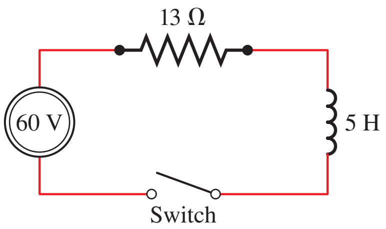
$I = I(t)$ (in amperes, A) t seconds after the switch is closed is $I = \frac{60}{13}(1 - e^{-13t/5})$.

(a) Use this equation to express the time t as a function of the current I.

(b) After how many seconds is the current 2 A?

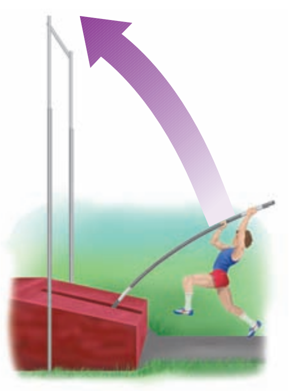


102. Learning Curve A *learning curve* is a graph of a function $P(t)$ that measures the performance of someone learning a skill as a function of the training time t. At first, the rate of learning is rapid. Then, as performance increases and approaches a maximal value M, the rate of learning decreases. It has been found that the function

$$P(t) = M - Ce^{-kt}$$

where k and C are positive constants and $C < M$ is a reasonable model for learning.

(a) Express the learning time t as a function of the performance level P.



(b) For a pole-vaulter in training, the learning curve is given by

$$P(t) = 20 - 14e^{-0.024t}$$

where $P(t)$ is the height he is able to pole-vault after t months. After how many months of training is he able to vault 12 ft?

(c) Draw a graph of the learning curve in part (b).

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

103. DISCUSS: Estimating a Solution Without actually solving the equation, find two whole numbers between which the solution of $9^x = 20$ must lie. Do the same for $9^x = 100$. Explain how you reached your conclusions.

104. DISCUSS ■ DISCOVER: A Surprising Equation Take logarithms to show that the equation

$$x^{1/\log x} = 5$$

has no solution. For what values of k does the equation

$$x^{1/\log x} = k$$

have a solution? What does this tell us about the graph of the function $f(x) = x^{1/\log x}$? Confirm your answer using a graphing device.

105. DISCUSS: Disguised Equations Each of these equations can be transformed into an equation of linear or quadratic type by applying the hint. Solve each equation.

(a) $(x - 1)^{\log(x-1)} = 100(x - 1)$
[*Hint:* Take log of each side.]

(b) $\log_2 x + \log_4 x + \log_8 x = 11$
[*Hint:* Change all logs to base 2.]

(c) $4^x - 2^{x+1} = 3$
[*Hint:* Write as a quadratic in 2^x.]

4.6 MODELING WITH EXPONENTIAL FUNCTIONS

■ Exponential Growth (Doubling Time) ■ Exponential Growth (Relative Growth Rate) ■ Radioactive Decay ■ Newton's Law of Cooling

Many processes that occur in nature, such as population growth, radioactive decay, heat diffusion, and numerous others, can be modeled by using exponential functions. In this section we study exponential models.

■ Exponential Growth (Doubling Time)

Suppose we start with a single bacterium, which divides every hour. After one hour we have 2 bacteria, after two hours we have 2^2 or 4 bacteria, after three hours we have 2^3

or 8 bacteria, and so on (see Figure 1). We see that we can model the bacteria population after t hours by $f(t) = 2^t$.

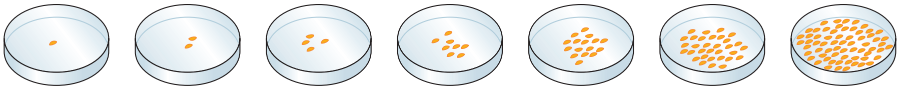


FIGURE 1 Bacteria population

If we start with 10 of these bacteria, then the population is modeled by $f(t) = 10 \cdot 2^t$. A slower-growing strain of bacteria doubles every 3 hours; in this case the population is modeled by $f(t) = 10 \cdot 2^{t/3}$. In general, we have the following.

