89. Height of Grass A home owner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period beginning on a Sunday.

90. Temperature Change You place a frozen pie in an oven and bake it for an hour. Then you take the pie out and let it cool before eating it. Sketch a rough graph of the temperature of the pie as a function of time.

91. Daily Temperature Change Temperature readings T (in °F) were recorded every 2 hours from midnight to noon in Atlanta, Georgia, on March 18, 2014. The time t was measured in hours from midnight. Sketch a rough graph of T as a function of t.

t	0	2	4	6	8	10	12
T	58	57	53	50	51	57	61

92. Population Growth The population P (in thousands) of San Jose, California, from 1980 to 2010 is shown in the table. (Midyear estimates are given.) Draw a rough graph of P as a function of time t.

t	1980	1985	1990	1995	2000	2005	2010
P	629	714	782	825	895	901	946

Source: U.S. Census Bureau

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

93. DISCUSS: Examples of Functions At the beginning of this section we discussed three examples of everyday, ordinary functions: Height is a function of age, temperature is a function of date, and postage cost is a function of weight. Give three other examples of functions from everyday life.

94. DISCUSS: Four Ways to Represent a Function In the box on page 154 we represented four different functions verbally, algebraically, visually, and numerically. Think of a function that can be represented in all four ways, and give the four representations.

95. DISCUSS: Piecewise Defined Functions In Exercises 85–88 we worked with real-world situations modeled by piecewise defined functions. Find other examples of real-world situations that can be modeled by piecewise defined functions, and express the models in function notation.

2.2 GRAPHS OF FUNCTIONS

■ Graphing Functions by Plotting Points ■ Graphing Functions with a Graphing Calculator ■ Graphing Piecewise Defined Functions ■ The Vertical Line Test: Which Graphs Represent Functions? ■ Which Equations Represent Functions?

The most important way to visualize a function is through its graph. In this section we investigate in more detail the concept of graphing functions.

■ Graphing Functions by Plotting Points

To graph a function f, we plot the points $(x, f(x))$ in a coordinate plane. In other words, we plot the points (x, y) whose x-coordinate is an input and whose y-coordinate is the corresponding output of the function.

THE GRAPH OF A FUNCTION

If f is a function with domain A, then the **graph** of f is the set of ordered pairs

$$\{(x, f(x)) \mid x \in A\}$$

plotted in a coordinate plane. In other words, the graph of f is the set of all points (x, y) such that $y = f(x)$; that is, the graph of f is the graph of the equation $y = f(x)$.

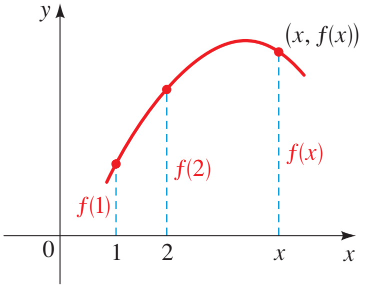


FIGURE 1 The height of the graph above the point x is the value of $f(x)$.

The graph of a function f gives a picture of the behavior or "life history" of the function. We can read the value of $f(x)$ from the graph as being the height of the graph above the point x (see Figure 1).

A function f of the form $f(x) = mx + b$ is called a **linear function** because its graph is the graph of the equation $y = mx + b$, which represents a line with slope m and y-intercept b. A special case of a linear function occurs when the slope is $m = 0$. The function $f(x) = b$, where b is a given number, is called a **constant function** because all its values are the same number, namely, b. Its graph is the horizontal line $y = b$. Figure 2 shows the graphs of the constant function $f(x) = 3$ and the linear function $f(x) = 2x + 1$.

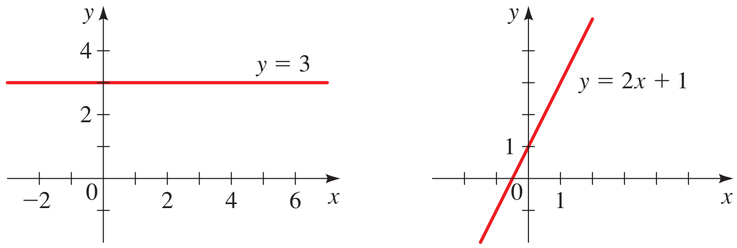


FIGURE 2 The constant function $f(x) = 3$ The linear function $f(x) = 2x + 1$

Functions of the form $f(x) = x^n$ are called **power functions**, and functions of the form $f(x) = x^{1/n}$ are called **root functions**. In the next example we graph two power functions and a root function.

EXAMPLE 1 ■ Graphing Functions by Plotting Points

Sketch graphs of the following functions.

