87. DISCOVER: Graph of the Absolute Value of a Function

(a) Draw graphs of the functions

$$f(x) = x^2 + x - 6$$

and $$g(x) = |x^2 + x - 6|$$

How are the graphs of f and g related?

(b) Draw graphs of the functions $f(x) = x^4 - 6x^2$ and $g(x) = |x^4 - 6x^2|$. How are the graphs of f and g related?

(c) In general, if $g(x) = |f(x)|$, how are the graphs of f and g related? Draw graphs to illustrate your answer.

2.3 GETTING INFORMATION FROM THE GRAPH OF A FUNCTION

■ Values of a Function; Domain and Range ■ Comparing Function Values: Solving Equations and Inequalities Graphically ■ Increasing and Decreasing Functions ■ Local Maximum and Minimum Values of a Function

Many properties of a function are more easily obtained from a graph than from the rule that describes the function. We will see in this section how a graph tells us whether the values of a function are increasing or decreasing and also where the maximum and minimum values of a function are.

■ Values of a Function; Domain and Range

A complete graph of a function contains all the information about a function, because the graph tells us which input values correspond to which output values. To analyze the graph of a function, we must keep in mind that *the height of the graph is the value of the function*. So we can read off the values of a function from its graph.

EXAMPLE 1 ■ Finding the Values of a Function from a Graph

The function T graphed in Figure 1 gives the temperature between noon and 6:00 P.M. at a certain weather station.

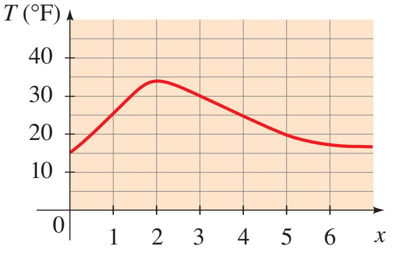


FIGURE 1 Temperature function

(a) Find $T(1)$, $T(3)$, and $T(5)$.

(b) Which is larger, $T(2)$ or $T(4)$?

(c) Find the value(s) of x for which $T(x) = 25$.

(d) Find the value(s) of x for which $T(x) \geq 25$.

(e) Find the net change in temperature from 1 P.M. to 3 P.M.

SOLUTION

(a) $T(1)$ is the temperature at 1:00 P.M. It is represented by the height of the graph above the x-axis at $x = 1$. Thus $T(1) = 25$. Similarly, $T(3) = 30$ and $T(5) = 20$.

(b) Since the graph is higher at $x = 2$ than at $x = 4$, it follows that $T(2)$ is larger than $T(4)$.

(c) The height of the graph is 25 when x is 1 and when x is 4. In other words, the temperature is 25 at 1:00 P.M. and 4:00 P.M.

(d) The graph is higher than 25 for x between 1 and 4. In other words, the temperature was 25 or greater between 1:00 P.M. and 4:00 P.M.

Net change is defined on page 151.

(e) The net change in temperature is

$$T(3) - T(1) = 30 - 25 = 5$$

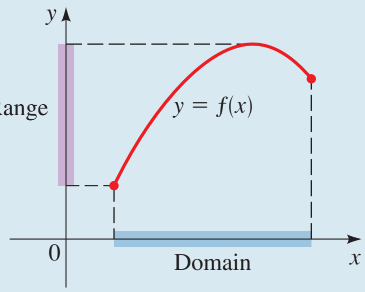
So there was a net increase of 5°F from 1 P.M. to 3 P.M.

Now Try Exercises 7 and 55 ■

The graph of a function helps us to picture the domain and range of the function on the x-axis and y-axis, as shown in the box below.

DOMAIN AND RANGE FROM A GRAPH

The **domain** and **range** of a function $y = f(x)$ can be obtained from a graph of f as shown in the figure. The domain is the set of all x-values for which f is defined, and the range is all the corresponding y-values.



See Appendix C, *Graphing with a Graphing Calculator,* for guidelines on using a graphing calculator. See Appendix D, *Using the TI-83/84 Graphing Calculator,* for specific graphing instructions. Go to **www.stewartmath.com**.

EXAMPLE 2 ■ Finding the Domain and Range from a Graph

(a) Use a graphing calculator to draw the graph of $f(x) = \sqrt{4 - x^2}$.

(b) Find the domain and range of f.

SOLUTION

(a) The graph is shown in Figure 2.

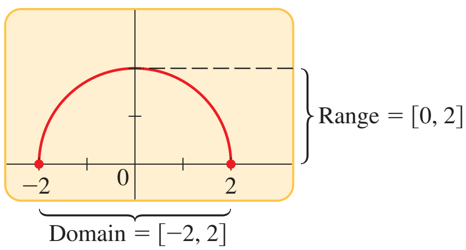


FIGURE 2 Graph of $f(x) = \sqrt{4 - x^2}$

(b) From the graph in Figure 2 we see that the domain is $[-2, 2]$ and the range is $[0, 2]$.

Now Try Exercise 21 ■

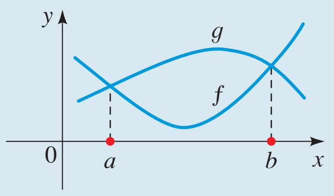
Comparing Function Values: Solving Equations and Inequalities Graphically

We can compare the values of two functions f and g visually by drawing their graphs. The points at which the graphs intersect are the points where the values of the two functions are equal. So the solutions of the equation $f(x) = g(x)$ are the values of x at which the two graphs intersect. The points at which the graph of g is higher than the graph of f are the points where the values of g are greater than the values of f. So the solutions of the inequality $f(x) < g(x)$ are the values of x at which the graph of g is *higher than* the graph of f.

SOLVING EQUATIONS AND INEQUALITIES GRAPHICALLY

The **solution(s) of the equation** $f(x) = g(x)$ are the values of x where the graphs of f and g intersect.