EXPONENTIAL GROWTH (DOUBLING TIME)

If the initial size of a population is n_0 and the doubling time is a, then the size of the population at time t is

$$n(t) = n_0 2^{t/a}$$

where a and t are measured in the same time units (minutes, hours, days, years, and so on).

EXAMPLE 1 ■ Bacteria Population

Under ideal conditions a certain bacteria population doubles every three hours. Initially, there are 1000 bacteria in a colony.

(a) Find a model for the bacteria population after t hours.

(b) How many bacteria are in the colony after 15 hours?

(c) After how many hours will the bacteria count reach 100,000?

SOLUTION

(a) The population at time t is modeled by

$$n(t) = 1000 \cdot 2^{t/3}$$

where t is measured in hours.

(b) After 15 hours the number of bacteria is

$$n(15) = 1000 \cdot 2^{15/3} = 32{,}000$$

(c) We set $n(t) = 100{,}000$ in the model that we found in part (a) and solve the resulting exponential equation for t.

$$100{,}000 = 1000 \cdot 2^{t/3} \qquad n(t) = 1000 \cdot 2^{t/3}$$

$$100 = 2^{t/3} \qquad \text{Divide by 1000}$$

$$\log 100 = \log 2^{t/3} \qquad \text{Take log of each side}$$

$$2 = \frac{t}{3}\log 2 \qquad \text{Properties of log}$$

$$t = \frac{6}{\log 2} \approx 19.93 \qquad \text{Solve for } t$$

The bacteria level reaches 100,000 in about 20 hours.

Now Try Exercise 1 ■

EXAMPLE 2 ■ Rabbit Population

A certain breed of rabbit was introduced onto a small island 8 months ago. The current rabbit population on the island is estimated to be 4100 and doubling every 3 months.

(a) What was the initial size of the rabbit population?

(b) Estimate the population 1 year after the rabbits were introduced to the island.

(c) Sketch a graph of the rabbit population.

SOLUTION

(a) The doubling time is $a = 3$, so the population at time t is

$$n(t) = n_0 2^{t/3} \qquad \text{Model}$$

where n_0 is the initial population. Since the population is 4100 when t is 8 months, we have

$$\begin{aligned} n(8) &= n_0 2^{8/3} && \text{From model} \\ 4100 &= n_0 2^{8/3} && \text{Because } n(8) = 4100 \\ n_0 &= \frac{4100}{2^{8/3}} && \text{Divide by } 2^{8/3} \text{ and switch sides} \\ n_0 &\approx 645 && \text{Calculator} \end{aligned}$$

Thus we estimate that 645 rabbits were introduced onto the island.

(b) From part (a) we know that the initial population is $n_0 = 645$, so we can model the population after t months by

$$n(t) = 645 \cdot 2^{t/3} \qquad \text{Model}$$

After 1 year $t = 12$, so

$$n(12) = 645 \cdot 2^{12/3} = 10{,}320$$

So after 1 year there would be about 10,000 rabbits.

(c) We first note that the domain is $t \geq 0$. The graph is shown in Figure 2.

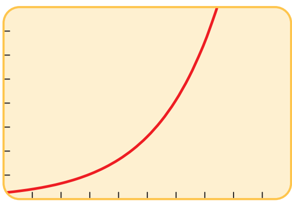


FIGURE 2 $n(t) = 645 \cdot 2^{t/3}$

Now Try Exercise 3 ■

■ Exponential Growth (Relative Growth Rate)

The growth of a population with relative growth rate r is analogous to the growth of an investment with continuously compounded interest rate r.

We have used an exponential function with base 2 to model population growth (in terms of the doubling time). We could also model the same population with an exponential function with base 3 (in terms of the tripling time). In fact, we can find an exponential model with any base. If we use the base e, we get a population model in terms of the **relative growth rate** r: the rate of population growth expressed as a proportion of the population at any time. In this case r is the "instantaneous" growth rate. (In calculus the concept of instantaneous rate is given a precise meaning.) For instance, if $r = 0.02$, then at any time t the growth rate is 2% of the population at time t.

