74. Biodiversity Some biologists model the number of species S in a fixed area A (such as an island) by the species-area relationship

$$\log S = \log c + k \log A$$

where c and k are positive constants that depend on the type of species and habitat.

(a) Solve the equation for S.

(b) Use part (a) to show that if $k = 3$, then doubling the area increases the number of species eightfold.

75. Magnitude of Stars The magnitude M of a star is a measure of how bright a star appears to the human eye. It is defined by

$$M = -2.5 \log\left(\frac{B}{B_0}\right)$$

where B is the actual brightness of the star and B_0 is a constant.

(a) Expand the right-hand side of the equation.

(b) Use part (a) to show that the brighter a star, the less its magnitude.

(c) Betelgeuse is about 100 times brighter than Albiero. Use part (a) to show that Betelgeuse is 5 magnitudes less bright than Albiero.

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

76. DISCUSS: True or False? Discuss each equation, and determine whether it is true for all possible values of the variables. (Ignore values of the variables for which any term is undefined.)

(a) $\log\left(\frac{x}{y}\right) = \frac{\log x}{\log y}$

(b) $\log_2(x - y) = \log_2 x - \log_2 y$

(c) $\log_5\left(\frac{a}{b^2}\right) = \log_5 a - 2\log_5 b$

(d) $\log 2^z = z \log 2$

(e) $(\log P)(\log Q) = \log P + \log Q$

(f) $\frac{\log a}{\log b} = \log a - \log b$

(g) $(\log_2 7)^x = x \log_2 7$

(h) $\log_a a^a = a$

(i) $\log(x - y) = \frac{\log x}{\log y}$

(j) $-\ln\left(\frac{1}{A}\right) = \ln A$

77. DISCUSS: Find the Error What is wrong with the following argument?

$$\begin{aligned} \log 0.1 &< 2 \log 0.1 \\ &= \log(0.1)^2 \\ &= \log 0.01 \\ \log 0.1 &< \log 0.01 \\ 0.1 &< 0.01 \end{aligned}$$

78. PROVE: Shifting, Shrinking, and Stretching Graphs of Functions Let $f(x) = x^2$. Show that $f(2x) = 4f(x)$, and explain how this shows that shrinking the graph of f horizontally has the same effect as stretching it vertically. Then use the identities $e^{2+x} = e^2e^x$ and $\ln(2x) = \ln 2 + \ln x$ to show that for $g(x) = e^x$ a horizontal shift is the same as a vertical stretch and for $h(x) = \ln x$ a horizontal shrinking is the same as a vertical shift.

4.5 EXPONENTIAL AND LOGARITHMIC EQUATIONS

■ Exponential Equations ■ Logarithmic Equations ■ Compound Interest

In this section we solve equations that involve exponential or logarithmic functions. The techniques that we develop here will be used in the next section for solving applied problems.

■ Exponential Equations

An *exponential equation* is one in which the variable occurs in the exponent. Some exponential equations can be solved by using the fact that exponential functions are one-to-one. This means that

$$a^x = a^y \quad \Rightarrow \quad x = y$$

We use this property in the next example.

EXAMPLE 1 ■ Exponential Equations

Solve the exponential equation.

(a) $5^x = 125$ **(b)** $5^{2x} = 5^{x+1}$

SOLUTION

(a) We first express 125 as a power of 5 and then use the fact that the exponential function $f(x) = 5^x$ is one-to-one.

$$5^x = 125 \qquad \text{Given equation}$$
$$5^x = 5^3 \qquad \text{Because } 125 = 5^3$$
$$x = 3 \qquad \text{One-to-one property}$$

The solution is $x = 3$.

(b) We first use the fact that the function $f(x) = 5^x$ is one-to-one.

$$5^{2x} = 5^{x+1} \qquad \text{Given equation}$$
$$2x = x + 1 \qquad \text{One-to-one property}$$
$$x = 1 \qquad \text{Solve for } x$$

The solution is $x = 1$.

Now Try Exercises 3 and 7 ■

Law 3: $\log_a A^C = C \log_a A$

The equations in Example 1 were solved by comparing exponents. This method is not suitable for solving an equation like $5^x = 160$ because 160 is not easily expressed as a power of the base 5. To solve such equations, we take the logarithm of each side and use Law 3 of logarithms to "bring down the exponent." The following guidelines describe the process.