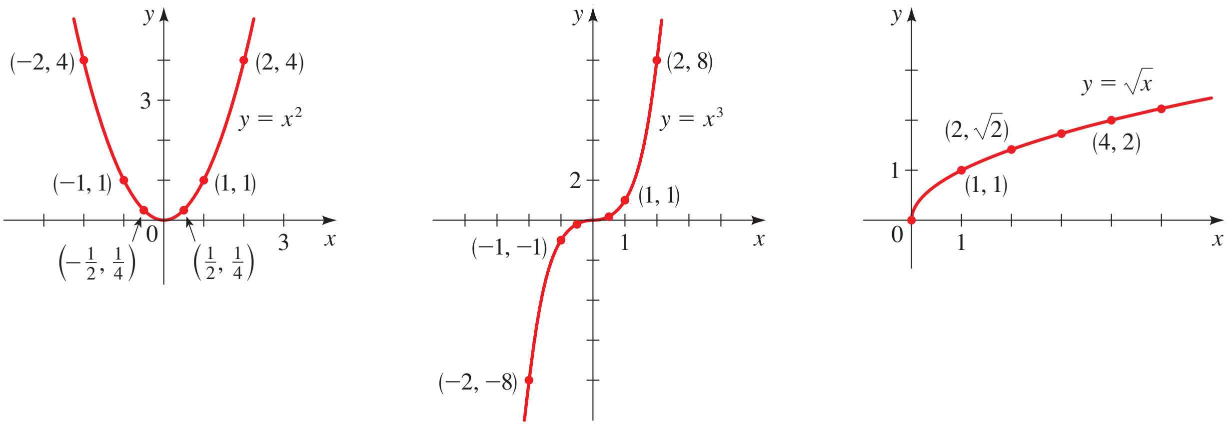
(a) $f(x) = x^2$ **(b)** $g(x) = x^3$ **(c)** $h(x) = \sqrt{x}$

SOLUTION We first make a table of values. Then we plot the points given by the table and join them by a smooth curve to obtain the graph. The graphs are sketched in Figure 3.

x	$f(x) = x^2$
0	0
$\pm\frac{1}{2}$	$\frac{1}{4}$
± 1	1
± 2	4
± 3	9

x	$g(x) = x^3$
0	0
$\frac{1}{2}$	$\frac{1}{8}$
1	1
2	8
$-\frac{1}{2}$	$-\frac{1}{8}$
-1	-1
-2	-8

x	$h(x) = \sqrt{x}$
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$



(a) $f(x) = x^2$ (b) $g(x) = x^3$ (c) $h(x) = \sqrt{x}$

FIGURE 3

Now Try Exercises 9, 15, and 19 ■

Graphing Functions with a Graphing Calculator

See Appendix C, *Graphing with a Graphing Calculator,* for general guidelines on using a graphing calculator. See Appendix D, *Using the TI-83/84 Graphing Calculator,* for specific instructions. Go to **www.stewartmath.com**.

A convenient way to graph a function is to use a graphing calculator. To graph the function f, we use a calculator to graph the equation $y = f(x)$.

EXAMPLE 2 ■ Graphing a Function with a Graphing Calculator

Use a graphing calculator to graph the function $f(x) = x^3 - 8x^2$ in an appropriate viewing rectangle.

SOLUTION To graph the function $f(x) = x^3 - 8x^2$, we must graph the equation $y = x^3 - 8x^2$. On the TI-83 graphing calculator the default viewing rectangle gives the graph in Figure 4(a). But this graph appears to spill over the top and bottom of the screen. We need to expand the vertical axis to get a better representation of the graph. The viewing rectangle $[-4, 10]$ by $[-100, 100]$ gives a more complete picture of the graph, as shown in Figure 4(b).

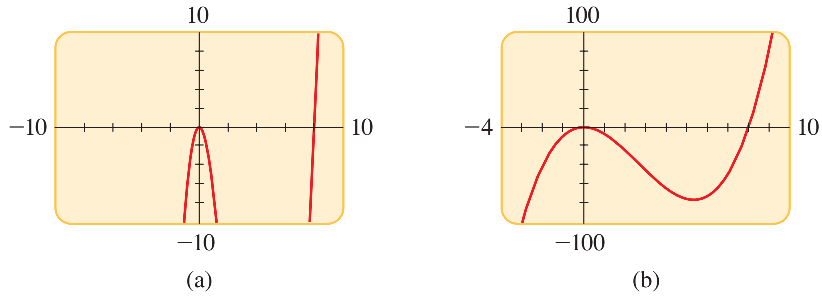


FIGURE 4 Graphing the function $f(x) = x^3 - 8x^2$

Now Try Exercise 29 ■

EXAMPLE 3 ■ A Family of Power Functions

(a) Graph the functions $f(x) = x^n$ for $n = 2, 4,$ and 6 in the viewing rectangle $[-2, 2]$ by $[-1, 3]$.

(b) Graph the functions $f(x) = x^n$ for $n = 1, 3,$ and 5 in the viewing rectangle $[-2, 2]$ by $[-2, 2]$.

(c) What conclusions can you draw from these graphs?

SOLUTION To graph the function $f(x) = x^n$, we graph the equation $y = x^n$. The graphs for parts (a) and (b) are shown in Figure 5.