EXPONENTIAL GROWTH (RELATIVE GROWTH RATE)

A population that experiences **exponential growth** increases according to the model

$$n(t) = n_0 e^{rt}$$

where $n(t) =$ population at time t

$n_0 =$ initial size of the population

$r =$ relative rate of growth (expressed as a proportion of the population)

$t =$ time

Notice that the formula for population growth is the same as that for continuously compounded interest. In fact, the same principle is at work in both cases: The growth of a population (or an investment) per time period is proportional to the size of the population (or the amount of the investment). A population of 1,000,000 will increase more in one year than a population of 1000; in exactly the same way, an investment of $1,000,000 will increase more in one year than an investment of $1000.

In the following examples we assume that the populations grow exponentially.

EXAMPLE 3 ■ Predicting the Size of a Population

The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per hour.

(a) Find a function that models the number of bacteria after t hours.

(b) What is the estimated count after 10 hours?

(c) After how many hours will the bacteria count reach 80,000?

(d) Sketch a graph of the function $n(t)$.

SOLUTION

(a) We use the exponential growth model with $n_0 = 500$ and $r = 0.4$ to get

$$n(t) = 500e^{0.4t}$$

where t is measured in hours.

(b) Using the function in part (a), we find that the bacterium count after 10 hours is

$$n(10) = 500e^{0.4(10)} = 500e^{4} \approx 27{,}300$$

(c) We set $n(t) = 80{,}000$ and solve the resulting exponential equation for t.

$$80{,}000 = 500 \cdot e^{0.4t} \qquad n(t) = 500 \cdot e^{0.4t}$$

$$160 = e^{0.4t} \qquad \text{Divide by 500}$$

$$\ln 160 = 0.4t \qquad \text{Take ln of each side}$$

$$t = \frac{\ln 160}{0.4} \approx 12.68 \qquad \text{Solve for } t$$

The bacteria level reaches 80,000 in about 12.7 hours.

(d) The graph is shown in Figure 3.

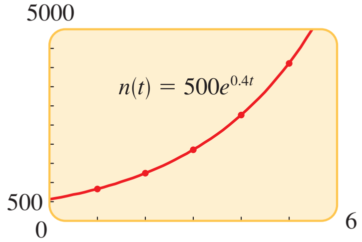


FIGURE 3

Now Try Exercise 5

EXAMPLE 4 ■ Comparing Different Rates of Population Growth

The relative growth of world population has been declining over the past few decades—from 2% in 1995 to 1.1% in 2013.

Standing Room Only

The population of the world was about 6.1 billion in 2000 and was increasing at 1.4% per year. Assuming that each person occupies an average of 4 ft^2 of the surface of the earth, the exponential model for population growth projects that by the year 2801 there will be standing room only! (The total land surface area of the world is about 1.8×10^{15} ft^2.)

In 2000 the population of the world was 6.1 billion, and the relative rate of growth was 1.4% per year. It is claimed that a rate of 1.0% per year would make a significant difference in the total population in just a few decades. Test this claim by estimating the population of the world in the year 2050 using a relative rate of growth of (a) 1.4% per year and (b) 1.0% per year.

Graph the population functions for the next 100 years for the two relative growth rates in the same viewing rectangle.

SOLUTION

(a) By the exponential growth model we have

$$n(t) = 6.1e^{0.014t}$$

where $n(t)$ is measured in billions and t is measured in years since 2000. Because the year 2050 is 50 years after 2000, we find

$$n(50) = 6.1e^{0.014(50)} = 6.1e^{0.7} \approx 12.3$$

The estimated population in the year 2050 is about 12.3 billion.

(b) We use the function

$$n(t) = 6.1e^{0.010t}$$

and find

$$n(50) = 6.1e^{0.010(50)} = 6.1e^{0.50} \approx 10.1$$

The estimated population in the year 2050 is about 10.1 billion.

The graphs in Figure 4 show that a small change in the relative rate of growth will, over time, make a large difference in population size.

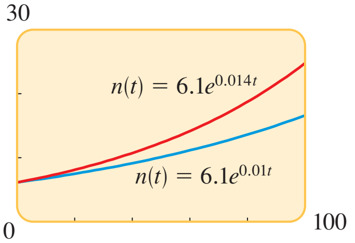


FIGURE 4

Now Try Exercise 7 ■

EXAMPLE 5 ■ Expressing a Model in Terms of *e*

A culture starts with 10,000 bacteria, and the number doubles every 40 minutes.