The **solution(s) of the inequality** $f(x) < g(x)$ are the values of x where the graph of g is higher than the graph of f.



The solutions of $f(x) = g(x)$ are the values a and b.

The solution of $f(x) < g(x)$ is the interval (a, b).

We can use these observations to solve equations and inequalities graphically, as the next example illustrates.

EXAMPLE 3 ■ Solving Graphically

Solve the given equation or inequality graphically.

(a) $2x^2 + 3 = 5x + 6$

(b) $2x^2 + 3 \leq 5x + 6$

(c) $2x^2 + 3 > 5x + 6$

You can also solve the equations and inequalities algebraically. Check that your solutions match the solutions we obtained graphically.

SOLUTION We first define functions f and g that correspond to the left-hand side and to the right-hand side of the equation or inequality. So we define

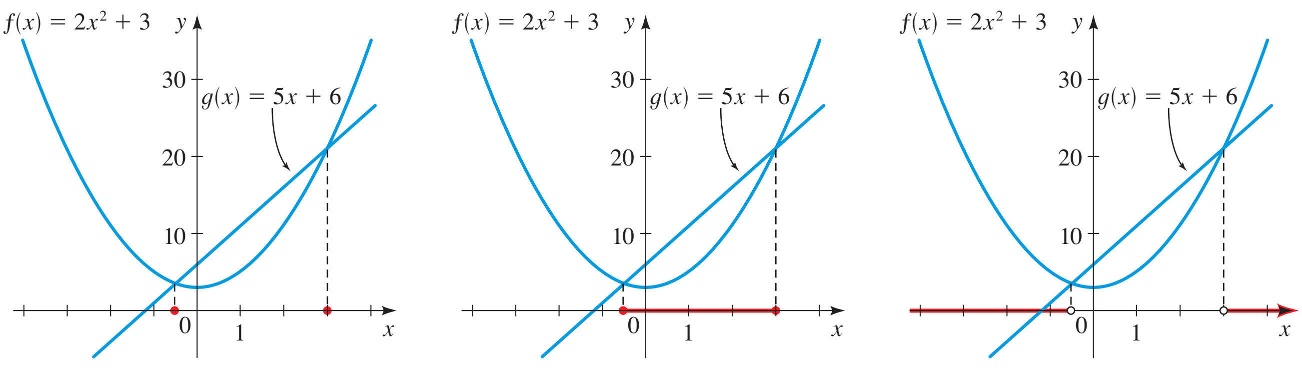
$$f(x) = 2x^2 + 3 \qquad \text{and} \qquad g(x) = 5x + 6$$

Next, we sketch graphs of f and g on the same set of axes.

(a) The given equation is equivalent to $f(x) = g(x)$. From the graph in Figure 3(a) we see that the solutions of the equation are $x = -0.5$ and $x = 3$.

(b) The given inequality is equivalent to $f(x) \leq g(x)$. From the graph in Figure 3(b) we see that the solution is the interval $[-0.5, 3]$.

(c) The given inequality is equivalent to $f(x) > g(x)$. From the graph in Figure 3(c) we see that the solution is $(-\infty, -0.5) \cup (3, \infty)$.



(a) Solution: $x = -0.5, 3$ (b) Solution: $[-0.5, 3]$ (c) Solution: $(-\infty, -0.5) \cup (3, \infty)$

FIGURE 3 Graphs of $f(x) = 2x^2 + 3$ and $g(x) = 5x + 6$

Now Try Exercises 9 and 23

To solve an equation graphically, we can first move all terms to one side of the equation and then graph the function that corresponds to the nonzero side of the equation. In this case the solutions of the equation are the x-intercepts of the graph. We can use this same method to solve inequalities graphically, as the following example shows.

EXAMPLE 4 ■ Solving Graphically

Solve the given equation or inequality graphically.

(a) $x^3 + 6 = 2x^2 + 5x$

(b) $x^3 + 6 \geq 2x^2 + 5x$

SOLUTION We first move all terms to one side to obtain an equivalent equation (or inequality). For the equation in part (a) we obtain

$$x^3 - 2x^2 - 5x + 6 = 0 \qquad \text{Move terms to LHS}$$

Then we define a function f by

$$f(x) = x^3 - 2x^2 - 5x + 6 \qquad \text{Define } f$$

Next, we use a graphing calculator to graph f, as shown in Figure 4.

(a) The given equation is the same as $f(x) = 0$, so the solutions are the x-intercepts of the graph. From Figure 4(a) we see that the solutions are $x = -2$, $x = 1$, and $x = 3$.

(b) The given inequality is the same as $f(x) \geq 0$, so the solutions are the x-values at which the graph of f is on or above the x-axis. From Figure 4(b) we see the solution is $[-2, 1] \cup [3, \infty]$.

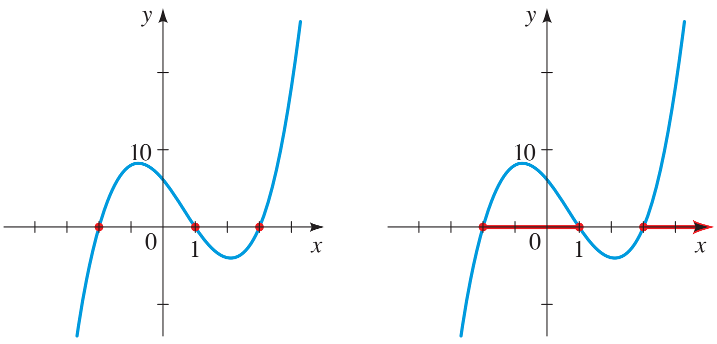


FIGURE 4 Graphs of $f(x) = x^3 - 2x^2 - 5x + 6$

Now Try Exercise 27

Increasing and Decreasing Functions

It is very useful to know where the graph of a function rises and where it falls. The graph shown in Figure 5 rises, falls, then rises again as we move from left to right: It rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be *increasing* when its graph rises and *decreasing* when its graph falls.

