MATH 195

§ 1.3 ALGEBRAIC EXPRESSIONS

13, 19, 21, 31, 34, 41, 49, 51, 52
63, 65, 69, 71, 75, 77, 125

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13. 4-TERM-POLYNOMIAL, TERMS: $x, -x^2, x^3, -x^4$, 4TH DEGREE

19. $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2)$

$5x^3 + 4x^2 - 3x - x^2 - 7x - 2$

$\boxed{5x^3 + 3x^2 - 10x - 2}$

21. $8(2x + 5) - 7(x - 9)$

$16x + 40 - 7x + 63 = \boxed{9x + 103}$

31. $(5x + 1)^2 = (5x)^2 + 2(5x)(1) + (1)^2$

$= \boxed{25x^2 + 10x + 1}$

34. $(x - 3y)^2 = (x)^2 - 2(x)(3y) + (3y)^2 = \boxed{x^2 - 6xy + 9y^2}$

or $= (x)^2 + 2(x)(-3y) + (-3y)^2 =$ ↷

41. $(\sqrt{x} + 2)(\sqrt{x} - 2) = (\sqrt{x})^2 - (2)^2 = \boxed{x - 4}$

49. $(2x - 5)(x^2 - x + 1) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5$

$= \boxed{2x^3 - 7x^2 + 7x - 5}$

51. $\sqrt{x}(x-\sqrt{x}) = x\sqrt{x} - \sqrt{x}^2 = \boxed{x\sqrt{x} - x}$

52. $x^{3/2}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = x^{3/2}\left(x^{1/2} - x^{-1/2}\right)$

$= x^{\frac{3}{2}+\frac{1}{2}} - x^{\frac{3}{2}-\frac{1}{2}} = \boxed{x^2 - x}$

63. $-2x^3 + x = \boxed{x(-2x^2 + 1)}$

65. $y(y-6) + 9(y-6) = \boxed{(y+9)(y-6)}$

69. $x^2 + 8x + 7$ $\quad \square \cdot \square = 7$, $\square + \square = 8$ $\quad$ (7 AND 1)

$\boxed{(x+7)(x+1)}$

71. $8x^2 - 14x - 15$

GUESS & CHECK THE 8 CAN ONLY BE FACTORED 2 WAYS: $8 = 8\cdot 1 = 4\cdot 2$

AND 15 CAN ONLY BE FACTORED 2 WAYS: $15 = 15\cdot 1 = 5\cdot 3$

SO THE ONLY POSSIBILITIES ARE:

$(8x \pm 15)(x \pm 1)$, $(4x \pm 15)(2x \pm 1)$

$(8x \pm 1)(x \pm 15)$, $(4x \pm 1)(2x \pm 15)$

$(8x \pm 5)(x \pm 3)$, $(4x \pm 5)(2x \pm 3)$

$(8x \pm 3)(x \pm 5)$, $(4x \pm 3)(2x \pm 5)$

AND SINCE THE PRODUCT IS -15 (NEGATIVE!) WE NEED ONE + AND ONE $-$.

THE ONLY ONE THAT WORKS IS $\boxed{(4x+3)(2x-5)}$

ALTERNATIVELY, WE CAN USE THE METHOD OF SPLITTING THE MIDDLE TERM:

$8x^2 - 14x - 15$

$(8)(-15) = -120$

$-20 \cdot 6 = -120$

$-20 + 6 = -14$

$8x^2 - 14x - 15 = \underline{8x^2 - 20x} + \underline{6x - 15}$ (FACTOR BY GROUPING)

$4x(2x-5) + 3(2x-5)$

$\boxed{(4x+3)(2x-5)}$

75. $(3x+2)^2 + 8(3x+2) + 12$

LET $w = 3x+2 \rightsquigarrow w^2 + 8w + 12$

$= (w+6)(w+2)$

$\rightsquigarrow (3x+2+6)(3x+2+2)$

$= \boxed{(3x+8)(3x+4)}$

77. $9a^2 - 16 = (3a)^2 - (4)^2 = \boxed{(3a+4)(3a-4)}$

125. $(a^2+1)^2 - 7(a^2+1) + 10$, LET $w = a^2+1$

$\rightsquigarrow w^2 - 7w + 10 = (w-5)(w-2)$

$\rightsquigarrow (a^2+1-5)(a^2+1-2) = (a^2-4)(a^2-1)$ (DIFFERENCES OF SQUARES!)

$= \boxed{(a+2)(a-2)(a+1)(a-1)}$