GUIDELINES FOR SOLVING EXPONENTIAL EQUATIONS

1. Isolate the exponential expression on one side of the equation.
2. Take the logarithm of each side, then use the Laws of Logarithms to "bring down the exponent."
3. Solve for the variable.

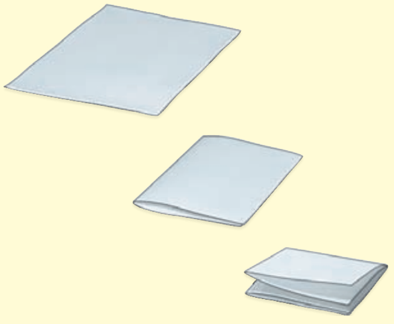
EXAMPLE 2 ■ Solving an Exponential Equation

Consider the exponential equation $3^{x+2} = 7$.

(a) Find the exact solution of the equation expressed in terms of logarithms.

(b) Use a calculator to find an approximation to the solution rounded to six decimal places.

DISCOVERY PROJECT



Super Origami

Origami is the traditional Japanese art of folding paper to create illustrations. In this project we explore some thought experiments about folding paper. Suppose that you fold a sheet of paper in half, then fold it in half again, and continue to fold the paper in half. How many folds are needed to obtain a mile-high stack of paper? To answer this question, we need to solve an exponential equation. In this project we use logarithms to answer this and other thought questions about folding paper. You can find the project at **www.stewartmath.com**.

SOLUTION

(a) We take the common logarithm of each side and use Law 3.

$$\begin{aligned} 3^{x+2} &= 7 && \text{Given equation} \\ \log(3^{x+2}) &= \log 7 && \text{Take log of each side} \\ (x+2)\log 3 &= \log 7 && \text{Law 3 (bring down exponent)} \\ x + 2 &= \frac{\log 7}{\log 3} && \text{Divide by log 3} \\ x &= \frac{\log 7}{\log 3} - 2 && \text{Subtract 2} \end{aligned}$$

The exact solution is $x = \dfrac{\log 7}{\log 3} - 2$.

(b) Using a calculator, we find the decimal approximation $x \approx -0.228756$.

We could have used natural logarithms instead of common logarithms. In fact, using the same steps, we get

$$x = \frac{\ln 7}{\ln 3} - 2 \approx -0.228756$$

CHECK YOUR ANSWER

Substituting $x = -0.228756$ into the original equation and using a calculator, we get

$$3^{(-0.228756)+2} \approx 7 \quad ✓$$

Now Try Exercise 15 ■

EXAMPLE 3 ■ Solving an Exponential Equation

Solve the equation $8e^{2x} = 20$.

SOLUTION We first divide by 8 to isolate the exponential term on one side of the equation.

$$\begin{aligned} 8e^{2x} &= 20 && \text{Given equation} \\ e^{2x} &= \tfrac{20}{8} && \text{Divide by 8} \\ \ln e^{2x} &= \ln 2.5 && \text{Take ln of each side} \\ 2x &= \ln 2.5 && \text{Property of ln} \\ x &= \frac{\ln 2.5}{2} && \text{Divide by 2 (exact solution)} \\ &\approx 0.458 && \text{Calculator (approximate solution)} \end{aligned}$$

CHECK YOUR ANSWER

Substituting $x = 0.458$ into the original equation and using a calculator, we get

$$8e^{2(0.458)} \approx 20 \quad ✓$$

Now Try Exercise 17 ■

EXAMPLE 4 ■ Solving an Exponential Equation Algebraically and Graphically

Solve the equation $e^{3-2x} = 4$ algebraically and graphically.

SOLUTION 1: Algebraic

Since the base of the exponential term is e, we use natural logarithms to solve this equation.

$$\begin{aligned} e^{3-2x} &= 4 && \text{Given equation} \\ \ln(e^{3-2x}) &= \ln 4 && \text{Take ln of each side} \\ 3 - 2x &= \ln 4 && \text{Property of ln} \\ -2x &= -3 + \ln 4 && \text{Subtract 3} \\ x &= \tfrac{1}{2}(3 - \ln 4) \approx 0.807 && \text{Multiply by } -\tfrac{1}{2} \end{aligned}$$

You should check that this answer satisfies the original equation.