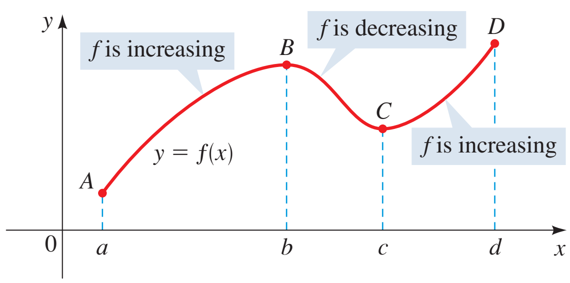


FIGURE 5 f is increasing on (a, b) and (c, d); f is decreasing on (b, c)

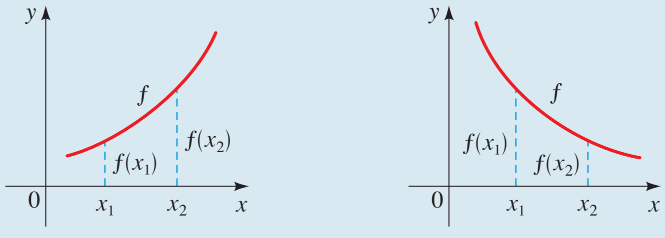
We have the following definition.

From the definition we see that a function increases or decreases *on an interval*. It does not make sense to apply these definitions at a single point.

DEFINITION OF INCREASING AND DECREASING FUNCTIONS

f is **increasing** on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

f is **decreasing** on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.



f is increasing   *f* is decreasing

EXAMPLE 5 ■ Intervals on Which a Function Increases or Decreases

The graph in Figure 6 gives the weight W of a person at age x. Determine the intervals on which the function W is increasing and on which it is decreasing.

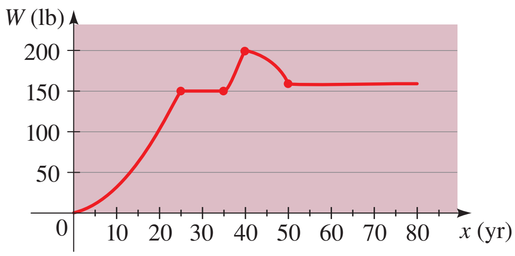


FIGURE 6 Weight as a function of age

SOLUTION The function W is increasing on $(0, 25)$ and $(35, 40)$. It is decreasing on $(40, 50)$. The function W is constant (neither increasing nor decreasing) on $(25, 35)$ and $(50, 80)$. This means that the person gained weight until age 25, then gained weight again between ages 35 and 40. He lost weight between ages 40 and 50.

Now Try Exercise 57 ■

By convention we write the intervals on which a function is increasing or decreasing as open intervals. (It would also be true to say that the function is increasing or decreasing on the corresponding closed interval. So for instance, it is also correct to say that the function W in Example 5 is decreasing on $[40, 50]$.)

EXAMPLE 6 ■ Finding Intervals on Which a Function Increases or Decreases

(a) Sketch a graph of the function $f(x) = 12x^2 + 4x^3 - 3x^4$.

(b) Find the domain and range of f.

(c) Find the intervals on which f is increasing and on which f is decreasing.

SOLUTION

(a) We use a graphing calculator to sketch the graph in Figure 7.

(b) The domain of f is $\mathbb{R}$ because f is defined for all real numbers. Using the TRACE feature on the calculator, we find that the highest value is $f(2) = 32$. So the range of f is $(-\infty, 32]$.

(c) From the graph we see that f is increasing on the intervals $(-\infty, -1)$ and $(0, 2)$ and is decreasing on $(-1, 0)$ and $(2, \infty)$.

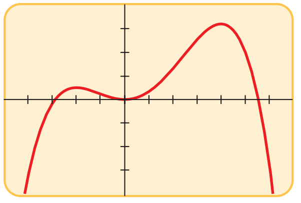


FIGURE 7 Graph of $f(x) = 12x^2 + 4x^3 - 3x^4$

Now Try Exercise 35 ■

EXAMPLE 7 ■ Finding Intervals Where a Function Increases and Decreases

(a) Sketch the graph of the function $f(x) = x^{2/3}$.

(b) Find the domain and range of the function.

(c) Find the intervals on which f is increasing and on which f is decreasing.

SOLUTION

(a) We use a graphing calculator to sketch the graph in Figure 8.

(b) From the graph we observe that the domain of f is $\mathbb{R}$ and the range is $[0, \infty)$.

(c) From the graph we see that f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

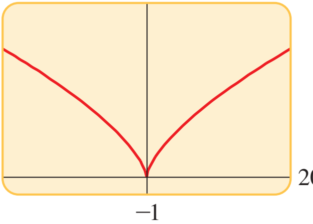


FIGURE 8 Graph of $f(x) = x^{2/3}$

Now Try Exercise 41 ■

■ Local Maximum and Minimum Values of a Function

Finding the largest or smallest values of a function is important in many applications. For example, if a function represents revenue or profit, then we are interested in its maximum value. For a function that represents cost, we would want to find its minimum value. (See *Focus on Modeling: Modeling with Functions* on pages 237–244 for many such examples.) We can easily find these values from the graph of a function. We first define what we mean by a local maximum or minimum.

