9.2 EXERCISES

1–2 ▪ Find dy/dx.

1. $x = t - t^3, \quad y = 2 - 5t$

2. $x = te^t, \quad y = t + e^t$

3–6 ▪ Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

3. $x = t^4 + 1, \quad y = t^3 + t; \quad t = -1$

4. $x = 2t^2 + 1, \quad y = \frac{1}{3}t^3 - t; \quad t = 3$

5. $x = e^{\sqrt{t}}, \quad y = t - \ln t^2; \quad t = 1$

6. $x = \cos\theta + \sin 2\theta, \quad y = \sin\theta + \cos 2\theta; \quad \theta = 0$

7. Find an equation of the tangent to the curve $x = e^t$, $y = (t - 1)^2$ at the point $(1, 1)$ by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.

8. Find equations of the tangents to the curve $x = \sin t$, $y = \sin(t + \sin t)$ at the origin. Then graph the curve and the tangents.

9–12 ▪ Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

9. $x = 4 + t^2, \quad y = t^2 + t^3$

10. $x = t^3 - 12t, \quad y = t^2 - 1$

11. $x = t - e^t, \quad y = t + e^{-t}$

12. $x = t + \ln t, \quad y = t - \ln t$

13–16 ▪ Find the points on the curve where the tangent is horizontal or vertical. If you have a graphing device, graph the curve to check your work.

13. $x = 10 - t^2, \quad y = t^3 - 12t$

14. $x = 2t^3 + 3t^2 - 12t, \quad y = 2t^3 + 3t^2 + 1$

15. $x = 2\cos\theta, \quad y = \sin 2\theta$

16. $x = \cos 3\theta, \quad y = 2\sin\theta$

17. Use a graph to estimate the coordinates of the leftmost point on the curve $x = t^4 - t^2$, $y = t + \ln t$. Then use calculus to find the exact coordinates.

18. Try to estimate the coordinates of the highest point and the leftmost point on the curve $x = te^t$, $y = te^{-t}$. Then find the exact coordinates. What are the asymptotes of this curve?

19–20 ▪ Graph the curve in a viewing rectangle that displays all the important aspects of the curve.

19. $x = t^4 - 2t^3 - 2t^2, \quad y = t^3 - t$

20. $x = t^4 + 4t^3 - 8t^2, \quad y = 2t^2 - t$

21. Show that the curve $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0, 0)$ and find their equations. Sketch the curve.

22. At what point does the curve $x = 1 - 2\cos^2 t$, $y = (\tan t)(1 - 2\cos^2 t)$ cross itself? Find the equations of both tangents at that point.

23. (a) Find the slope of the tangent line to the trochoid $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$ in terms of θ. (See Exercise 34 in Section 9.1.)
(b) Show that if $d < r$, then the trochoid does not have a vertical tangent.

24. (a) Find the slope of the tangent to the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in terms of θ.
(b) At what points is the tangent horizontal or vertical?
(c) At what points does the tangent have slope 1 or -1?

25. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equations $x = -7t$, $y = 12t - 5$?

26. Find equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point $(4, 3)$.

27. Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \leqslant \theta \leqslant 2\pi$, to find the area that it encloses.

28. Find the area bounded by the curve $x = t - 1/t$, $y = t + 1/t$ and the line $y = 2.5$.

29. Find the area bounded by the curve $x = \cos t$, $y = e^t$, $0 \leqslant t \leqslant \pi/2$, and the lines $y = 1$ and $x = 0$.

30. Find the area of the region enclosed by the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$.

31. Find the area under one arch of the trochoid of Exercise 34 in Section 9.1 for the case $d < r$.

32. Let $\mathcal{R}$ be the region enclosed by the loop of the curve in Example 1.
(a) Find the area of $\mathcal{R}$.
(b) If $\mathcal{R}$ is rotated about the x-axis, find the volume of the resulting solid.
(c) Find the centroid of $\mathcal{R}$.

33–36 ▪ Set up, but do not evaluate, an integral that represents the length of the curve.

