

§ 2.7 COMBINING FUNCTIONS

6/26/2016

# 27 - 50

$$f(x) = 2x - 3, \quad g(x) = 4 - x^2$$

$$\underline{27.} \quad (a) \quad f(g(0)) = f(4 - 0^2) = f(4) = 2 \cdot 4 - 3 = \boxed{5}$$

$$(b) \quad g(f(0)) = g(2 \cdot 0 - 3) = g(-3) = 4 - (-3)^2 = \boxed{-5}$$

$$\underline{28.} \quad (a) \quad f(f(2)) = f(2 \cdot 2 - 3) = f(1) = 2 \cdot 1 - 3 = \boxed{-1}$$

$$(b) \quad g(g(3)) = g(4 - 3^2) = g(-5) = 4 - (-5)^2 = \boxed{-21}$$

$$\underline{29.} \quad (a) \quad (f \circ g)(-2) = f(g(-2)) = f(4 - 2^2) = f(0) = 2 \cdot 0 - 3 = \boxed{-3}$$

$$(b) \quad (g \circ f)(-2) = g(f(-2)) = g(2(-2) - 3) = g(-7) = 4 - 7^2 = \boxed{-45}$$

$$\underline{30.} \quad (a) \quad (f \circ f)(-1) = 2(2(-1) - 3) - 3 = \boxed{-13}$$

$$(b) \quad (g \circ g)(-1) = 4 - (4 - (-1)^2)^2 = 4 - 3^2 = \boxed{-5}$$

$$\underline{31.} \quad (a) \quad (f \circ g)(x) = 2(g(x)) - 3 = 2(4 - x^2) - 3$$

$$= \boxed{-2x^2 + 5}$$

$$(b) \quad (g \circ f)(x) = 4 - (f(x))^2 = 4 - (2x - 3)^2$$

$$= 4 - 4x^2 + 12x - 9 = \boxed{-4x^2 + 12x - 5}$$

$$\underline{32.} \quad (f \circ f)(x) = 2f(x) - 3 = 2(2x - 3) - 3 = \boxed{4x - 9}$$

$$(g \circ g)(x) = 4 - g(x)^2 = 4 - (4 - x^2)^2 = 4 - 16 + 8x^2 - x^4$$
$$= \boxed{-x^4 + 8x^2 - 12}$$

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$$\underline{33.} \quad f(g(2)) = f(5) = \boxed{4} \quad \underline{36.} \quad (f \circ g)(0) = f(3) = \boxed{0}$$

$$\underline{34.} \quad g(f(0)) = g(0) = \boxed{3} \quad \underline{37.} \quad (g \circ g)(-2) = g(1) = \boxed{4}$$

$$\underline{35.} \quad (g \circ f)(4) = g(2) = \boxed{5} \quad \underline{38.} \quad (f \circ f)(4) = f(2) = \boxed{-2}$$

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$$\underline{39.} \quad f(g(2)) = f(5) = \boxed{6} \quad \underline{43.} \quad f(g(6)) = f(4) = \boxed{1}$$

$$\underline{40.} \quad g(f(2)) = g(3) = \boxed{6} \quad \underline{44.} \quad g(f(2)) = g(3) = \boxed{6}$$

$$\underline{41.} \quad f(f(1)) = f(2) = \boxed{3} \quad \underline{45.} \quad f(f(5)) = f(6) = \boxed{3}$$

$$\underline{42.} \quad g(g(2)) = g(5) = \boxed{1} \quad \underline{46.} \quad g(g(2)) = g(5) = \boxed{1}$$

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47.  $f(x) = 2x + 3$  ,  $g(x) = 4x - 1$

$$f(g(x)) = 2(4x - 1) + 3 = 8x + 1 \text{ , DOMAIN } \mathbb{R}$$

$$g(f(x)) = 4(2x + 3) - 1 = 8x + 11 \text{ , DOMAIN } \mathbb{R}$$

$$f(f(x)) = 2(2x + 3) + 3 = 4x + 9 \text{ , DOMAIN } \mathbb{R}$$

$$g(g(x)) = 4(4x - 1) - 1 = 16x - 5 \text{ , DOMAIN } \mathbb{R}$$

48.  $f(x) = 6x - 5$  ,  $g(x) = \frac{x}{2}$

$$f(g(x)) = 6\left(\frac{x}{2}\right) - 5 = 3x - 5$$

$$g(f(x)) = \frac{6x - 5}{2}$$

$$f(f(x)) = 6(6x - 5) - 5 = 36x - 35$$

$$g(g(x)) = \frac{\frac{x}{2}}{2} = \frac{x}{4}$$

DOMAIN  $\mathbb{R}$

49.  $f(x) = x^2$  ,  $g(x) = x + 1$

$$f(g(x)) = (x + 1)^2 = x^2 + 2x + 1$$

$$g(f(x)) = x^2 + 1$$

$$f(f(x)) = (x^2)^2 = x^4$$

$$g(g(x)) = (x + 1) + 1 = x + 2$$

DOMAIN  $\mathbb{R}$ .