LOCAL MAXIMA AND MINIMA OF A FUNCTION

1. The function value $f(a)$ is a **local maximum value** of f if

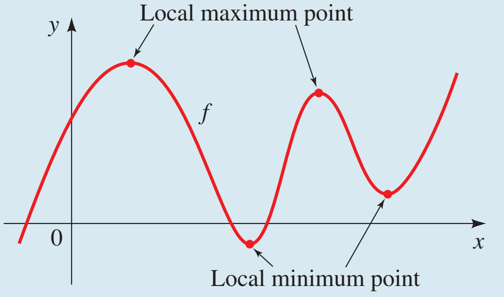
$$f(a) \geq f(x) \quad \text{when } x \text{ is near } a$$

(This means that $f(a) \geq f(x)$ for all x in some open interval containing a.) In this case we say that f has a **local maximum** at $x = a$.

2. The function value $f(a)$ is a **local minimum value** of f if

$$f(a) \leq f(x) \quad \text{when } x \text{ is near } a$$

(This means that $f(a) \leq f(x)$ for all x in some open interval containing a.) In this case we say that f has a **local minimum** at $x = a$.



We can find the local maximum and minimum values of a function using a graphing calculator. If there is a viewing rectangle such that the point $(a, f(a))$ is the highest point on the graph of f *within* the viewing rectangle (not on the edge), then the number $f(a)$ is a local maximum value of f (see Figure 9). Notice that $f(a) \geq f(x)$ for all numbers x that are close to a.

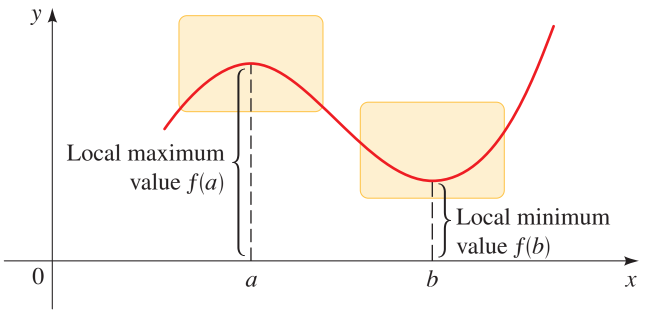


FIGURE 9

Similarly, if there is a viewing rectangle such that the point $(b, f(b))$ is the lowest point on the graph of f within the viewing rectangle, then the number $f(b)$ is a local minimum value of f. In this case $f(b) \leq f(x)$ for all numbers x that are close to b.

EXAMPLE 8 ■ Finding Local Maxima and Minima from a Graph

Find the local maximum and minimum values of the function $f(x) = x^3 - 8x + 1$, rounded to three decimal places.

SOLUTION The graph of f is shown in Figure 10. There appears to be one local maximum between $x = -2$ and $x = -1$, and one local minimum between $x = 1$ and $x = 2$.

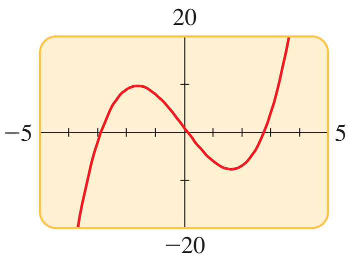


FIGURE 10 Graph of $f(x) = x^3 - 8x + 1$

Let's find the coordinates of the local maximum point first. We zoom in to enlarge the area near this point, as shown in Figure 11. Using the TRACE feature on the graphing device, we move the cursor along the curve and observe how the y-coordinates change. The local maximum value of y is 9.709, and this value occurs when x is -1.633, correct to three decimal places.

We locate the minimum value in a similar fashion. By zooming in to the viewing rectangle shown in Figure 12, we find that the local minimum value is about -7.709, and this value occurs when $x \approx 1.633$.

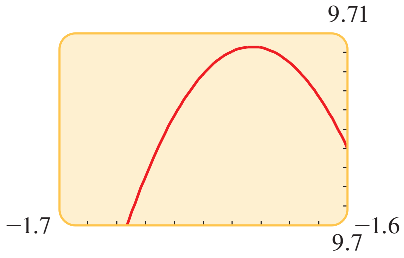


FIGURE 11

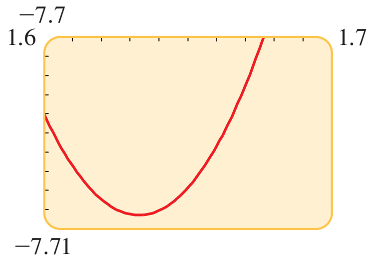


FIGURE 12

Now Try Exercise 47 ■

The `maximum` and `minimum` commands on a TI-83 or TI-84 calculator provide another method for finding extreme values of functions. We use this method in the next example.

EXAMPLE 9 ■ A Model for Managing Traffic

See the *Discovery Project* referenced in Chapter 3, on page 295, for how this model is obtained.

A highway engineer develops a formula to estimate the number of cars that can safely travel a particular highway at a given speed. She assumes that each car is 17 ft long, travels at a speed of x mi/h, and follows the car in front of it at the safe following distance for that speed. She finds that the number N of cars that can pass a given point per minute is modeled by the function

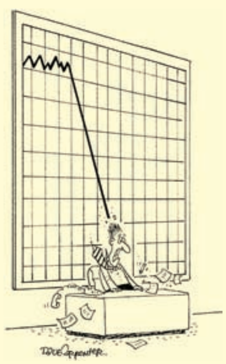
$$N(x) = \frac{88x}{17 + 17\left(\dfrac{x}{20}\right)^2}$$

Graph the function in the viewing rectangle $[0, 100]$ by $[0, 60]$.

(a) Find the intervals on which the function N is increasing and on which it is decreasing.

(b) Find the maximum value of N. What is the maximum carrying capacity of the road, and at what speed is it achieved?

DISCOVERY PROJECT



Dave Carpenter/www.CartoonStock.com

Every Graph Tells a Story

A graph can often describe a real-world "story" much more quickly and effectively than many words. For example, the stock market crash of 1929 is effectively described by a graph of the Dow Jones Industrial Average. No words are needed to convey the message in the cartoon shown here. In this project we describe, or tell the story that corresponds to, a given graph as well as make graphs that correspond to a real-world "story." You can find the project at **www.stewartmath.com**.