33. $x = t - t^2, \quad y = \frac{4}{3}t^{3/2}, \quad 1 \leqslant t \leqslant 2$

34. $x = 1 + e^t, \quad y = t^2, \quad -3 \leqslant t \leqslant 3$

35. $x = t + \cos t, \quad y = t - \sin t, \quad 0 \leqslant t \leqslant 2\pi$

36. $x = \ln t, \quad y = \sqrt{t + 1}, \quad 1 \leqslant t \leqslant 5$

37–40 ▪ Find the length of the curve.

37. $x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leqslant t \leqslant 1$

38. $x = a(\cos\theta + \theta\sin\theta), \quad y = a(\sin\theta - \theta\cos\theta),$
$0 \leqslant \theta \leqslant \pi$

39. $x = \dfrac{t}{1 + t}, \quad y = \ln(1 + t), \quad 0 \leqslant t \leqslant 2$

40. $x = e^t + e^{-t}, \quad y = 5 - 2t, \quad 0 \leqslant t \leqslant 3$

41–43 ▪ Graph the curve and find its length.

41. $x = e^t\cos t, \quad y = e^t\sin t, \quad 0 \leqslant t \leqslant \pi$

42. $x = \cos t + \ln\left(\tan\frac{1}{2}t\right), \quad y = \sin t, \quad \pi/4 \leqslant t \leqslant 3\pi/4$

43. $x = e^t - t, \quad y = 4e^{t/2}, \quad -8 \leqslant t \leqslant 3$

44. Find the length of the loop of the curve $x = 3t - t^3$, $y = 3t^2$.

45. Use Simpson's Rule with $n = 6$ to estimate the length of the curve $x = t - e^t$, $y = t + e^t$, $-6 \leqslant t \leqslant 6$.

46. In Exercise 36 in Section 9.1 you were asked to derive the parametric equations $x = 2a\cot\theta$, $y = 2a\sin^2\theta$ for the curve called the witch of Maria Agnesi. Use Simpson's Rule with $n = 4$ to estimate the length of the arc of this curve given by $\pi/4 \leqslant \theta \leqslant \pi/2$.

47–48 ▪ Find the distance traveled by a particle with position (x, y) as t varies in the given time interval. Compare with the length of the curve.

47. $x = \sin^2 t, \quad y = \cos^2 t, \quad 0 \leqslant t \leqslant 3\pi$

48. $x = \cos^2 t, \quad y = \cos t, \quad 0 \leqslant t \leqslant 4\pi$

49. Show that the total length of the ellipse $x = a\sin\theta$, $y = b\cos\theta$, $a > b > 0$, is

$$L = 4a\int_0^{\pi/2}\sqrt{1 - e^2\sin^2\theta}\,d\theta$$

where e is the eccentricity of the ellipse $\left(e = c/a, \text{ where } c = \sqrt{a^2 - b^2}\right)$.

50. Find the total length of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$, where $a > 0$.

CAS **51.** (a) Graph the **epitrochoid** with equations

$$x = 11\cos t - 4\cos(11t/2)$$
$$y = 11\sin t - 4\sin(11t/2)$$

What parameter interval gives the complete curve?

(b) Use your CAS to find the approximate length of this curve.

CAS **52.** A curve called **Cornu's spiral** is defined by the parametric equations

$$x = C(t) = \int_0^t \cos(\pi u^2/2)\,du$$

$$y = S(t) = \int_0^t \sin(\pi u^2/2)\,du$$

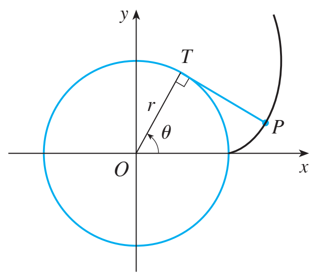
where C and S are the Fresnel functions that were introduced in Chapter 4.

(a) Graph this curve. What happens as $t \to \infty$ and as $t \to -\infty$?

(b) Find the length of Cornu's spiral from the origin to the point with parameter value t.

53. A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the **involute** of the circle. If the circle has radius r and center O and the initial position of P is $(r, 0)$, and if the parameter θ is chosen as in the figure, show that parametric equations of the involute are

$$x = r(\cos\theta + \theta\sin\theta) \qquad y = r(\sin\theta - \theta\cos\theta)$$



54. A cow is tied to a silo with radius r by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow.