6.2 EXERCISES

1–34 ▪ Evaluate the integral.

1. $\int \sin^3 x \cos^2 x \, dx$
2. $\int \sin^6 x \cos^3 x \, dx$
3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$
4. $\int_0^{\pi/2} \cos^5 x \, dx$
5. $\int_0^{\pi/2} \cos^2 \theta \, d\theta$
6. $\int \sin^3(mx) \, dx$
7. $\int_0^{\pi} \sin^4(3t) \, dt$
8. $\int_0^{\pi/2} \sin^2(2\theta) \, d\theta$
9. $\int (1 + \cos \theta)^2 \, d\theta$
10. $\int_0^{\pi} \cos^6 \theta \, d\theta$
11. $\int_0^{\pi/4} \sin^4 x \cos^2 x \, dx$
12. $\int x \cos^2 x \, dx$
13. $\int \cos^2 x \tan^3 x \, dx$
14. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$
15. $\int \frac{1 - \sin x}{\cos x} \, dx$
16. $\int \cos^2 x \sin 2x \, dx$
17. $\int \sec^2 x \tan x \, dx$
18. $\int_0^{\pi/2} \sec^4(t/2) \, dt$
19. $\int \tan^2 x \, dx$
20. $\int \tan^4 x \, dx$
21. $\int \sec^6 t \, dt$
22. $\int_0^{\pi/4} \sec^4 \theta \tan^4 \theta \, d\theta$
23. $\int_0^{\pi/3} \tan^5 x \sec^4 x \, dx$
24. $\int \tan^3(2x) \sec^5(2x) \, dx$
25. $\int \tan^3 x \sec x \, dx$
26. $\int_0^{\pi/3} \tan^5 x \sec^6 x \, dx$
27. $\int \tan^5 x \, dx$
28. $\int \tan^6(ay) \, dy$
29. $\int_{\pi/6}^{\pi/2} \cot^2 x \, dx$
30. $\int_{\pi/4}^{\pi/2} \cot^3 x \, dx$
31. $\int \cot^3 \alpha \csc^3 \alpha \, d\alpha$
32. $\int \csc^4 x \cot^6 x \, dx$
33. $\int \csc x \, dx$
34. $\int \frac{1 - \tan^2 x}{\sec^2 x} \, dx$

35. (a) Use the formulas for $\cos(A + B)$ and $\cos(A - B)$ to show that

$$\sin A \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$

(b) Use part (a) to evaluate $\int \sin 5x \sin 2x \, dx$.

36. (a) Use the formulas for $\sin(A + B)$ and $\sin(A - B)$ to show that

$$\sin A \cos B = \tfrac{1}{2}[\sin(A - B) + \sin(A + B)]$$

(b) Use part (a) to evaluate $\int \sin 3x \cos x \, dx$.

37–39 ▪ Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

37. $\int \frac{1}{x^2\sqrt{x^2 - 9}} \, dx; \quad x = 3 \sec \theta$
38. $\int x^3\sqrt{9 - x^2} \, dx; \quad x = 3 \sin \theta$
39. $\int \frac{x^3}{\sqrt{x^2 + 9}} \, dx; \quad x = 3 \tan \theta$

40–58 ▪ Evaluate the integral.

40. $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16 - x^2}} \, dx$
41. $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2 - 1}} \, dt$
42. $\int_0^2 x^3\sqrt{x^2 + 4} \, dx$
43. $\int \frac{1}{x^2\sqrt{25 - x^2}} \, dx$
44. $\int \frac{\sqrt{x^2 - a^2}}{x^4} \, dx$
45. $\int \frac{dx}{\sqrt{x^2 + 16}}$
46. $\int \frac{t^5}{\sqrt{t^2 + 2}} \, dt$
47. $\int \sqrt{1 - 4x^2} \, dx$
48. $\int_0^1 x\sqrt{x^2 + 4} \, dx$
49. $\int \frac{\sqrt{x^2 - 9}}{x^3} \, dx$
50. $\int \frac{du}{u\sqrt{5 - u^2}}$
51. $\int \frac{x^2}{(a^2 - x^2)^{3/2}} \, dx$
52. $\int \frac{dx}{x^2\sqrt{16x^2 - 9}}$
53. $\int \frac{x}{\sqrt{x^2 - 7}} \, dx$
54. $\int_0^1 \sqrt{x^2 + 1} \, dx$
55. $\int \frac{\sqrt{1 + x^2}}{x} \, dx$
56. $\int \frac{t}{\sqrt{25 - t^2}} \, dt$
57. $\int x\sqrt{1 - x^4} \, dx$
58. $\int_0^{\pi/2} \frac{\cos t}{\sqrt{1 + \sin^2 t}} \, dt$