SOLUTION The graph is shown in Figure 13(a).

(a) From the graph we see that the function N is increasing on $(0, 20)$ and decreasing on $(20, \infty)$.

(b) There appears to be a maximum between $x = 19$ and $x = 21$. Using the `maximum` command, as shown in Figure 13(b), we see that the maximum value of N is about 51.78, and it occurs when x is 20. So the maximum carrying capacity is about 52 cars per minute at a speed of 20 mi/h.

See Appendix D, *Using the TI-83/84 Graphing Calculator,* for specific instructions on using the `maximum` command. Go to **www.stewartmath.com**.

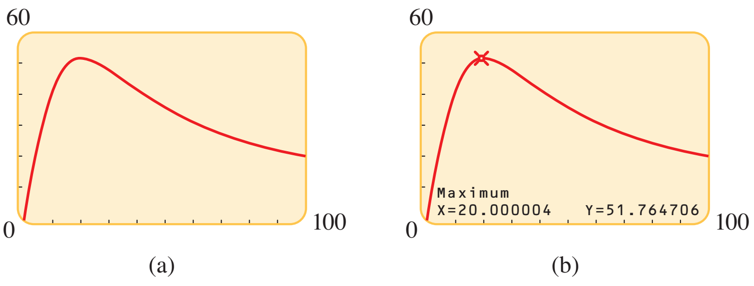


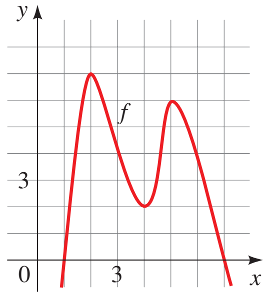
FIGURE 13 Highway capacity at speed x

Now Try Exercise 65

2.3 EXERCISES

CONCEPTS

1–5 ■ The function f graphed below is defined by a polynomial expression of degree 4. Use the graph to solve the exercises.



1. To find a function value $f(a)$ from the graph of f, we find the height of the graph above the x-axis at $x =$ ________. From the graph of f we see that $f(3) =$ ________ and $f(1) =$ _____. The net change in f between $x = 1$ and $x = 3$ is $f($_____$) - f($_____$) =$ _____.

2. The domain of the function f is all the ____-values of the points on the graph, and the range is all the corresponding ____-values. From the graph of f we see that the domain of f is the interval ________ and the range of f is the interval ________.

3. (a) If f is increasing on an interval, then the y-values of the points on the graph ________ as the x-values increase. From the graph of f we see that f is increasing on the intervals ________ and ________.

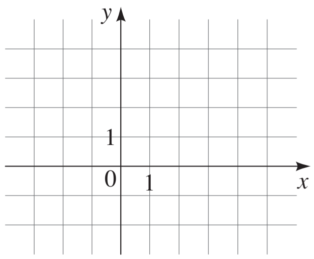
(b) If f is decreasing on an interval, then the y-values of the points on the graph ________ as the x-values increase. From the graph of f we see that f is decreasing on the intervals ________ and ________.

4. (a) A function value $f(a)$ is a local maximum value of f if $f(a)$ is the ________ value of f on some open interval containing a. From the graph of f we see that there are two local maximum values of f: One local maximum is ________, and it occurs when $x = 2$; the other local maximum is ________, and it occurs when $x =$ ________.

(b) The function value $f(a)$ is a local minimum value of f if $f(a)$ is the ________ value of f on some open interval containing a. From the graph of f we see that there is one local minimum value of f. The local minimum value is ________, and it occurs when $x =$ ________.

5. The solutions of the equation $f(x) = 0$ are the _____-intercepts of the graph of f. The solution of the inequality $f(x) \geq 0$ is the set of x-values at which the graph of f is on or above the _____-axis. From the graph of f we find that the solutions of the equation $f(x) = 0$ are $x =$ _____ and $x =$ _____, and the solution of the inequality $f(x) \geq 0$ is ____________.

6. (a) To solve the equation $2x + 1 = -x + 4$ graphically, we graph the functions $f(x) =$ ____________ and $g(x) =$ ____________ on the same set of axes and determine the values of x at which the graphs of f and g intersect. Graph f and g below, and use the graphs to solve the equation. The solution is $x =$ ________.



(b) To solve the inequality $2x + 1 < -x + 4$ graphically, we graph the functions $f(x) =$ ______________ and $g(x) =$ ______________ on the same set of axes and find the values of x at which the graph of g is ______________ (higher/lower) than the graph of f. From the graphs in part (a) we see that the solution of the inequality is the interval (____, ____).

SKILLS

7. Values of a Function The graph of a function h is given.

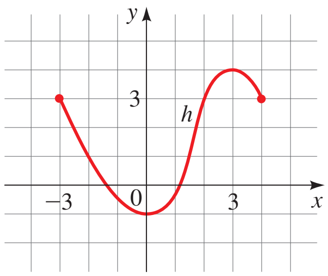
(a) Find $h(-2)$, $h(0)$, $h(2)$, and $h(3)$.

(b) Find the domain and range of h.

(c) Find the values of x for which $h(x) = 3$.

(d) Find the values of x for which $h(x) \leq 3$.

(e) Find the net change in h between $x = -3$ and $x = 3$.



8. Values of a Function The graph of a function g is given.

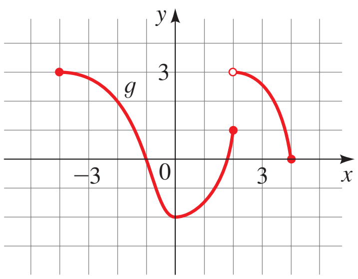
(a) Find $g(-4)$, $g(-2)$, $g(0)$, $g(2)$, and $g(4)$.

(b) Find the domain and range of g.

(c) Find the values of x for which $g(x) = 3$.

(d) Estimate the values of x for which $g(x) \leq 0$.

(e) Find the net change in g between $x = -1$ and $x = 2$.



9. Solving Equations and Inequalities Graphically Graphs of the functions f and g are given.

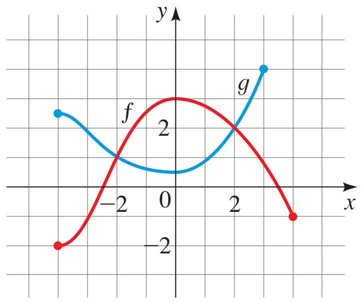
(a) Which is larger, $f(0)$ or $g(0)$?

(b) Which is larger, $f(-3)$ or $g(-3)$?

(c) For which values of x is $f(x) = g(x)$?

(d) Find the values of x for which $f(x) \leq g(x)$.

(e) Find the values of x for which $f(x) > g(x)$.



10. Solving Equations and Inequalities Graphically Graphs of the functions f and g are given.

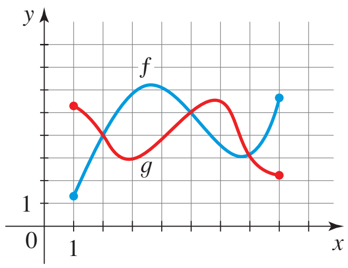
(a) Which is larger, $f(6)$ or $g(6)$?

(b) Which is larger, $f(3)$ or $g(3)$?

(c) Find the values of x for which $f(x) = g(x)$.

(d) Find the values of x for which $f(x) \leq g(x)$.

(e) Find the values of x for which $f(x) > g(x)$.



11–16 ■ Domain and Range from a Graph A function f is given. **(a)** Sketch a graph of f. **(b)** Use the graph to find the domain and range of f.

11. $f(x) = 2x + 3$

12. $f(x) = 3x - 2$

13. $f(x) = x - 2, \quad -2 \leq x \leq 5$

14. $f(x) = 4 - 2x, \quad 1 < x < 4$

15. $f(x) = x^2 - 1, \quad -3 \leq x \leq 3$

16. $f(x) = 3 - x^2, \quad -3 \leq x \leq 3$

17–22 ■ Finding Domain and Range Graphically A function f is given. **(a)** Use a graphing calculator to draw the graph of f. **(b)** Find the domain and range of f from the graph.

17. $f(x) = x^2 + 4x + 3$

18. $f(x) = -x^2 + 2x + 1$

19. $f(x) = \sqrt{x - 1}$

20. $f(x) = \sqrt{x + 2}$

21. $f(x) = \sqrt{16 - x^2}$

22. $f(x) = -\sqrt{25 - x^2}$

23–26 ■ Solving Equations and Inequalities Graphically Solve the given equation or inequality graphically.

23. **(a)** $x - 2 = 4 - x$ **(b)** $x - 2 > 4 - x$

24. **(a)** $-2x + 3 = 3x - 7$ **(b)** $-2x + 3 \le 3x - 7$

25. **(a)** $x^2 = 2 - x$ **(b)** $x^2 \le 2 - x$

26. **(a)** $-x^2 = 3 - 4x$ **(b)** $-x^2 \ge 3 - 4x$

27–30 ■ Solving Equations and Inequalities Graphically Solve the given equation or inequality graphically. State your answers rounded to two decimals.

27. **(a)** $x^3 + 3x^2 = -x^2 + 3x + 7$
(b) $x^3 + 3x^2 \ge -x^2 + 3x + 7$

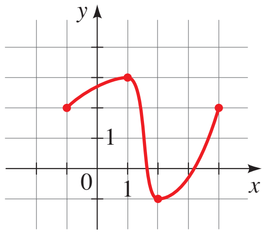
28. **(a)** $5x^2 - x^3 = -x^2 + 3x + 4$
(b) $5x^2 - x^3 \le -x^2 + 3x + 4$

29. **(a)** $16x^3 + 16x^2 = x + 1$
(b) $16x^3 + 16x^2 \ge x + 1$

30. **(a)** $1 + \sqrt{x} = \sqrt{x^2 + 1}$
(b) $1 + \sqrt{x} > \sqrt{x^2 + 1}$

31–34 ■ Increasing and Decreasing The graph of a function f is given. Use the graph to estimate the following. **(a)** The domain and range of f. **(b)** The intervals on which f is increasing and on which f is decreasing.

31.



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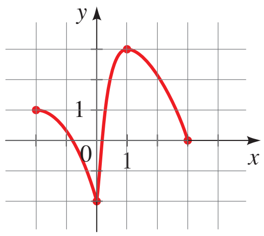

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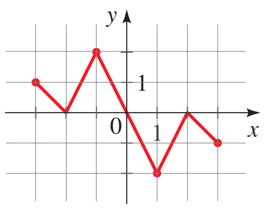

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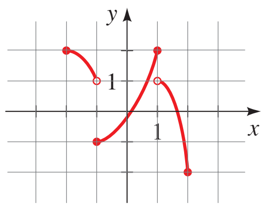


35–42 ■ Increasing and Decreasing A function f is given. **(a)** Use a graphing calculator to draw the graph of f. **(b)** Find the domain and range of f. **(c)** State approximately the intervals on which f is increasing and on which f is decreasing.

35. $f(x) = x^2 - 5x$

36. $f(x) = x^3 - 4x$

37. $f(x) = 2x^3 - 3x^2 - 12x$

38. $f(x) = x^4 - 16x^2$

39. $f(x) = x^3 + 2x^2 - x - 2$

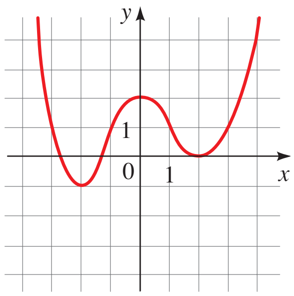
40. $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$

41. $f(x) = x^{2/5}$

42. $f(x) = 4 - x^{2/3}$

43–46 ■ Local Maximum and Minimum Values The graph of a function f is given. Use the graph to estimate the following. **(a)** All the local maximum and minimum values of the function and the value of x at which each occurs. **(b)** The intervals on which the function is increasing and on which the function is decreasing.