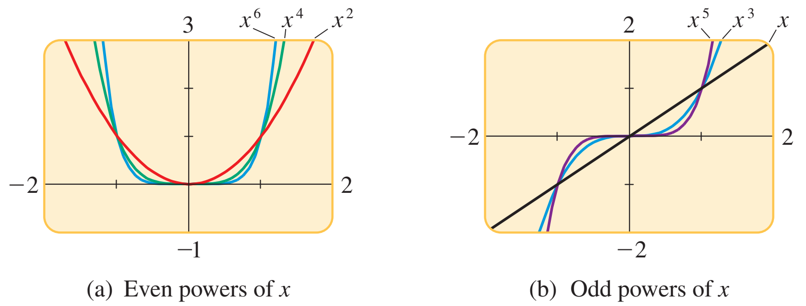


FIGURE 5 A family of power functions: $f(x) = x^n$

(c) We see that the general shape of the graph of $f(x) = x^n$ depends on whether n is even or odd.

If n is even, the graph of $f(x) = x^n$ is similar to the parabola $y = x^2$.
If n is odd, the graph of $f(x) = x^n$ is similar to that of $y = x^3$.

Now Try Exercise 69 ■

Notice from Figure 5 that as n increases, the graph of $y = x^n$ becomes flatter near 0 and steeper when $x > 1$. When $0 < x < 1$, the lower powers of x are the "bigger" functions. But when $x > 1$, the higher powers of x are the dominant functions.

Graphing Piecewise Defined Functions

A piecewise defined function is defined by different formulas on different parts of its domain. As you might expect, the graph of such a function consists of separate pieces.

EXAMPLE 4 ■ Graph of a Piecewise Defined Function

Sketch the graph of the function

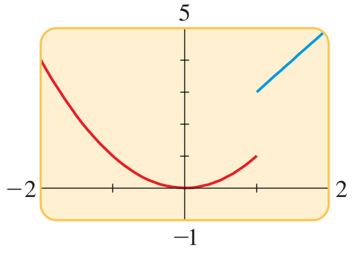
$$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

SOLUTION If $x \le 1$, then $f(x) = x^2$, so the part of the graph to the left of $x = 1$ coincides with the graph of $y = x^2$, which we sketched in Figure 3. If $x > 1$, then $f(x) = 2x + 1$, so the part of the graph to the right of $x = 1$ coincides with the line $y = 2x + 1$, which we graphed in Figure 2. This enables us to sketch the graph in Figure 6.

The solid dot at $(1, 1)$ indicates that this point is included in the graph; the open dot at $(1, 3)$ indicates that this point is excluded from the graph.

On many graphing calculators the graph in Figure 6 can be produced by using the logical functions in the calculator. For example, on the TI-83 the following equation gives the required graph:

```
Y1=(X≤1)X²+(X>1)(2X+1)
```



(To avoid the extraneous vertical line between the two parts of the graph, put the calculator in `Dot` mode.)

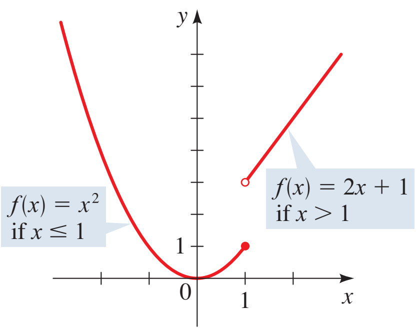


FIGURE 6

$$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

Now Try Exercise 35 ■

EXAMPLE 5 ■ Graph of the Absolute Value Function

Sketch a graph of the absolute value function $f(x) = |x|$.

SOLUTION Recall that

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

Using the same method as in Example 4, we note that the graph of f coincides with the line $y = x$ to the right of the y-axis and coincides with the line $y = -x$ to the left of the y-axis (see Figure 7).

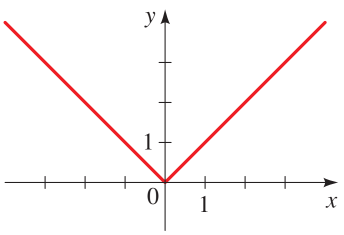


FIGURE 7 Graph of $f(x) = |x|$

Now Try Exercise 23 ■

The **greatest integer function** is defined by

$$[\![x]\!] = \text{greatest integer less than or equal to } x$$

For example, $[\![2]\!] = 2$, $[\![2.3]\!] = 2$, $[\![1.999]\!] = 1$, $[\![0.002]\!] = 0$, $[\![-3.5]\!] = -4$, and $[\![-0.5]\!] = -1$.

EXAMPLE 6 ■ Graph of the Greatest Integer Function

Sketch a graph of $f(x) = [\![x]\!]$.

SOLUTION The table shows the values of f for some values of x. Note that $f(x)$ is constant between consecutive integers, so the graph between integers is a horizontal line segment, as shown in Figure 8.

x	$[\![x]\!]$
⋮	⋮
$-2 \le x < -1$	-2
$-1 \le x < 0$	-1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2
⋮	⋮

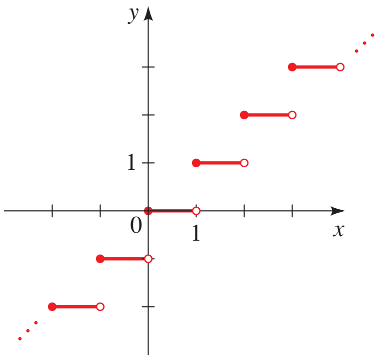


FIGURE 8 The greatest integer function, $y = [\![x]\!]$

The greatest integer function is an example of a **step function**. The next example gives a real-world example of a step function.