(a) Find a function $n(t) = n_0 2^{t/a}$ that models the number of bacteria after t hours.

(b) Find a function $n(t) = n_0 e^{rt}$ that models the number of bacteria after t hours.

(c) Sketch a graph of the number of bacteria at time t.

SOLUTION

(a) The initial population is $n_0 = 10{,}000$. The doubling time is $a = 40$ min $= 2/3$ h. Since $1/a = 3/2 = 1.5$, the model is

$$n(t) = 10{,}000 \cdot 2^{1.5t}$$

(b) The initial population is $n_0 = 10{,}000$. We need to find the relative growth rate r. Since there are 20,000 bacteria when $t = 2/3$ h, we have

$$20{,}000 = 10{,}000e^{r(2/3)} \qquad n(t) = 10{,}000e^{rt}$$

$$2 = e^{r(2/3)} \qquad \text{Divide by 10,000}$$

$$\ln 2 = \ln e^{r(2/3)} \qquad \text{Take ln of each side}$$

$$\ln 2 = r(2/3) \qquad \text{Property of ln}$$

$$r = \frac{3 \ln 2}{2} \approx 1.0397 \qquad \text{Solve for } r$$

Now that we know the relative growth rate r, we can find the model:

$$n(t) = 10{,}000e^{1.0397t}$$

(c) We can graph the model in part (a) or the one in part (b). The graphs are identical. See Figure 5.

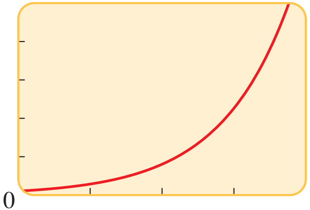


FIGURE 5 Graphs of $y = 10{,}000 \cdot 2^{1.5t}$ and $y = 10{,}000e^{1.0397t}$

Now Try Exercise 9

Radioactive Decay

Radioactive substances decay by spontaneously emitting radiation. The rate of decay is proportional to the mass of the substance. This is analogous to population growth except that the mass *decreases*. Physicists express the rate of decay in terms of **half-life**, the time it takes for a sample of the substance to decay to half its original mass. For example, the half-life of radium-226 is 1600 years, so a 100-g sample decays to 50 g $\left(\text{or } \frac{1}{2} \times 100 \text{ g}\right)$ in 1600 years, then to 25 g $\left(\text{or } \frac{1}{2} \times \frac{1}{2} \times 100 \text{ g}\right)$ in 3200 years, and so on. In general, for a radioactive substance with mass m_0 and half-life h, the amount remaining at time t is modeled by

$$m(t) = m_0 2^{-t/h}$$

where h and t are measured in the same time units (minutes, hours, days, years, and so on).

To express this model in the form $m(t) = m_0 e^{rt}$, we need to find the relative decay rate r. Since h is the half-life, we have

$$m(t) = m_0 e^{-rt} \quad \text{Model}$$

$$\frac{m_0}{2} = m_0 e^{-rh} \quad h \text{ is the half-life}$$

$$\frac{1}{2} = e^{-rh} \quad \text{Divide by } m_0$$

$$\ln \frac{1}{2} = -rh \quad \text{Take ln of each side}$$

$$r = \frac{\ln 2}{h} \quad \text{Solve for } r$$

This last equation allows us to find the relative decay rate r from the half-life h.

The half-lives of **radioactive elements** vary from very long to very short. Here are some examples.

Element	Half-life
Thorium-232	14.5 billion years
Uranium-235	4.5 billion years
Thorium-230	80,000 years
Plutonium-239	24,360 years
Carbon-14	5,730 years
Radium-226	1,600 years
Cesium-137	30 years
Strontium-90	28 years
Polonium-210	140 days
Thorium-234	25 days
Iodine-135	8 days
Radon-222	3.8 days
Lead-211	3.6 minutes
Krypton-91	10 seconds

DISCOVERY PROJECT

Modeling Radiation with Coins and Dice

Radioactive elements decay when their atoms spontaneously emit radiation and change into smaller, stable atoms. But if atoms decay randomly, how is it possible to find a function that models their behavior? We'll try to answer this question by experimenting with randomly tossing coins and rolling dice. The experiments allow us to experience how a very large number of random events can result in predictable exponential results. You can find the project at **www.stewartmath.com**.