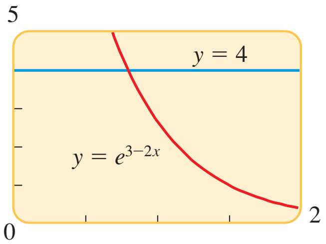


FIGURE 1

SOLUTION 2: Graphical

We graph the equations $y = e^{3-2x}$ and $y = 4$ in the same viewing rectangle as in Figure 1. The solutions occur where the graphs intersect. Zooming in on the point of intersection of the two graphs, we see that $x \approx 0.81$.

Now Try Exercise 21 ■

EXAMPLE 5 ■ An Exponential Equation of Quadratic Type

Solve the equation $e^{2x} - e^x - 6 = 0$.

SOLUTION To isolate the exponential term, we factor.

$$e^{2x} - e^x - 6 = 0 \quad \text{Given equation}$$
$$(e^x)^2 - e^x - 6 = 0 \quad \text{Law of Exponents}$$
$$(e^x - 3)(e^x + 2) = 0 \quad \text{Factor (a quadratic in } e^x)$$
$$e^x - 3 = 0 \quad \text{or} \quad e^x + 2 = 0 \quad \text{Zero-Product Property}$$
$$e^x = 3 \qquad e^x = -2$$

If we let $w = e^x$, we get the quadratic equation

$$w^2 - w - 6 = 0$$

which factors as

$$(w - 3)(w + 2) = 0$$

The equation $e^x = 3$ leads to $x = \ln 3$. But the equation $e^x = -2$ has no solution because $e^x > 0$ for all x. Thus $x = \ln 3 \approx 1.0986$ is the only solution. You should check that this answer satisfies the original equation.

Now Try Exercise 39 ■

EXAMPLE 6 ■ An Equation Involving Exponential Functions

Solve the equation $3xe^x + x^2e^x = 0$.

SOLUTION First we factor the left side of the equation.

$$3xe^x + x^2e^x = 0 \quad \text{Given equation}$$
$$x(3 + x)e^x = 0 \quad \text{Factor out common factors}$$
$$x(3 + x) = 0 \quad \text{Divide by } e^x \text{ (because } e^x \neq 0)$$
$$x = 0 \quad \text{or} \quad 3 + x = 0 \quad \text{Zero-Product Property}$$

Thus the solutions are $x = 0$ and $x = -3$.

CHECK YOUR ANSWER

$x = 0$:

$3(0)e^0 + 0^2e^0 = 0$ ✓

$x = -3$:

$3(-3)e^{-3} + (-3)^2e^{-3}$

$= -9e^{-3} + 9e^{-3} = 0$ ✓

Now Try Exercise 45 ■

■ Logarithmic Equations

A *logarithmic equation* is one in which a logarithm of the variable occurs. Some logarithmic equations can be solved by using the fact that logarithmic functions are one-to-one. This means that

$$\log_a x = \log_a y \quad \Rightarrow \quad x = y$$

We use this property in the next example.

EXAMPLE 7 ■ Solving a Logarithmic Equation

Solve the equation $\log(x^2 + 1) = \log(x - 2) + \log(x + 3)$.

SOLUTION First we combine the logarithms on the right-hand side, and then we use the one-to-one property of logarithms.

$\log_5(x^2 + 1) = \log_5(x - 2) + \log_5(x + 3)$	Given equation
$\log_5(x^2 + 1) = \log_5[(x - 2)(x + 3)]$	Law 1: $\log_a AB = \log_a A + \log_a B$
$\log_5(x^2 + 1) = \log_5(x^2 + x - 6)$	Expand
$x^2 + 1 = x^2 + x - 6$	log is one-to-one (or raise 5 to each side)
$x = 7$	Solve for x

The solution is $x = 7$. (You can check that $x = 7$ satisfies the original equation.)

Now Try Exercise 49 ■

The method of Example 7 is not suitable for solving an equation like $\log_5 x = 13$ because the right-hand side is not expressed as a logarithm (base 5). To solve such equations, we use the following guidelines.

