Name: *ANSWER KEY* 6/17/2014

Math 202 Calculus II Quiz 2

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

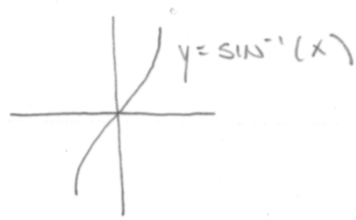
1. (a) (4 points) Show that if $y = \sin^{-1}(x)$ then $y' = \frac{1}{\sqrt{1-x^2}}$.

$y = \sin^{-1}(x) \iff \sin(y) = x$

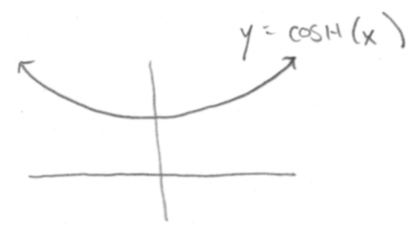
$\frac{d}{dx}: \quad \cos(y)\, y' = 1$

$$y' = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-x^2}}$$

POSITIVE BECAUSE $\sin^{-1}$ IS ALWAYS INCREASING



(b) (4 points) Show that if $y = \sinh^{-1}(x)$ then $y' = \frac{1}{\sqrt{1+x^2}}$.



$y = \sinh^{-1}(x) \iff \sinh(y) = x$

$\frac{d}{dx}: \quad \cosh(y)\, y' = 1$

$$y' = \frac{1}{\cosh(y)}$$

POSITIVE BECAUSE COSH IS ALWAYS POSITIVE

SINCE $\cosh^2(y) - \sinh^2(y) = 1$, WE HAVE $\cosh(y) = \sqrt{1 + \sinh^2(y)}$

THEREFORE $$y' = \frac{1}{\sqrt{1+\sinh^2(y)}} = \frac{1}{\sqrt{1+x^2}}$$

2. (4 points) Find $\dfrac{dy}{dx}$ when $y = \arctan\left(\dfrac{x}{a}\right) + \ln\sqrt{\dfrac{x-a}{x+a}}$.

$$y = \text{ARCTAN}\left(\frac{x}{a}\right) + \frac{1}{2}\left(\ln(x-a) - \ln(x+a)\right)$$

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{1}{a} + \frac{1}{2}\left(\frac{1}{x-a} - \frac{1}{x+a}\right)$$

$$= \frac{1}{a + \frac{x^2}{a}} + \frac{(x+a)-(x-a)}{2(x+a)(x-a)}$$

$$= \frac{a}{a^2+x^2} + \frac{a}{x^2-a^2} = \boxed{\frac{2ax^2}{x^4-a^4}}$$

3. (4 points) Show that the following identity is true for all real numbers n.

$$(\cosh(x) + \sinh(x))^n = \cosh(nx) + \sinh(nx)$$

Hint: use the definition of cosh and sinh *to show that both the left and right hand side simplify to the same (simple) expression.*

LHS: $\left(\dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2}\right)^n = (e^x)^n = e^{nx}$

RHS: $\dfrac{e^{nx} + e^{-nx}}{2} + \dfrac{e^{nx} - e^{-nx}}{2} = e^{nx}$

SAME ✓

4. Evaluate the limit. If the limit is of indeterminate form, be sure to indicate which type it is.

(a) (4 points) $\lim_{x\to\infty} \frac{x}{\ln(1+2^x)}$: $\frac{\infty}{\infty}$

$$\text{L'HÔ} \longrightarrow \lim_{x\to\infty} \frac{1}{\frac{1}{1+2^x}\cdot 2^x \ln(2)} = \lim_{x\to\infty} \frac{1+2^x}{2^x \ln(2)} : \frac{\infty}{\infty}$$

$$\text{L'HÔ} \longrightarrow \lim_{x\to\infty} \frac{\cancel{2^x \ln(2)}}{\cancel{2^x \ln(2)} \ln(2)} = \boxed{\frac{1}{\ln(2)}}$$

(b) (4 points) $\lim_{x\to 0}(1-x)^{1/x}$: 1^{∞}

SET $L = \lim_{x\to 0} (1-x)^{1/x}$

$$\ln(L) = \lim_{x\to 0} \frac{\ln(1-x)}{x} : \frac{0}{0}$$

$$\text{L'HÔ} \longrightarrow \ln(L) = \lim_{x\to 0} \frac{-1}{1-x} = -1$$

$$\therefore L = e^{-1} = \boxed{\frac{1}{e}}$$