RADIOACTIVE DECAY MODEL

If m_0 is the initial mass of a radioactive substance with half-life h, then the mass remaining at time t is modeled by the function

$$m(t) = m_0 e^{-rt}$$

where $r = \dfrac{\ln 2}{h}$ is the **relative decay rate**.

EXAMPLE 6 ■ Radioactive Decay

Polonium-210 (^{210}Po) has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

(a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t days.

(b) Find a function $m(t) = m_0 e^{-rt}$ that models the mass remaining after t days.

(c) Find the mass remaining after one year.

(d) How long will it take for the sample to decay to a mass of 200 mg?

(e) Draw a graph of the sample mass as a function of time.

SOLUTION

(a) We have $m_0 = 300$ and $h = 140$, so the amount remaining after t days is

$$m(t) = 300 \cdot 2^{-t/140}$$

(b) We have $m_0 = 300$ and $r = \ln 2/140 \approx -0.00495$, so the amount remaining after t days is

$$m(t) = 300 \cdot e^{-0.00495t}$$

In parts (c) and (d) we can also use the model found in part (a). Check that the result is the same using either model.

(c) We use the function we found in part (a) with $t = 365$ (1 year):

$$m(365) = 300e^{-0.00495(365)} \approx 49.256$$

Thus approximately 49 mg of ^{210}Po remains after 1 year.

(d) We use the function that we found in part (b) with $m(t) = 200$ and solve the resulting exponential equation for t:

$$300e^{-0.00495t} = 200 \qquad m(t) = m_0 e^{-rt}$$

$$e^{-0.00495t} = \tfrac{2}{3} \qquad \text{Divide by 300}$$

$$\ln e^{-0.00495t} = \ln \tfrac{2}{3} \qquad \text{Take ln of each side}$$

$$-0.00495t = \ln \tfrac{2}{3} \qquad \text{Property of ln}$$

$$t = -\frac{\ln \frac{2}{3}}{0.00495} \qquad \text{Solve for } t$$

$$t \approx 81.9 \qquad \text{Calculator}$$

The time required for the sample to decay to 200 mg is about 82 days.

(e) We can graph the model in part (a) or the one in part (b). The graphs are identical. See Figure 6.

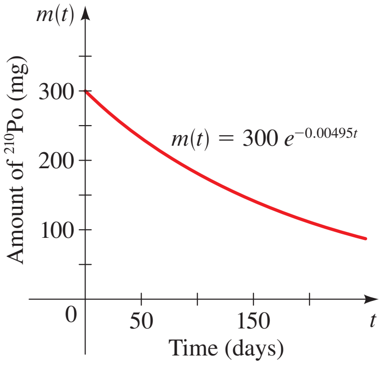


FIGURE 6

Now Try Exercise 17 ■

Joel W. Rogers/Encyclopedia/Corbis

Radioactive Waste

Harmful radioactive isotopes are produced whenever a nuclear reaction occurs, whether as the result of an atomic bomb test, a nuclear accident such as the one at Fukushima Daiichi in 2011, or the uneventful production of electricity at a nuclear power plant.

One radioactive material that is produced in atomic bombs is the isotope strontium-90 (^{90}Sr), with a half-life of 28 years. This is deposited like calcium in human bone tissue, where it can cause leukemia and other cancers. However, in the decades since atmospheric testing of nuclear weapons was halted, ^{90}Sr levels in the environment have fallen to a level that no longer poses a threat to health.

Nuclear power plants produce radioactive plutonium-239 (^{239}Pu), which has a half-life of 24,360 years. Because of its long half-life, ^{239}Pu could pose a threat to the environment for thousands of years. So great care must be taken to dispose of it properly. The difficulty of ensuring the safety of the disposed radioactive waste is one reason that nuclear power plants remain controversial.

Newton's Law of Cooling

Newton's Law of Cooling states that the rate at which an object cools is proportional to the temperature difference between the object and its surroundings, provided that the temperature difference is not too large. By using calculus, the following model can be deduced from this law.

NEWTON'S LAW OF COOLING

If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.