GUIDELINES FOR SOLVING LOGARITHMIC EQUATIONS

1. Isolate the logarithmic term on one side of the equation; you might first need to combine the logarithmic terms.
2. Write the equation in exponential form (or raise the base to each side of the equation).
3. Solve for the variable.

EXAMPLE 8 ■ Solving Logarithmic Equations

Solve each equation for x.

(a) $\ln x = 8$

(b) $\log_2(25 - x) = 3$

SOLUTION

(a)

$\ln x = 8$	Given equation
$x = e^8$	Exponential form

Therefore $x = e^8 \approx 2981$.

We can also solve this problem another way.

$\ln x = 8$	Given equation
$e^{\ln x} = e^8$	Raise e to each side
$x = e^8$	Property of ln

(b) The first step is to rewrite the equation in exponential form.

$\log_2(25 - x) = 3$	Given equation
$25 - x = 2^3$	Exponential form (or raise 2 to each side)
$25 - x = 8$	
$x = 25 - 8 = 17$	

CHECK YOUR ANSWER

If $x = 17$, we get

$\log_2(25 - 17) = \log_2 8 = 3$ ✓

Now Try Exercises 55 and 59 ■

EXAMPLE 9 ■ Solving a Logarithmic Equation

Solve the equation $4 + 3\log(2x) = 16$.

SOLUTION We first isolate the logarithmic term. This allows us to write the equation in exponential form.

$$\begin{aligned} 4 + 3\log(2x) &= 16 && \text{Given equation} \\ 3\log(2x) &= 12 && \text{Subtract 4} \\ \log(2x) &= 4 && \text{Divide by 3} \\ 2x &= 10^4 && \text{Exponential form (or raise 10 to each side)} \\ x &= 5000 && \text{Divide by 2} \end{aligned}$$

CHECK YOUR ANSWER

If $x = 5000$, we get

$$\begin{aligned} 4 + 3\log 2(5000) &= 4 + 3\log 10{,}000 \\ &= 4 + 3(4) \\ &= 16 \quad ✓ \end{aligned}$$

Now Try Exercise 61 ■

EXAMPLE 10 ■ Solving a Logarithmic Equation Algebraically and Graphically

Solve the equation $\log(x + 2) + \log(x - 1) = 1$ algebraically and graphically.

SOLUTION 1: Algebraic

We first combine the logarithmic terms, using the Laws of Logarithms.

$$\begin{aligned} \log[(x + 2)(x - 1)] &= 1 && \text{Law 1} \\ (x + 2)(x - 1) &= 10 && \text{Exponential form (or raise 10 to each side)} \\ x^2 + x - 2 &= 10 && \text{Expand left side} \\ x^2 + x - 12 &= 0 && \text{Subtract 10} \\ (x + 4)(x - 3) &= 0 && \text{Factor} \end{aligned}$$

$$x = -4 \quad \text{or} \quad x = 3$$

CHECK YOUR ANSWERS

$x = -4$:

$$\begin{aligned} &\log(-4 + 2) + \log(-4 - 1) \\ &\quad = \log(-2) + \log(-5) \\ &\quad \text{undefined} \quad ✗ \end{aligned}$$

$x = 3$:

$$\begin{aligned} &\log(3 + 2) + \log(3 - 1) \\ &\quad = \log 5 + \log 2 = \log(5 \cdot 2) \\ &\quad = \log 10 = 1 \quad ✓ \end{aligned}$$

We check these potential solutions in the original equation and find that $x = -4$ is not a solution (because logarithms of negative numbers are undefined), but $x = 3$ is a solution. (See *Check Your Answers*.)

SOLUTION 2: Graphical

We first move all terms to one side of the equation:

$$\log(x + 2) + \log(x - 1) - 1 = 0$$

Then we graph

$$y = \log(x + 2) + \log(x - 1) - 1$$

as in Figure 2. The solutions are the x-intercepts of the graph. Thus the only solution is $x \approx 3$.

