89. Flea Market The manager of a weekend flea market knows from past experience that if she charges x dollars for a rental space at the flea market, then the number y of spaces she can rent is given by the equation $y = 200 - 4x$.

(a) Sketch a graph of this linear equation. (Remember that the rental charge per space and the number of spaces rented must both be nonnegative quantities.)

(b) What do the slope, the y-intercept, and the x-intercept of the graph represent?

90. Production Cost A small-appliance manufacturer finds that if he produces x toaster ovens in a month, his production cost is given by the equation

$$y = 6x + 3000$$

(where y is measured in dollars).

(a) Sketch a graph of this linear equation.

(b) What do the slope and y-intercept of the graph represent?

91. Temperature Scales The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the equation $F = \frac{9}{5}C + 32$.

(a) Complete the table to compare the two scales at the given values.

(b) Find the temperature at which the scales agree. [*Hint:* Suppose that a is the temperature at which the scales agree. Set $F = a$ and $C = a$. Then solve for a.]

C	F
−30°	
−20°	
−10°	
0°	
	50°
	68°
	86°

92. Crickets and Temperature Biologists have observed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 120 chirps per minute at 70°F and 168 chirps per minute at 80°F.

(a) Find the linear equation that relates the temperature t and the number of chirps per minute n.

(b) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

93. Depreciation A small business buys a computer for \$4000. After 4 years the value of the computer is expected to be \$200. For accounting purposes the business uses *linear depreciation* to assess the value of the computer at a given time. This means that if V is the value of the computer at time t, then a linear equation is used to relate V and t.

(a) Find a linear equation that relates V and t.

(b) Sketch a graph of this linear equation.

(c) What do the slope and V-intercept of the graph represent?

(d) Find the depreciated value of the computer 3 years from the date of purchase.

94. Pressure and Depth At the surface of the ocean the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Find an equation for the relationship between pressure and depth below the ocean surface.

(b) Sketch a graph of this linear equation.

(c) What do the slope and y-intercept of the graph represent?

(d) At what depth is the pressure 100 lb/in^2?

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

95. DISCUSS: What Does the Slope Mean? Suppose that the graph of the outdoor temperature over a certain period of time is a line. How is the weather changing if the slope of the line is positive? If it is negative? If it is zero?

96. DISCUSS: Collinear Points Suppose that you are given the coordinates of three points in the plane and you want to see whether they lie on the same line. How can you do this using slopes? Using the Distance Formula? Can you think of another method?