EXAMPLE 7 ■ Newton's Law of Cooling

A cup of coffee has a temperature of 200°F and is placed in a room that has a temperature of 70°F. After 10 min the temperature of the coffee is 150°F.

(a) Find a function that models the temperature of the coffee at time t.

(b) Find the temperature of the coffee after 15 min.

(c) After how long will the coffee have cooled to 100°F?

(d) Illustrate by drawing a graph of the temperature function.

SOLUTION

(a) The temperature of the room is $T_s = 70$°F, and the initial temperature difference is

$$D_0 = 200 - 70 = 130°\text{F}$$

So by Newton's Law of Cooling, the temperature after t minutes is modeled by the function

$$T(t) = 70 + 130e^{-kt}$$

We need to find the constant k associated with this cup of coffee. To do this, we use the fact that when $t = 10$, the temperature is $T(10) = 150$. So we have

$$\begin{aligned}
70 + 130e^{-10k} &= 150 && T_s + D_0 e^{-kt} = T(t) \\
130e^{-10k} &= 80 && \text{Subtract 70} \\
e^{-10k} &= \tfrac{8}{13} && \text{Divide by 130} \\
-10k &= \ln \tfrac{8}{13} && \text{Take ln of each side} \\
k &= -\tfrac{1}{10} \ln \tfrac{8}{13} && \text{Solve for } k \\
k &\approx 0.04855 && \text{Calculator}
\end{aligned}$$

Substituting this value of k into the expression for $T(t)$, we get

$$T(t) = 70 + 130e^{-0.04855t}$$

(b) We use the function that we found in part (a) with $t = 15$.

$$T(15) = 70 + 130e^{-0.04855(15)} \approx 133°\text{F}$$

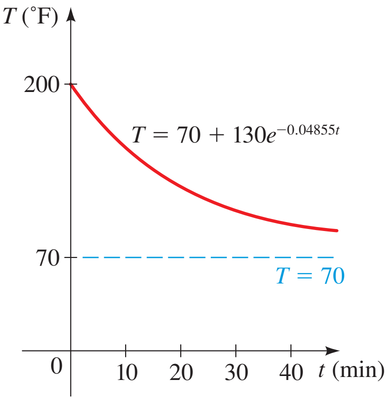


FIGURE 7 Temperature of coffee after t minutes

(c) We use the function that we found in part (a) with $T(t) = 100$ and solve the resulting exponential equation for t.

$$\begin{aligned} 70 + 130e^{-0.04855t} &= 100 && T_s + D_0e^{-kt} = T(t) \\ 130e^{-0.04855t} &= 30 && \text{Subtract 70} \\ e^{-0.04855t} &= \tfrac{3}{13} && \text{Divide by 130} \\ -0.04855t &= \ln \tfrac{3}{13} && \text{Take ln of each side} \\ t &= \frac{\ln \frac{3}{13}}{-0.04855} && \text{Solve for } t \\ t &\approx 30.2 && \text{Calculator} \end{aligned}$$

The coffee will have cooled to 100°F after about half an hour.

(d) The graph of the temperature function is sketched in Figure 7. Notice that the line $t = 70$ is a horizontal asymptote. (Why?)

Now Try Exercise 25 ■

4.6 EXERCISES

APPLICATIONS

1–16 ■ Population Growth These exercises use the population growth model.

1. Bacteria Culture A certain culture of the bacterium *Streptococcus A* initially has 10 bacteria and is observed to double every 1.5 hours.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the number of bacteria in the culture after t hours.

(b) Estimate the number of bacteria after 35 hours.

(c) After how many hours will the bacteria count reach 10,000?

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Streptococcus A
(12,000 × magnification)

2. Bacteria Culture A certain culture of the bacterium *Rhodobacter sphaeroides* initially has 25 bacteria and is observed to double every 5 hours.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the number of bacteria in the culture after t hours.

(b) Estimate the number of bacteria after 18 hours.

(c) After how many hours will the bacteria count reach 1 million?

3. Squirrel Population A grey squirrel population was introduced in a certain county of Great Britain 30 years ago. Biologists observe that the population doubles every 6 years, and now the population is 100,000.

(a) What was the initial size of the squirrel population?