EXAMPLE 7 ■ The Cost Function for a Global Data Plan

A global data plan costs \$25 a month for the first 100 megabytes and \$20 for each additional 100 megabytes (or portion thereof). Draw a graph of the cost C (in dollars) as a function of the number of megabytes x used per month.

SOLUTION Let $C(x)$ be the cost of using x megabytes of data in a month. Since $x \ge 0$, the domain of the function is $[0, \infty)$. From the given information we have

$$\begin{aligned}
C(x) &= 25 && \text{if } 0 < x \le 100\\
C(x) &= 25 + 20 = 45 && \text{if } 100 < x \le 200\\
C(x) &= 25 + 2(20) = 65 && \text{if } 200 < x \le 300\\
C(x) &= 25 + 3(20) = 85 && \text{if } 300 < x \le 400\\
&\vdots && \vdots
\end{aligned}$$

The graph is shown in Figure 9.

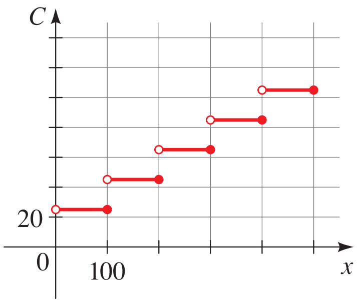


FIGURE 9 Cost of data usage

Now Try Exercise 83

DISCOVERY PROJECT

Relations and Functions

Many real-world relationships are functions, but many are not. For example, the rule that assigns to each student his or her school ID number is a function. But what about the rule that assigns to each date those persons born in Chicago on that date? Do you see why this "relation" is not a function? A set of ordered pairs is called a *relation*. In this project we explore the question of which relations are functions. You can find the project at **www.stewartmath.com**.

A function is called **continuous** if its graph has no "breaks" or "holes." The functions in Examples 1, 2, 3, and 5 are continuous; the functions in Examples 4, 6, and 7 are not continuous.

■ The Vertical Line Test: Which Graphs Represent Functions?

The graph of a function is a curve in the xy-plane. But the question arises: Which curves in the xy-plane are graphs of functions? This is answered by the following test.

THE VERTICAL LINE TEST

A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once.

We can see from Figure 10 why the Vertical Line Test is true. If each vertical line $x = a$ intersects a curve only once at (a, b), then exactly one functional value is defined by $f(a) = b$. But if a line $x = a$ intersects the curve twice, at (a, b) and at (a, c), then the curve cannot represent a function because a function cannot assign two different values to a.

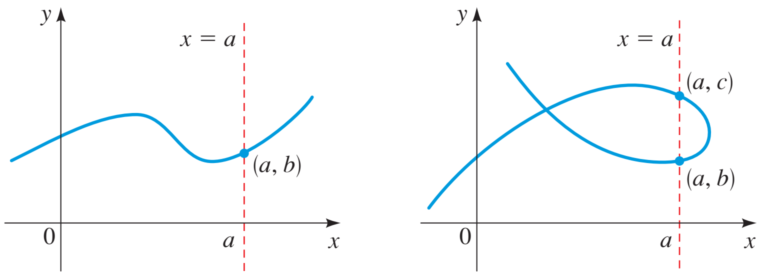


FIGURE 10 Vertical Line Test

Graph of a function   Not a graph of a function

EXAMPLE 8 ■ Using the Vertical Line Test

Using the Vertical Line Test, we see that the curves in parts (b) and (c) of Figure 11 represent functions, whereas those in parts (a) and (d) do not.

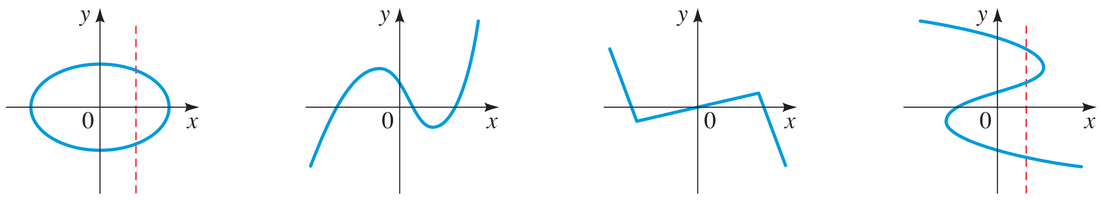


FIGURE 11

(a) (b) (c) (d)

Now Try Exercise 51 ■

■ Which Equations Represent Functions?

Any equation in the variables x and y defines a relationship between these variables. For example, the equation

$$y - x^2 = 0$$

BBVA/NOTIMEX/Newscom

DONALD KNUTH was born in Milwaukee in 1938 and is Professor Emeritus of Computer Science at Stanford University. When Knuth was a high school student, he became fascinated with graphs of functions and laboriously drew many hundreds of them because he wanted to see the behavior of a great variety of functions. (Today, of course, it is far easier to use computers and graphing calculators to do this.) While still a graduate student at Caltech, he started writing a monumental series of books entitled *The Art of Computer Programming*.

Knuth is famous for his invention of TEX, a system of computer-assisted typesetting. This system was used in the preparation of the manuscript for this textbook.