43.



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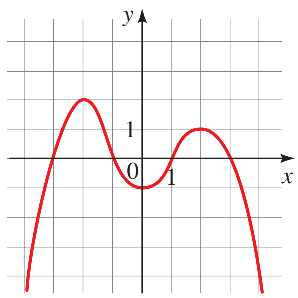

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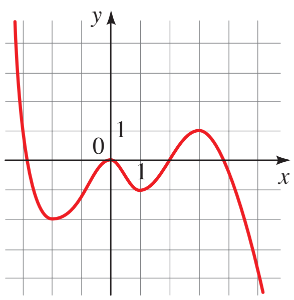

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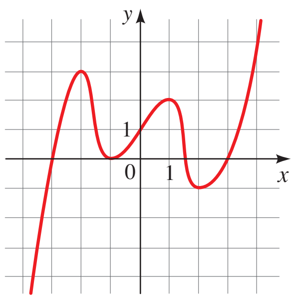


47–54 ■ Local Maximum and Minimum Values A function is given. **(a)** Find all the local maximum and minimum values of the function and the value of x at which each occurs. State each answer rounded to two decimal places. **(b)** Find the intervals on which the function is increasing and on which the function is decreasing. State each answer rounded to two decimal places.

47. $f(x) = x^3 - x$

48. $f(x) = 3 + x + x^2 - x^3$

49. $g(x) = x^4 - 2x^3 - 11x^2$

50. $g(x) = x^5 - 8x^3 + 20x$

51. $U(x) = x\sqrt{6 - x}$

52. $U(x) = x\sqrt{x - x^2}$

53. $V(x) = \dfrac{1 - x^2}{x^3}$

54. $V(x) = \dfrac{1}{x^2 + x + 1}$

APPLICATIONS

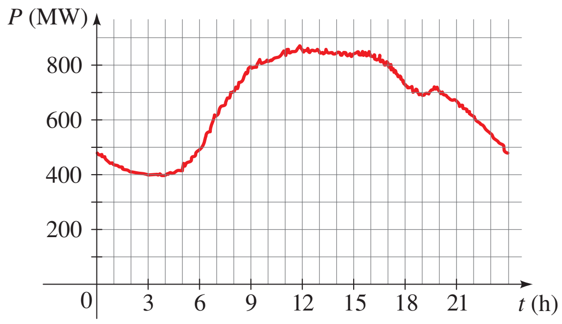
55. **Power Consumption** The figure shows the power consumption in San Francisco for a day in September (P is measured in megawatts; t is measured in hours starting at midnight).

(a) What was the power consumption at 6:00 A.M.? At 6:00 P.M.?

(b) When was the power consumption the lowest?

(c) When was the power consumption the highest?

(d) Find the net change in the power consumption from 9:00 A.M. to 7:00 P.M.



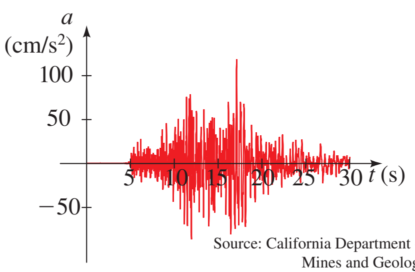
Source: Pacific Gas & Electric

56. Earthquake The graph shows the vertical acceleration of the ground from the 1994 Northridge earthquake in Los Angeles, as measured by a seismograph. (Here t represents the time in seconds.)

(a) At what time t did the earthquake first make noticeable movements of the earth?

(b) At what time t did the earthquake seem to end?

(c) At what time t was the maximum intensity of the earthquake reached?



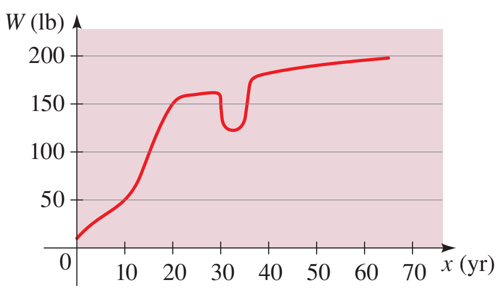
Source: California Department of Mines and Geology

57. Weight Function The graph gives the weight W of a person at age x.

(a) Determine the intervals on which the function W is increasing and those on which it is decreasing.

(b) What do you think happened when this person was 30 years old?

(c) Find the net change in the person's weight W from age 10 to age 20.

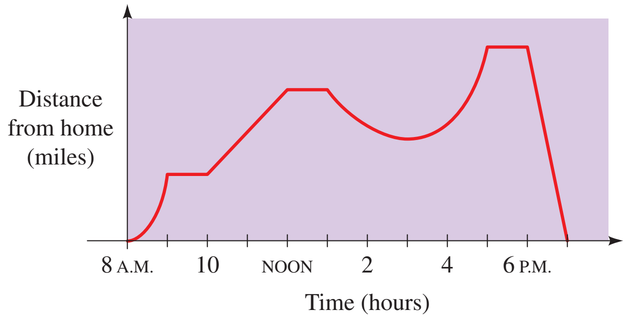


58. Distance Function The graph gives a sales representative's distance from his home as a function of time on a certain day.

(a) Determine the time intervals on which his distance from home was increasing and those on which it was decreasing.

(b) Describe in words what the graph indicates about his travels on this day.

(c) Find the net change in his distance from home between noon and 1:00 P.M.