59. Evaluate the integral

$$\int \frac{1}{\sqrt{9x^2 + 6x - 8}} \, dx$$

by first completing the square and using the substitution $u = 3x + 1$.

60–62 ▪ Evaluate the integral by first completing the square.

60. $\displaystyle\int \frac{dt}{\sqrt{t^2 - 6t + 13}}$

61. $\displaystyle\int \frac{dx}{(x^2 + 2x + 2)^2}$

62. $\displaystyle\int \frac{x^2}{\sqrt{4x - x^2}}\,dx$

63. A particle moves on a straight line with velocity function $v(t) = \sin \omega t \cos^2 \omega t$. Find its position function $s = f(t)$ if $f(0) = 0$.

64. Household electricity is supplied in the form of alternating current that varies from 155 V to -155 V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the equation

$$E(t) = 155 \sin(120\pi t)$$

where t is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle.

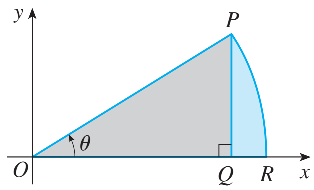
(a) Calculate the RMS voltage of household current.

(b) Many electric stoves require an RMS voltage of 220 V. Find the corresponding amplitude A needed for the voltage $E(t) = A \sin(120\pi t)$.

65. Find the average value of $f(x) = \sqrt{x^2 - 1}/x$, $1 \leqslant x \leqslant 7$.

66. Find the area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 3$.

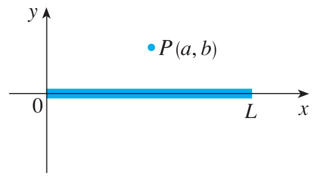
67. Prove the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector of a circle with radius r and central angle θ. [*Hint:* Assume $0 < \theta < \pi/2$ and place the center of the circle at the origin so it has the equation $x^2 + y^2 = r^2$. Then A is the sum of the area of the triangle POQ and the area of the region PQR in the figure.]



68. A charged rod of length L produces an electric field at point $P(a, b)$ given by

$$E(P) = \int_{-a}^{L-a} \frac{\lambda b}{4\pi\varepsilon_0(x^2 + b^2)^{3/2}}\,dx$$

where λ is the charge density per unit length on the rod and ε_0 is the free space permittivity (see the figure). Evaluate the integral to determine an expression for the electric field $E(P)$.



6.3 PARTIAL FRACTIONS

In this section we show how to integrate any rational function (a ratio of polynomials) by expressing it as a sum of simpler fractions, called *partial fractions,* that we already know how to integrate. To illustrate the method, observe that by taking the fractions $2/(x - 1)$ and $1/(x + 2)$ to a common denominator we obtain

$$\frac{2}{x-1} - \frac{1}{x+2} = \frac{2(x+2) - (x-1)}{(x-1)(x+2)} = \frac{x+5}{x^2+x-2}$$

If we now reverse the procedure, we see how to integrate the function on the right side of this equation:

$$\int \frac{x+5}{x^2+x-2}\,dx = \int \left(\frac{2}{x-1} - \frac{1}{x+2}\right) dx$$

$$= 2 \ln|x - 1| - \ln|x + 2| + C$$

To see how the method of partial fractions works in general, let's consider a rational function

$$f(x) = \frac{P(x)}{Q(x)}$$