Knuth has received numerous honors, among them election as an associate of the French Academy of Sciences, and as a Fellow of the Royal Society. President Carter awarded him the National Medal of Science in 1979.

defines a relationship between y and x. Does this equation define y as a *function* of x? To find out, we solve for y and get

$$y = x^2 \qquad \text{Equation form}$$

We see that the equation defines a rule, or function, that gives one value of y for each value of x. We can express this rule in function notation as

$$f(x) = x^2 \qquad \text{Function form}$$

But not every equation defines y as a function of x, as the following example shows.

EXAMPLE 9 ■ Equations That Define Functions

Does the equation define y as a function of x?

(a) $y - x^2 = 2$ **(b)** $x^2 + y^2 = 4$

SOLUTION

(a) Solving for y in terms of x gives

$$y - x^2 = 2$$
$$y = x^2 + 2 \qquad \text{Add } x^2$$

The last equation is a rule that gives one value of y for each value of x, so it defines y as a function of x. We can write the function as $f(x) = x^2 + 2$.

(b) We try to solve for y in terms of x.

$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2 \qquad \text{Subtract } x^2$$
$$y = \pm\sqrt{4 - x^2} \qquad \text{Take square roots}$$

The last equation gives two values of y for a given value of x. Thus the equation does not define y as a function of x.

Now Try Exercises 57 and 61 ■

The graphs of the equations in Example 9 are shown in Figure 12. The Vertical Line Test shows graphically that the equation in Example 9(a) defines a function but the equation in Example 9(b) does not.

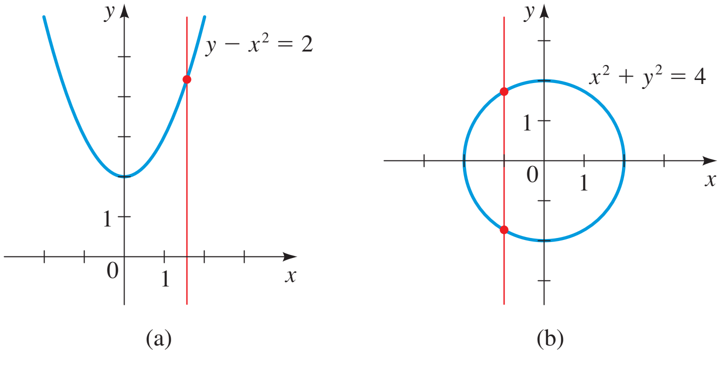


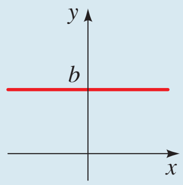
FIGURE 12

The following box shows the graphs of some functions that you will see frequently in this book.