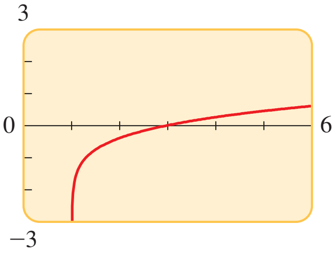


FIGURE 2

Now Try Exercise 63 ■

EXAMPLE 11 ■ Solving a Logarithmic Equation Graphically

In Example 11 it's not possible to isolate x algebraically, so we must solve the equation graphically.

Solve the equation $x^2 = 2\ln(x + 2)$.

SOLUTION We first move all terms to one side of the equation.

$$x^2 - 2\ln(x + 2) = 0$$

Then we graph

$$y = x^2 - 2\ln(x + 2)$$

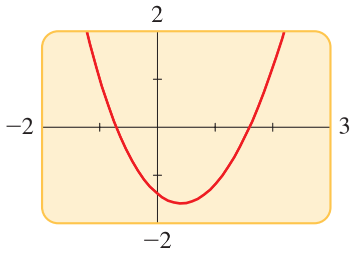


FIGURE 3

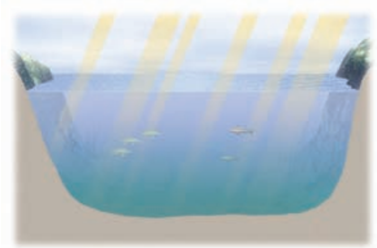
as in Figure 3. The solutions are the x-intercepts of the graph. Zooming in on the x-intercepts, we see that there are two solutions:

$$x \approx -0.71 \qquad \text{and} \qquad x \approx 1.60$$

Now Try Exercise 69 ■

Logarithmic equations are used in determining the amount of light that reaches various depths in a lake. (This information helps biologists to determine the types of life a lake can support.) As light passes through water (or other transparent materials such as glass or plastic), some of the light is absorbed. It's easy to see that the murkier the water, the more light is absorbed. The exact relationship between light absorption and the distance light travels in a material is described in the next example.

EXAMPLE 12 ■ Transparency of a Lake



The intensity of light in a lake diminishes with depth.

If I_0 and I denote the intensity of light before and after going through a material and x is the distance (in feet) the light travels in the material, then according to the **Beer-Lambert Law**,

$$-\frac{1}{k}\ln\left(\frac{I}{I_0}\right) = x$$

where k is a constant depending on the type of material.

(a) Solve the equation for I.

(b) For a certain lake $k = 0.025$, and the light intensity is $I_0 = 14$ lumens (lm). Find the light intensity at a depth of 20 ft.

SOLUTION

(a) We first isolate the logarithmic term.

$$-\frac{1}{k}\ln\left(\frac{I}{I_0}\right) = x \qquad \text{Given equation}$$

$$\ln\left(\frac{I}{I_0}\right) = -kx \qquad \text{Multiply by } -k$$

$$\frac{I}{I_0} = e^{-kx} \qquad \text{Exponential form}$$

$$I = I_0 e^{-kx} \qquad \text{Multiply by } I_0$$

(b) We find I using the formula from part (a).

$$I = I_0 e^{-kx} \qquad \text{From part (a)}$$

$$= 14e^{(-0.025)(20)} \qquad I_0 = 14,\ k = 0.025,\ x = 20$$

$$\approx 8.49 \qquad \text{Calculator}$$

The light intensity at a depth of 20 ft is about 8.5 lm.

Now Try Exercise 99 ■

■ Compound Interest

Recall the formulas for interest that we found in Section 4.1. If a principal P is invested at an interest rate r for a period of t years, then the amount A of the investment is given by

$$A = P(1 + r) \qquad \text{Simple interest (for one year)}$$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{Interest compounded } n \text{ times per year}$$

$$A(t) = Pe^{rt} \qquad \text{Interest compounded continuously}$$

Radiocarbon Dating is a method that archeologists use to determine the age of ancient objects. The carbon dioxide in the atmosphere always contains a fixed fraction of radioactive carbon, carbon-14 (^{14}C), with a half-life of about 5730 years. Plants absorb carbon dioxide from the atmosphere, which then makes its way to animals through the food chain. Thus, all living creatures contain the same fixed proportions of ^{14}C to nonradioactive ^{12}C as the atmosphere.

After an organism dies, it stops assimilating ^{14}C, and the amount of ^{14}C in it begins to decay exponentially. We can then determine the time that has elapsed since the death of the organism by measuring the amount of ^{14}C left in it.