(b) Estimate the squirrel population 10 years from now.

(c) Sketch a graph of the squirrel population.

4. Bird Population A certain species of bird was introduced in a certain county 25 years ago. Biologists observe that the population doubles every 10 years, and now the population is 13,000.

(a) What was the initial size of the bird population?

(b) Estimate the bird population 5 years from now.

(c) Sketch a graph of the bird population.

5. Fox Population The fox population in a certain region has a relative growth rate of 8% per year. It is estimated that the population in 2013 was 18,000.

(a) Find a function $n(t) = n_0 e^{rt}$ that models the population t years after 2013.

(b) Use the function from part (a) to estimate the fox population in the year 2021.

(c) After how many years will the fox population reach 25,000?

(d) Sketch a graph of the fox population function for the years 2013–2021.

6. Fish Population The population of a certain species of fish has a relative growth rate of 1.2% per year. It is estimated that the population in 2010 was 12 million.

(a) Find an exponential model $n(t) = n_0e^{rt}$ for the population t years after 2010.

(b) Estimate the fish population in the year 2015.

(c) After how many years will the fish population reach 14 million?

(d) Sketch a graph of the fish population.

7. Population of a Country The population of a country has a relative growth rate of 3% per year. The government is trying to reduce the growth rate to 2%. The population in 2011 was approximately 110 million. Find the projected population for the year 2036 for the following conditions.

(a) The relative growth rate remains at 3% per year.

(b) The relative growth rate is reduced to 2% per year.

8. Bacteria Culture It is observed that a certain bacteria culture has a relative growth rate of 12% per hour, but in the presence of an antibiotic the relative growth rate is reduced to 5% per hour. The initial number of bacteria in the culture is 22. Find the projected population after 24 hours for the following conditions.

(a) No antibiotic is present, so the relative growth rate is 12%.

(b) An antibiotic is present in the culture, so the relative growth rate is reduced to 5%.

9. Population of a City The population of a certain city was 112,000 in 2014, and the observed doubling time for the population is 18 years.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the population t years after 2014.

(b) Find an exponential model $n(t) = n_0e^{rt}$ for the population t years after 2014.

(c) Sketch a graph of the population at time t.

(d) Estimate how long it takes the population to reach 500,000.

10. Bat Population The bat population in a certain Midwestern county was 350,000 in 2012, and the observed doubling time for the population is 25 years.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the population t years after 2012.

(b) Find an exponential model $n(t) = n_0e^{rt}$ for the population t years after 2012.

(c) Sketch a graph of the population at time t.

(d) Estimate how long it takes the population to reach 2 million.

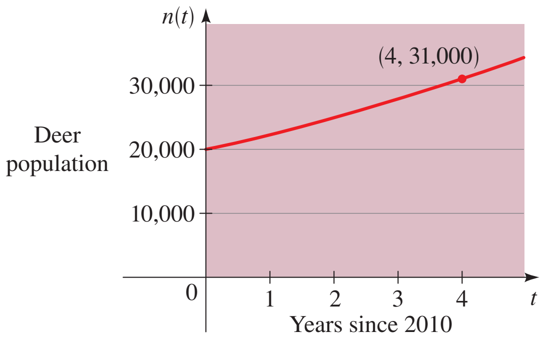
11. Deer Population The graph shows the deer population in a Pennsylvania county between 2010 and 2014. Assume that the population grows exponentially.

(a) What was the deer population in 2010?

(b) Find a function that models the deer population t years after 2010.

(c) What is the projected deer population in 2018?

(d) Estimate how long it takes the population to reach 100,000.



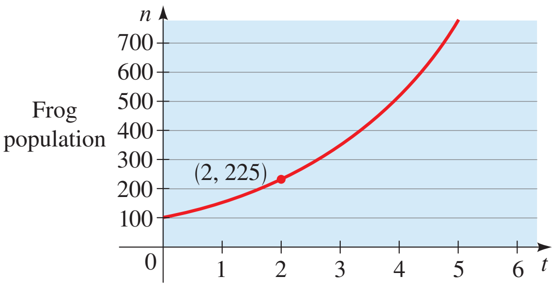
12. Frog Population Some bullfrogs were introduced into a small pond. The graph shows the bullfrog population for the next few years. Assume that the population grows exponentially.