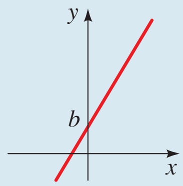
SOME FUNCTIONS AND THEIR GRAPHS

Linear functions

$f(x) = mx + b$



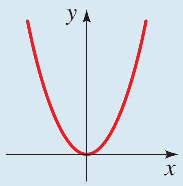
$f(x) = b$



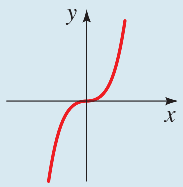
$f(x) = mx + b$

Power functions

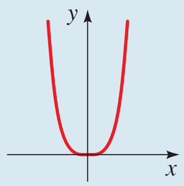
$f(x) = x^n$



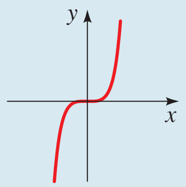
$f(x) = x^2$



$f(x) = x^3$



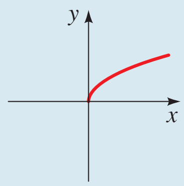
$f(x) = x^4$



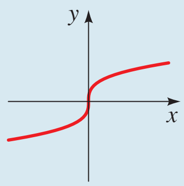
$f(x) = x^5$

Root functions

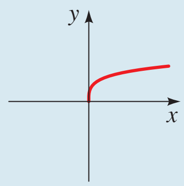
$f(x) = \sqrt[n]{x}$



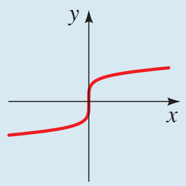
$f(x) = \sqrt{x}$



$f(x) = \sqrt[3]{x}$



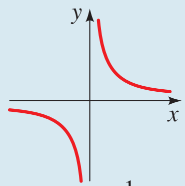
$f(x) = \sqrt[4]{x}$



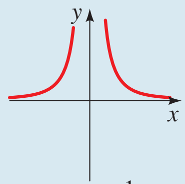
$f(x) = \sqrt[5]{x}$

Reciprocal functions

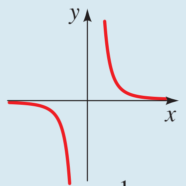
$f(x) = \dfrac{1}{x^n}$



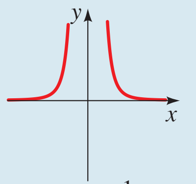
$f(x) = \dfrac{1}{x}$



$f(x) = \dfrac{1}{x^2}$



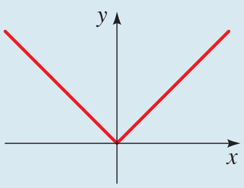
$f(x) = \dfrac{1}{x^3}$



$f(x) = \dfrac{1}{x^4}$

Absolute value function

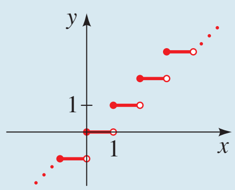
$f(x) = |x|$



$f(x) = |x|$

Greatest integer function

$f(x) = [\![x]\!]$



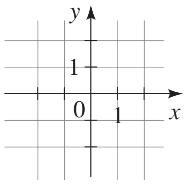
$f(x) = [\![x]\!]$

2.2 EXERCISES

CONCEPTS

1. To graph the function f, we plot the points $(x,$ ________ $)$ in a coordinate plane. To graph $f(x) = x^2 - 2$, we plot the points $(x,$ ________ $)$. So the point $(3,$ ________ $)$ is on the graph of f. The height of the graph of f above the x-axis when $x = 3$ is ________. Complete the table, and sketch a graph of f.

x	$f(x)$	(x, y)
-2		
-1		
0		
1		
2		



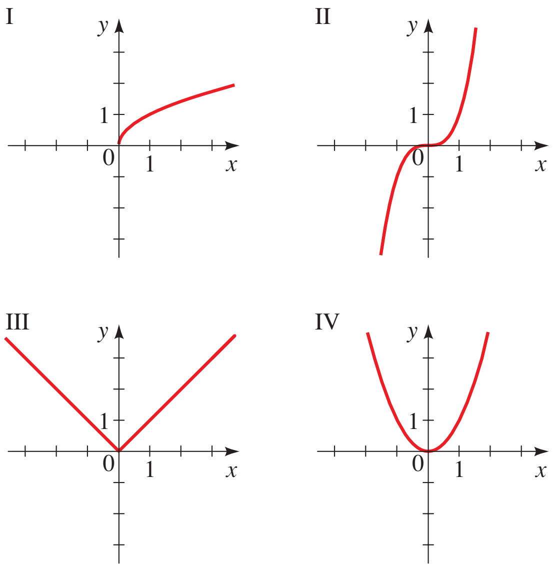
2. If $f(4) = 10$ then the point $(4, \_\_\_)$ is on the graph of f.

3. If the point $(3, 7)$ is on the graph of f, then $f(3) =$ ________.

4. Match the function with its graph.

(a) $f(x) = x^2$ **(b)** $f(x) = x^3$

(c) $f(x) = \sqrt{x}$ **(d)** $f(x) = |x|$



SKILLS

5–28 ■ Graphing Functions Sketch a graph of the function by first making a table of values.

5. $f(x) = x + 2$ **6.** $f(x) = 4 - 2x$

7. $f(x) = -x + 3, \quad -3 \le x \le 3$

8. $f(x) = \dfrac{x - 3}{2}, \quad 0 \le x \le 5$

9. $f(x) = -x^2$ **10.** $f(x) = x^2 - 4$

11. $g(x) = -(x + 1)^2$ **12.** $g(x) = x^2 + 2x + 1$

13. $r(x) = 3x^4$ **14.** $r(x) = 1 - x^4$

15. $g(x) = x^3 - 8$ **16.** $g(x) = (x - 1)^3$

17. $k(x) = \sqrt[3]{-x}$ **18.** $k(x) = -\sqrt[3]{x}$

19. $f(x) = 1 + \sqrt{x}$ **20.** $f(x) = \sqrt{x - 2}$

21. $C(t) = \dfrac{1}{t^2}$ **22.** $C(t) = -\dfrac{1}{t + 1}$

23. $H(x) = |2x|$ **24.** $H(x) = |x + 1|$

25. $G(x) = |x| + x$ **26.** $G(x) = |x| - x$

27. $f(x) = |2x - 2|$ **28.** $f(x) = \dfrac{x}{|x|}$

29–32 ■ Graphing Functions Graph the function in each of the given viewing rectangles, and select the one that produces the most appropriate graph of the function.

29. $f(x) = 8x - x^2$

(a) $[-5, 5]$ by $[-5, 5]$
(b) $[-10, 10]$ by $[-10, 10]$
(c) $[-2, 10]$ by $[-5, 20]$
(d) $[-10, 10]$ by $[-100, 100]$

30. $g(x) = x^2 - x - 20$

(a) $[-2, 2]$ by $[-5, 5]$
(b) $[-10, 10]$ by $[-10, 10]$
(c) $[-7, 7]$ by $[-25, 20]$
(d) $[-10, 10]$ by $[-100, 100]$

31. $h(x) = x^3 - 5x - 4$

(a) $[-2, 2]$ by $[-2, 2]$
(b) $[-3, 3]$ by $[-10, 10]$
(c) $[-3, 3]$ by $[-10, 5]$
(d) $[-10, 10]$ by $[-10, 10]$

32. $k(x) = \frac{1}{32}x^4 - x^2 + 2$

(a) $[-1, 1]$ by $[-1, 1]$
(b) $[-2, 2]$ by $[-2, 2]$
(c) $[-5, 5]$ by $[-5, 5]$
(d) $[-10, 10]$ by $[-10, 10]$

33–46 ■ Graphing Piecewise Defined Functions Sketch a graph of the piecewise defined function.

33. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$

34. $f(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x + 1 & \text{if } x > 1 \end{cases}$

35. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \ge 2 \end{cases}$

36. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \ge -2 \end{cases}$

37. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

38. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \ge -1 \end{cases}$

39. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$

40. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$

41. $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

42. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$

43. $f(x) = \begin{cases} 0 & \text{if } |x| \le 2 \\ 3 & \text{if } |x| > 2 \end{cases}$

44. $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

45. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x + 6 & \text{if } x > 2 \end{cases}$

46. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ 9 - x^2 & \text{if } 0 < x \le 3 \\ x - 3 & \text{if } x > 3 \end{cases}$

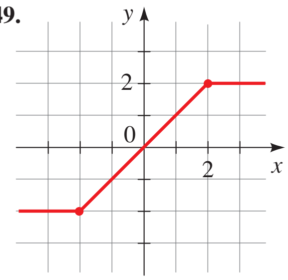
47–48 ■ Graphing Piecewise Defined Functions Use a graphing device to draw a graph of the piecewise defined function. (See the margin note on page 162.)

47. $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

48. $f(x) = \begin{cases} 2x - x^2 & \text{if } x > 1 \\ (x - 1)^3 & \text{if } x \le 1 \end{cases}$

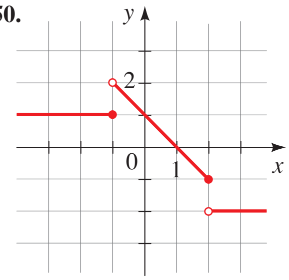
49–50 ■ Finding Piecewise Defined Functions A graph of a piecewise defined function is given. Find a formula for the function in the indicated form.

49.



$f(x) = \begin{cases} \quad & \text{if } x < -2 \\ \quad & \text{if } -2 \le x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$

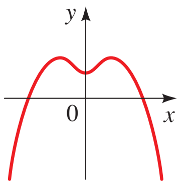
50.



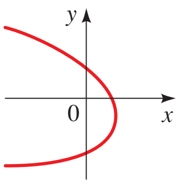
$f(x) = \begin{cases} \quad & \text{if } x \le -1 \\ \quad & \text{if } -1 < x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$

51–52 ■ Vertical Line Test Use the Vertical Line Test to determine whether the curve is a graph of a function of x.

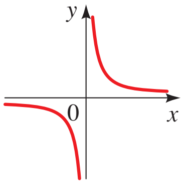
51. (a)



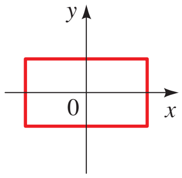
(b)



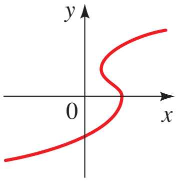
(c)



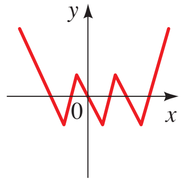
(d)



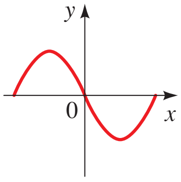
52. (a)



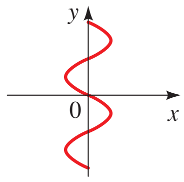
(b)



(c)

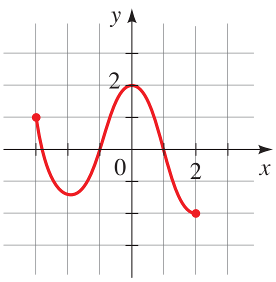


(d)

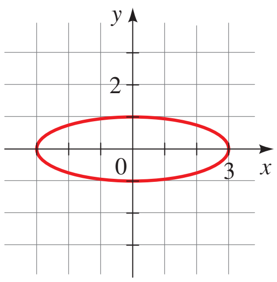


53–56 ■ Vertical Line Test: Domain and Range Use the Vertical Line Test to determine whether the curve is a graph of a function of x. If it is, state the domain and range of the function.

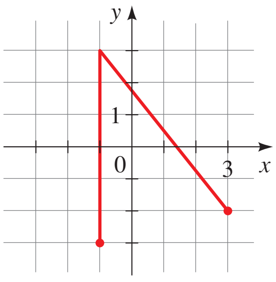
53.



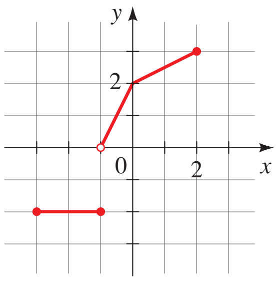
54.



55.



56.



57–68 ■ Equations That Define Functions Determine whether the equation defines y as a function of x. (See Example 9.)

57. $3x - 5y = 7$

58. $3x^2 - y = 5$

59. $x = y^2$

60. $x^2 + (y - 1)^2 = 4$

61. $2x - 4y^2 = 3$

62. $2x^2 - 4y^2 = 3$

63. $2xy - 5y^2 = 4$

64. $\sqrt{y} - x = 5$

65. $2|x| + y = 0$

66. $2x + |y| = 0$

67. $x = y^3$

68. $x = y^4$

69–74 ■ Families of Functions A family of functions is given. In parts (a) and (b) graph all the given members of the family in the viewing rectangle indicated. In part (c) state the conclusions that you can make from your graphs.