For example, if a donkey bone contains 73% as much ^{14}C as a living donkey and it died t years ago, then by the formula for radioactive decay (Section 4.6),

$$0.73 = (1.00)e^{-(t \ln 2)/5730}$$

We solve this exponential equation to find $t \approx 2600$, so the bone is about 2600 years old.

We can use logarithms to determine the time it takes for the principal to increase to a given amount.

EXAMPLE 13 ■ Finding the Term for an Investment to Double

A sum of \$5000 is invested at an interest rate of 5% per year. Find the time required for the money to double if the interest is compounded according to the following methods.

(a) Semiannually **(b)** Continuously

SOLUTION

(a) We use the formula for compound interest with $P = \$5000$, $A(t) = \$10{,}000$, $r = 0.05$, and $n = 2$, and solve the resulting exponential equation for t.

$$\begin{aligned}
5000\left(1 + \frac{0.05}{2}\right)^{2t} &= 10{,}000 && P\left(1+\frac{r}{n}\right)^{nt} = A\\
(1.025)^{2t} &= 2 && \text{Divide by 5000}\\
\log 1.025^{2t} &= \log 2 && \text{Take log of each side}\\
2t \log 1.025 &= \log 2 && \text{Law 3 (bring down the exponent)}\\
t &= \frac{\log 2}{2 \log 1.025} && \text{Divide by 2 log 1.025}\\
t &\approx 14.04 && \text{Calculator}
\end{aligned}$$

The money will double in 14.04 years.

(b) We use the formula for continuously compounded interest with $P = \$5000$, $A(t) = \$10{,}000$, and $r = 0.05$ and solve the resulting exponential equation for t.

$$\begin{aligned}
5000e^{0.05t} &= 10{,}000 && Pe^{rt} = A\\
e^{0.05t} &= 2 && \text{Divide by 5000}\\
\ln e^{0.05t} &= \ln 2 && \text{Take ln of each side}\\
0.05t &= \ln 2 && \text{Property of ln}\\
t &= \frac{\ln 2}{0.05} && \text{Divide by 0.05}\\
t &\approx 13.86 && \text{Calculator}
\end{aligned}$$

The money will double in 13.86 years.

Now Try Exercise 89 ■

EXAMPLE 14 ■ Time Required to Grow an Investment

A sum of \$1000 is invested at an interest rate of 4% per year. Find the time required for the amount to grow to \$4000 if interest is compounded continuously.

SOLUTION We use the formula for continuously compounded interest with $P = \$1000$, $A(t) = \$4000$, and $r = 0.04$ and solve the resulting exponential equation for t.

$$\begin{aligned}
1000e^{0.04t} &= 4000 && Pe^{rt} = A\\
e^{0.04t} &= 4 && \text{Divide by 1000}\\
0.04t &= \ln 4 && \text{Take ln of each side}\\
t &= \frac{\ln 4}{0.04} && \text{Divide by 0.04}\\
t &\approx 34.66 && \text{Calculator}
\end{aligned}$$

The amount will be \$4000 in about 34 years and 8 months.

Now Try Exercise 91 ■

4.5 EXERCISES

CONCEPTS

1. Let's solve the exponential equation $2e^x = 50$.

(a) First, we isolate e^x to get the equivalent equation ________.

(b) Next, we take ln of each side to get the equivalent equation ________________.

(c) Now we use a calculator to find $x \approx$ ________.

2. Let's solve the logarithmic equation

$$\log 3 + \log(x - 2) = \log x$$

(a) First, we combine the logarithms on the LHS to get the equivalent equation ________________.

(b) Next, we use the fact that log is one-to-one to get the equivalent equation ________________.

(c) Now we find $x =$ ________.

SKILLS

3–10 ■ Exponential Equations Find the solution of the exponential equation, as in Example 1.

3. $5^{x-1} = 125$ **4.** $e^{x^2} = e^9$

5. $5^{2x-3} = 1$ **6.** $10^{2x-3} = \frac{1}{10}$

7. $7^{2x-3} = 7^{6+5x}$ **8.** $e^{1-2x} = e^{3x-5}$

9. $6^{x^2-1} = 6^{1-x^2}$ **10.** $10^{2x^2-3} = 10^{9-x^2}$

11–38 ■ Exponential Equations **(a)** Find the exact solution of the exponential equation in terms of logarithms. **(b)** Use a calculator to find an approximation to the solution rounded to six decimal places.