50.  $f(x) = x^3 + 2$ ,  $g(x) = \sqrt[3]{x}$

$$f(g(x)) = (\sqrt[3]{x})^3 + 2 = x + 2$$

$$g(f(x)) = \sqrt[3]{x^3 + 2}$$

$$f(f(x)) = (x^3 + 2)^3 = x^9 + 6x^6 + 12x^3 + 8$$

$$g(g(x)) = \sqrt[3]{\sqrt[3]{x}} = \sqrt[9]{x}$$

DOMAIN  $\mathbb{R}$

51.  $f(x) = \frac{1}{x}$ ,  $g(x) = 2x + 4$

$$f(g(x)) = \frac{1}{2x+4} \quad \text{DOMAIN } (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$$

$$g(f(x)) = 2\left(\frac{1}{x}\right) + 4 = \frac{2+4x}{x} \quad \text{DOMAIN: } (-\infty, 0) \cup (0, \infty)$$

$$f(f(x)) = \frac{1}{\frac{1}{x}} = x \quad \text{DOMAIN } \mathbb{R}$$

$$g(g(x)) = 2(2x+4) + 4 = 4x + 12 \quad \text{DOMAIN } \mathbb{R}.$$

52.  $f(x) = x^2$ ,  $g(x) = \sqrt{x-3}$

$f(g(x)) = \sqrt{x-3}^2 = x-3$  DOMAIN  $x > 3$

$g(f(x)) = \sqrt{x^2-3}$  DOMAIN  $|x| \geq \sqrt{3}$

$f(f(x)) = x^4$  DOMAIN  $\mathbb{R}$

$g(g(x)) = \sqrt{\sqrt{x-3}-3}$  DOMAIN:  $\sqrt{x-3} - 3 \geq 0$   
 $\sqrt{x-3} \geq 3$   
 $x-3 \geq 9$   
 $x \geq 12$

53.  $f(x) = |x|$ ,  $g(x) = 2x+3$

$f(g(x)) = |2x+3|$ , DOMAIN  $\mathbb{R}$

$g(f(x)) = 2|x|+3$ , DOMAIN  $\mathbb{R}$

$f(f(x)) = ||x|| = |x|$ , DOMAIN  $\mathbb{R}$

$g(g(x)) = 2(2x+3)+3 = 4x+9$ , DOMAIN  $\mathbb{R}$

$$54. \quad f(x) = x - 4, \quad g(x) = |x + 4|$$

$$f(g(x)) = |x + 4| - 4 \quad \text{DOMAIN } \mathbb{R}$$

$$g(f(x)) = |(x - 4) + 4| = |x| \quad \text{DOMAIN } \mathbb{R}$$

$$f(f(x)) = (x - 4) - 4 = x - 8 \quad \text{DOMAIN } \mathbb{R}$$

$$g(g(x)) = ||x + 4| + 4| = |x + 4| + 4, \quad \text{DOMAIN } \mathbb{R}$$

$$55. \quad f(x) = \frac{x}{x+1}, \quad g(x) = 2x - 1$$

$$f(g(x)) = \frac{2x - 1}{(2x - 1) + 1} = \frac{2x - 1}{2x} \quad \text{DOMAIN } x \neq 0$$

$$g(f(x)) = 2\left(\frac{x}{x+1}\right) - 1 = \frac{x - 1}{x + 1} \quad \text{DOMAIN } x \neq -1$$

$$f(f(x)) = \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} = \frac{x}{x + x + 1} = \frac{x}{2x + 1} \quad \text{DOMAIN } x \neq -1, -\frac{1}{2}$$

$$g(g(x)) = 2(2x - 1) - 1 = 4x - 3, \quad \text{DOMAIN } \mathbb{R}$$

56.  $f(x) = \frac{1}{\sqrt{x}}$  ,  $g(x) = x^2 - 4x$

$f(g(x)) = \frac{1}{\sqrt{x^2 - 4x}}$  , DOMAIN :  $x^2 - 4x > 0$

$x(x-4) > 0$

$(-\infty, 0) \cup (4, \infty)$



$f(f(x)) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = \sqrt[4]{x}$  , DOMAIN  $x > 0$

$g(g(x)) = (x^2 - 4x)^2 - 4(x^2 - 4x)$

$= x^4 - 8x^3 + 16x^2 - 4x^2 + 16x$

$= x^4 - 8x^3 + 12x^2 + 16x$  , DOMAIN  $\mathbb{R}$

57.  $f(x) = \frac{x}{x+1}$  ,  $g(x) = \frac{1}{x}$

$f(g(x)) = \frac{\frac{1}{x}}{\frac{1}{x} + 1} = \frac{1}{1+x}$  DOMAIN  $x \neq 0, -1$

$g(f(x)) = \frac{1}{\frac{x}{x+1}} = \frac{x+1}{x}$  DOMAIN :  $x \neq 0, -1$

$f(f(x)) = \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} = \frac{x}{2x+1}$  , DOMAIN :  $x \neq -1, -\frac{1}{2}$

$$g(g(x)) = \frac{1}{\frac{1}{x}} = x, \text{ DOMAIN } x \neq 0$$

58.  $f(x) = \frac{2}{x}$ ,  $g(x) = \frac{x}{x+2}$

$$f(g(x)) = \frac{2}{\frac{x}{x+2}} = \frac{2x+4}{x} \quad \text{DOMAIN: } x \neq -2, 0$$

$$g(g(x)) = \frac{\frac{x}{x+2}}{\frac{x}{x+2} + 2} = \frac{x}{3x+4} \quad \text{DOMAIN: } x \neq -2, -\frac{4}{3}$$

$$f(f(x)) = \frac{2}{\frac{2}{x}} = x, \text{ DOMAIN } x \neq 0$$