69. $f(x) = x^2 + c$

(a) $c = 0, 2, 4, 6$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 0, -2, -4, -6$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

70. $f(x) = (x - c)^2$

(a) $c = 0, 1, 2, 3$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 0, -1, -2, -3$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

71. $f(x) = (x - c)^3$

(a) $c = 0, 2, 4, 6$; $[-10, 10]$ by $[-10, 10]$

(b) $c = 0, -2, -4, -6$; $[-10, 10]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

72. $f(x) = cx^2$

(a) $c = 1, \frac{1}{2}, 2, 4$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 1, -1, -\frac{1}{2}, -2$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

73. $f(x) = x^c$

(a) $c = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}$; $[-1, 4]$ by $[-1, 3]$

(b) $c = 1, \frac{1}{3}, \frac{1}{5}$; $[-3, 3]$ by $[-2, 2]$

(c) How does the value of c affect the graph?

74. $f(x) = \dfrac{1}{x^n}$

(a) $n = 1, 3$; $[-3, 3]$ by $[-3, 3]$

(b) $n = 2, 4$; $[-3, 3]$ by $[-3, 3]$

(c) How does the value of n affect the graph?

SKILLS Plus

75–78 ■ Finding Functions for Certain Curves Find a function whose graph is the given curve.

75. The line segment joining the points $(-2, 1)$ and $(4, -6)$

76. The line segment joining the points $(-3, -2)$ and $(6, 3)$

77. The top half of the circle $x^2 + y^2 = 9$

78. The bottom half of the circle $x^2 + y^2 = 9$

APPLICATIONS

79. Weather Balloon As a weather balloon is inflated, the thickness T of its rubber skin is related to the radius of the balloon by

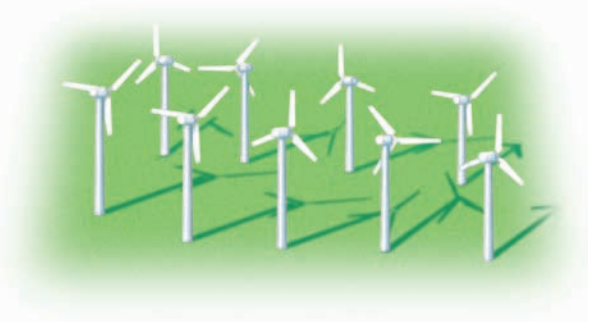
$$T(r) = \frac{0.5}{r^2}$$

where T and r are measured in centimeters. Graph the function T for values of r between 10 and 100.

80. Power from a Wind Turbine The power produced by a wind turbine depends on the speed of the wind. If a windmill has blades 3 meters long, then the power P produced by the turbine is modeled by

$$P(v) = 14.1v^3$$

where P is measured in watts (W) and v is measured in meters per second (m/s). Graph the function P for wind speeds between 1 m/s and 10 m/s.



81. Utility Rates Westside Energy charges its electric customers a base rate of \$6.00 per month, plus 10¢ per kilowatt-hour (kWh) for the first 300 kWh used and 6¢ per kWh for all usage over 300 kWh. Suppose a customer uses x kWh of electricity in one month.

(a) Express the monthly cost E as a piecewise defined function of x.

(b) Graph the function E for $0 \le x \le 600$.

82. Taxicab Function A taxi company charges \$2.00 for the first mile (or part of a mile) and 20 cents for each succeeding tenth of a mile (or part). Express the cost C (in dollars) of a ride as a piecewise defined function of the distance x traveled (in miles) for $0 < x < 2$, and sketch a graph of this function.

83. Postage Rates The 2014 domestic postage rate for first-class letters weighing 3.5 oz or less is 49 cents for the first ounce (or less), plus 21 cents for each additional ounce (or part of an ounce). Express the postage P as a piecewise defined function of the weight x of a letter, with $0 < x \le 3.5$, and sketch a graph of this function.

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

84. DISCOVER: When Does a Graph Represent a Function? For every integer n, the graph of the equation $y = x^n$ is the graph of a function, namely $f(x) = x^n$. Explain why the graph of $x = y^2$ is *not* the graph of a function of x. Is the graph of $x = y^3$ the graph of a function of x? If so, of what function of x is it the graph? Determine for what integers n the graph of $x = y^n$ is a graph of a function of x.

85. DISCUSS: Step Functions In Example 7 and Exercises 82 and 83 we are given functions whose graphs consist of horizontal line segments. Such functions are often called *step functions*, because their graphs look like stairs. Give some other examples of step functions that arise in everyday life.

86. DISCOVER: Stretched Step Functions Sketch graphs of the functions $f(x) = [\![x]\!]$, $g(x) = [\![2x]\!]$, and $h(x) = [\![3x]\!]$ on separate graphs. How are the graphs related? If n is a positive integer, what does a graph of $k(x) = [\![nx]\!]$ look like?