11. $10^x = 25$ **12.** $10^{-x} = 4$

13. $e^{-5x} = 10$ **14.** $e^{0.4x} = 8$

15. $2^{1-x} = 3$ **16.** $3^{2x-1} = 5$

17. $3e^x = 10$ **18.** $2e^{12x} = 17$

19. $300(1.025)^{12t} = 1000$ **20.** $10(1.375)^{10t} = 50$

21. $e^{1-4x} = 2$ **22.** $e^{3-5x} = 16$

23. $2^{5-7x} = 15$ **24.** $2^{3x} = 34$

25. $3^{x/14} = 0.1$ **26.** $5^{-x/100} = 2$

27. $4(1 + 10^{5x}) = 9$ **28.** $2(5 + 3^{x+1}) = 100$

29. $8 + e^{1-4x} = 20$ **30.** $1 + e^{4x+1} = 20$

31. $4^x + 2^{1+2x} = 50$ **32.** $125^x + 5^{3x+1} = 200$

33. $5^x = 4^{x+1}$ **34.** $10^{1-x} = 6^x$

35. $2^{3x+1} = 3^{x-2}$ **36.** $7^{x/2} = 5^{1-x}$

37. $\dfrac{50}{1 + e^{-x}} = 4$ **38.** $\dfrac{10}{1 + e^{-x}} = 2$

39–44 ■ Exponential Equations of Quadratic Type Solve the equation.

39. $e^{2x} - 3e^x + 2 = 0$ **40.** $e^{2x} - e^x - 6 = 0$

41. $e^{4x} + 4e^{2x} - 21 = 0$ **42.** $3^{4x} - 3^{2x} - 6 = 0$

43. $2^x - 10(2^{-x}) + 3 = 0$ **44.** $e^x + 15e^{-x} - 8 = 0$

45–48 ■ Equations Involving Exponential Functions Solve the equation.

45. $x^2 2^x - 2^x = 0$ **46.** $x^2 10^x - x10^x = 2(10^x)$

47. $4x^3e^{-3x} - 3x^4e^{-3x} = 0$ **48.** $x^2e^x + xe^x - e^x = 0$

49–54 ■ Logarithmic Equations Solve the logarithmic equation for x, as in Example 7.

49. $\log x + \log(x - 1) = \log(4x)$

50. $\log_5 x + \log_5(x + 1) = \log_5 20$

51. $2 \log x = \log 2 + \log(3x - 4)$

52. $\ln\left(x - \frac{1}{2}\right) + \ln 2 = 2 \ln x$

53. $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x - 2)$

54. $\log_4(x + 2) + \log_4 3 = \log_4 5 + \log_4(2x - 3)$

55–68 ■ Logarithmic Equations Solve the logarithmic equation for x.

55. $\ln x = 10$ **56.** $\ln(2 + x) = 1$

57. $\log x = -2$ **58.** $\log(x - 4) = 3$

59. $\log(3x + 5) = 2$ **60.** $\log_3(2 - x) = 3$

61. $4 - \log(3 - x) = 3$

62. $\log_2(x^2 - x - 2) = 2$

63. $\log_2 x + \log_2(x - 3) = 2$

64. $\log x + \log(x - 3) = 1$

65. $\log_9(x - 5) + \log_9(x + 3) = 1$

66. $\ln(x - 1) + \ln(x + 2) = 1$

67. $\log_5(x + 1) - \log_5(x - 1) = 2$

68. $\log_3(x + 15) - \log_3(x - 1) = 2$

69–76 ■ Solving Equations Graphically Use a graphing device to find all solutions of the equation, rounded to two decimal places.

69. $\ln x = 3 - x$ **70.** $\log x = x^2 - 2$

71. $x^3 - x = \log(x + 1)$ **72.** $x = \ln(4 - x^2)$

73. $e^x = -x$ **74.** $2^{-x} = x - 1$

75. $4^{-x} = \sqrt{x}$ **76.** $e^{x^2} - 2 = x^3 - x$

77–78 ■ More Exponential and Logarithmic Equations Solve the equation for x.