(a) What was the initial bullfrog population?

(b) Find a function that models the bullfrog population t years since the bullfrogs were put into the pond.

(c) What is the projected bullfrog population after 15 years?

(d) Estimate how long it takes the population to reach 75,000.



13. Bacteria Culture A culture starts with 8600 bacteria. After 1 hour the count is 10,000.

(a) Find a function that models the number of bacteria $n(t)$ after t hours.

(b) Find the number of bacteria after 2 hours.

(c) After how many hours will the number of bacteria double?

14. Bacteria Culture The count in a culture of bacteria was 400 after 2 hours and 25,600 after 6 hours.

(a) What is the relative rate of growth of the bacteria population? Express your answer as a percentage.

(b) What was the initial size of the culture?

(c) Find a function that models the number of bacteria $n(t)$ after t hours.

(d) Find the number of bacteria after 4.5 hours.

(e) After how many hours will the number of bacteria reach 50,000?

15. Population of California The population of California was 29.76 million in 1990 and 33.87 million in 2000. Assume that the population grows exponentially.

(a) Find a function that models the population t years after 1990.

(b) Find the time required for the population to double.

(c) Use the function from part (a) to predict the population of California in the year 2010. Look up California's actual population in 2010, and compare.

16. World Population The population of the world was 7.1 billion in 2013, and the observed relative growth rate was 1.1% per year.

(a) Estimate how long it takes the population to double.

(b) Estimate how long it takes the population to triple.

17–24 ■ Radioactive Decay These exercises use the radioactive decay model.

17. Radioactive Radium The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.

(a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years.

(b) Find a function $m(t) = m_0 e^{-rt}$ that models the mass remaining after t years.

(c) How much of the sample will remain after 4000 years?

(d) After how many years will only 18 mg of the sample remain?

18. Radioactive Cesium The half-life of cesium-137 is 30 years. Suppose we have a 10-g sample.

(a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years.

(b) Find a function $m(t) = m_0 e^{-rt}$ that models the mass remaining after t years.

(c) How much of the sample will remain after 80 years?

(d) After how many years will only 2 g of the sample remain?

19. Radioactive Strontium The half-life of strontium-90 is 28 years. How long will it take a 50-mg sample to decay to a mass of 32 mg?

20. Radioactive Radium Radium-221 has a half-life of 30 s. How long will it take for 95% of a sample to decay?

21. Finding Half-Life If 250 mg of a radioactive element decays to 200 mg in 48 hours, find the half-life of the element.

22. Radioactive Radon After 3 days a sample of radon-222 has decayed to 58% of its original amount.

(a) What is the half-life of radon-222?

(b) How long will it take the sample to decay to 20% of its original amount?

23. Carbon-14 Dating A wooden artifact from an ancient tomb contains 65% of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)

24. Carbon-14 Dating The burial cloth of an Egyptian mummy is estimated to contain 59% of the carbon-14 it contained originally. How long ago was the mummy buried? (The half-life of carbon-14 is 5730 years.)

25–28 ■ Law of Cooling These exercises use Newton's Law of Cooling.

25. Cooling Soup A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling, so its temperature at time t is given by

$$T(t) = 65 + 145e^{-0.05t}$$

where t is measured in minutes and T is measured in °F.

(a) What is the initial temperature of the soup?

(b) What is the temperature after 10 min?

(c) After how long will the temperature be 100°F?

26. Time of Death Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death, the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling is approximately $k = 0.1947$, assuming that time is measured in hours. Suppose that the temperature of the surroundings is 60°F.

(a) Find a function $T(t)$ that models the temperature t hours after death.

(b) If the temperature of the body is now 72°F, how long ago was the time of death?

27. Cooling Turkey A roasted turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F.

(a) If the temperature of the turkey is 150°F after half an hour, what is its temperature after 45 min?

(b) After how many hours will the turkey cool to 100°F?

28. Boiling Water A kettle full of water is brought to a boil in a room with temperature 20°C. After 15 min the temperature of the water has decreased from 100°C to 75°C. Find the temperature after another 10 min. Illustrate by graphing the temperature function.