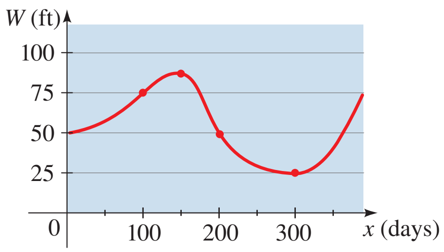


59. Changing Water Levels The graph shows the depth of water W in a reservoir over a one-year period as a function of the number of days x since the beginning of the year.

(a) Determine the intervals on which the function W is increasing and on which it is decreasing.

(b) At what value of x does W achieve a local maximum? A local minimum?

(c) Find the net change in the depth W from 100 days to 300 days.

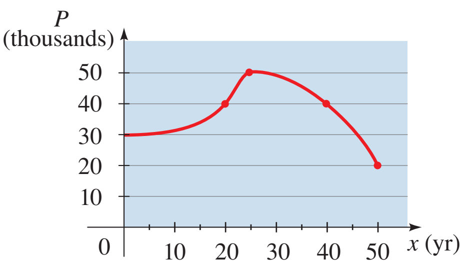


60. Population Growth and Decline The graph shows the population P in a small industrial city from 1950 to 2000. The variable x represents the number of years since 1950.

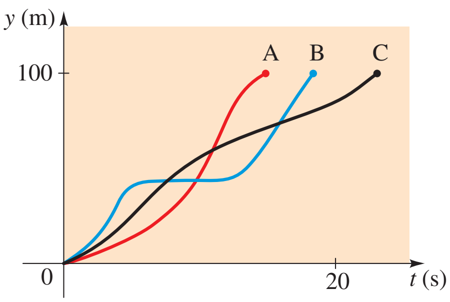
(a) Determine the intervals on which the function P is increasing and on which it is decreasing.

(b) What was the maximum population, and in what year was it attained?

(c) Find the net change in the population P from 1970 to 1990.



61. Hurdle Race Three runners compete in a 100-meter hurdle race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race? What do you think happened to Runner B?



62. Gravity Near the Moon We can use Newton's Law of Gravity to measure the gravitational attraction between the moon and an algebra student in a spaceship located a distance x above the moon's surface:

$$F(x) = \frac{350}{x^2}$$

Here F is measured in newtons (N), and x is measured in millions of meters.

(a) Graph the function F for values of x between 0 and 10.

(b) Use the graph to describe the behavior of the gravitational attraction F as the distance x increases.

63. Radii of Stars Astronomers infer the radii of stars using the Stefan Boltzmann Law:

$$E(T) = (5.67 \times 10^{-8})T^4$$

where E is the energy radiated per unit of surface area measured in watts (W) and T is the absolute temperature measured in kelvins (K).

(a) Graph the function E for temperatures T between 100 K and 300 K.

(b) Use the graph to describe the change in energy E as the temperature T increases.

64. Volume of Water Between 0°C and 30°C, the volume V (in cubic centimeters) of 1 kg of water at a temperature T is given by the formula

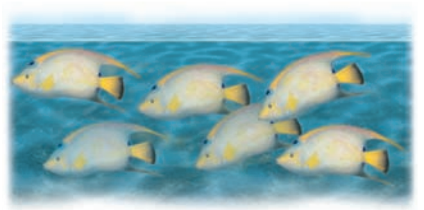
$$V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$$

Find the temperature at which the volume of 1 kg of water is a minimum.
[*Source: Physics,* by D. Halliday and R. Resnick]

65. Migrating Fish A fish swims at a speed v relative to the water, against a current of 5 mi/h. Using a mathematical model of energy expenditure, it can be shown that the total energy E required to swim a distance of 10 mi is given by

$$E(v) = 2.73v^3 \frac{10}{v - 5}$$

Biologists believe that migrating fish try to minimize the total energy required to swim a fixed distance. Find the value of v that minimizes energy required.
[*Note:* This result has been verified; migrating fish swim against a current at a speed 50% greater than the speed of the current.]



66. Coughing When a foreign object that is lodged in the trachea (windpipe) forces a person to cough, the diaphragm thrusts upward, causing an increase in pressure in the lungs. At the same time, the trachea contracts, causing the expelled air to move faster and increasing the pressure on the foreign object. According to a mathematical model of coughing, the velocity v (in cm/s) of the airstream through an average-sized person's trachea is related to the radius r of the trachea (in cm) by the function

$$v(r) = 3.2(1 - r)r^2 \qquad \tfrac{1}{2} \le r \le 1$$

Determine the value of r for which v is a maximum.

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

67. DISCUSS: Functions That Are Always Increasing or Decreasing Sketch rough graphs of functions that are defined for all real numbers and that exhibit the indicated behavior (or explain why the behavior is impossible).

(a) f is always increasing, and $f(x) > 0$ for all x

(b) f is always decreasing, and $f(x) > 0$ for all x

(c) f is always increasing, and $f(x) < 0$ for all x

(d) f is always decreasing, and $f(x) < 0$ for all x

68. DISCUSS: Maximum and Minimum Values In Example 9 we saw a real-world situation in which the maximum value of a function is important. Name several other everyday situations in which a maximum or minimum value is important.

69. DISCUSS ■ DISCOVER: Minimizing a Distance When we seek a minimum or maximum value of a function, it is sometimes easier to work with a simpler function instead.

(a) Suppose

$$g(x) = \sqrt{f(x)}$$

where $f(x) \ge 0$ for all x. Explain why the local minima and maxima of f and g occur at the same values of x.

(b) Let $g(x)$ be the distance between the point $(3, 0)$ and the point (x, x^2) on the graph of the parabola $y = x^2$. Express g as a function of x.

(c) Find the minimum value of the function g that you found in part (b). Use the principle described in part (a) to simplify your work.