77. $2^{2/\log_5 x} = \frac{1}{16}$ **78.** $\log_2(\log_3 x) = 4$

SKILLS Plus

79–82 ■ Solving Inequalities Solve the inequality.

79. $\log(x - 2) + \log(9 - x) < 1$

80. $3 \le \log_2 x \le 4$

81. $2 < 10^x < 5$

82. $x^2 e^x - 2e^x < 0$

83–86 ■ Inverse Functions Find the inverse function of f.

83. $f(x) = 2^{2x}$

84. $f(x) = 3^{x+1}$

85. $f(x) = \log_2(x - 1)$

86. $f(x) = \log 3x$

87–88 ■ Special Logarithmic Equations Find the value(s) of x for which the equation is true.

87. $\log(x + 3) = \log x + \log 3$

88. $(\log x)^3 = 3 \log x$

APPLICATIONS

89. Compound Interest A man invests \$5000 in an account that pays 8.5% interest per year, compounded quarterly.

(a) Find the amount after 3 years.

(b) How long will it take for the investment to double?

90. Compound Interest A woman invests \$6500 in an account that pays 6% interest per year, compounded continuously.

(a) What is the amount after 2 years?

(b) How long will it take for the amount to be \$8000?

91. Compound Interest Find the time required for an investment of \$5000 to grow to \$8000 at an interest rate of 7.5% per year, compounded quarterly.

92. Compound Interest Nancy wants to invest \$4000 in saving certificates that bear an interest rate of 9.75% per year, compounded semiannually. How long a time period should she choose to save an amount of \$5000?

93. Doubling an Investment How long will it take for an investment of \$1000 to double in value if the interest rate is 8.5% per year, compounded continuously?

94. Interest Rate A sum of \$1000 was invested for 4 years, and the interest was compounded semiannually. If this sum amounted to \$1435.77 in the given time, what was the interest rate?

95. Radioactive Decay A 15-g sample of radioactive iodine decays in such a way that the mass remaining after t days is given by $m(t) = 15e^{-0.087t}$, where $m(t)$ is measured in grams. After how many days are there only 5 g remaining?

96. Sky Diving The velocity of a sky diver t seconds after jumping is given by $v(t) = 80(1 - e^{-0.2t})$. After how many seconds is the velocity 70 ft/s?

97. Fish Population A small lake is stocked with a certain species of fish. The fish population is modeled by the function

$$P = \frac{10}{1 + 4e^{-0.8t}}$$

where P is the number of fish in thousands and t is measured in years since the lake was stocked.

(a) Find the fish population after 3 years.

(b) After how many years will the fish population reach 5000 fish?

98. Transparency of a Lake Environmental scientists measure the intensity of light at various depths in a lake to find the "transparency" of the water. Certain levels of transparency are required for the biodiversity of the submerged macrophyte population. In a certain lake the intensity of light at depth x is given by

$$I = 10e^{-0.008x}$$

where I is measured in lumens and x in feet.

(a) Find the intensity I at a depth of 30 ft.

(b) At what depth has the light intensity dropped to $I = 5$?

99. Atmospheric Pressure Atmospheric pressure P (in kilopascals, kPa) at altitude h (in kilometers, km) is governed by the formula

$$\ln\left(\frac{P}{P_0}\right) = -\frac{h}{k}$$

where $k = 7$ and $P_0 = 100$ kPa are constants.

(a) Solve the equation for P.

(b) Use part (a) to find the pressure P at an altitude of 4 km.

100. Cooling an Engine Suppose you're driving your car on a cold winter day (20°F outside) and the engine overheats (at about 220°F). When you park, the engine begins to cool down. The temperature T of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{T - 20}{200}\right) = -0.11t$$

(a) Solve the equation for T.

(b) Use part (a) to find the temperature of the engine after 20 min ($t = 20$).

101. Electric Circuits An electric circuit contains a battery that produces a voltage of 60 volts (V), a resistor with a resistance of 13 ohms (Ω), and an inductor with an inductance of 5 henrys (H), as shown in the figure on the following page. Using calculus, it can be shown that the current