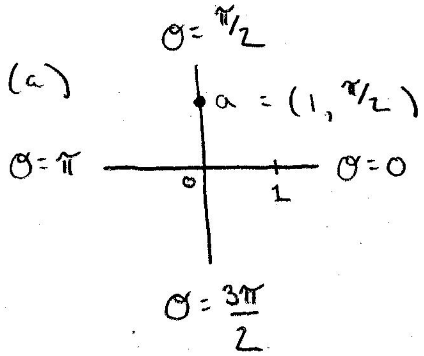
JOHN ADAMSKI
7/17/2012

MATH 202

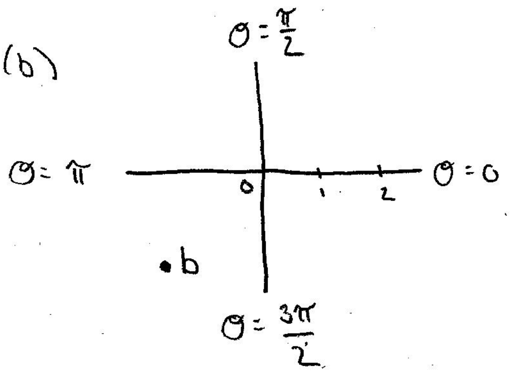
§ 9.3

1 (a)



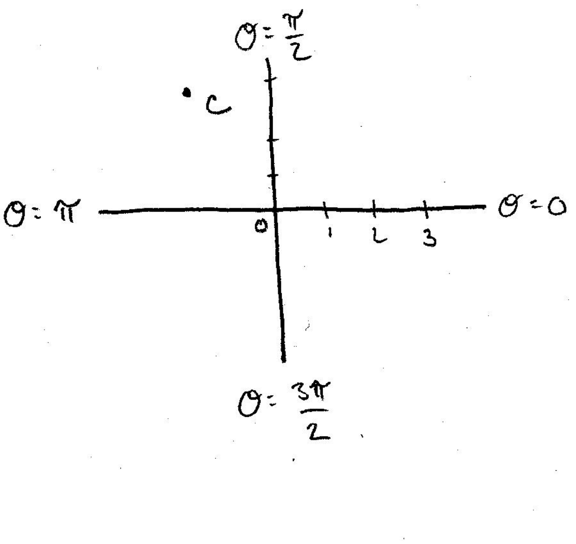
$a = (1, \pi/2) = (1, \frac{5\pi}{2}) = (-1, \frac{3\pi}{2})$

(b)



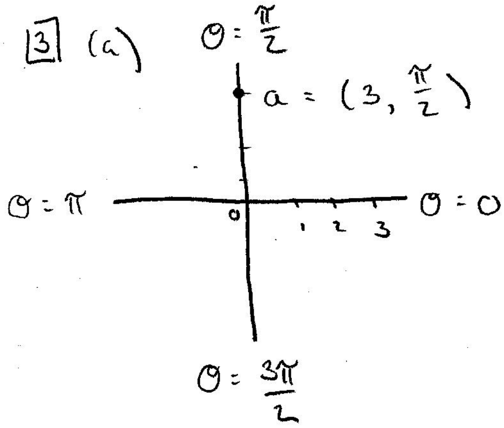
$b = (-2, \frac{\pi}{4}) = (2, -\frac{3\pi}{4}) = (-2, -\frac{7\pi}{4})$

(c) NOTE: $2 \approx \frac{2\pi}{3}$ (KINDA)



$c = (3, 2) = (3, 2 + 2\pi) = (-3, 2 + \pi)$

3 (a)



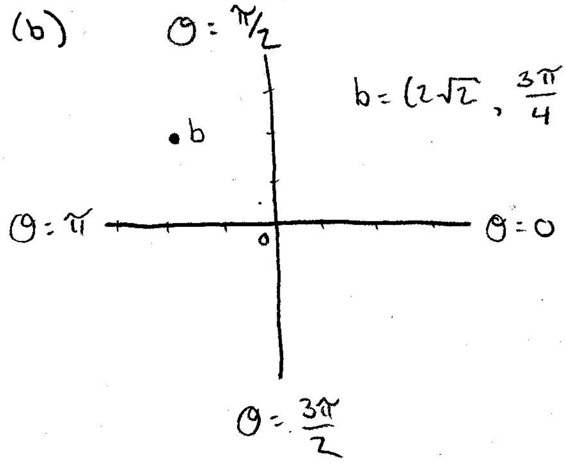
$a = (3, \frac{\pi}{2})$

$x = r\cos\theta = (3)\cos(\frac{\pi}{2}) = 0$

$y = r\sin\theta = (3)\sin(\frac{\pi}{2}) = 3$

$\rightarrow \boxed{(0, 3)}$

(b)



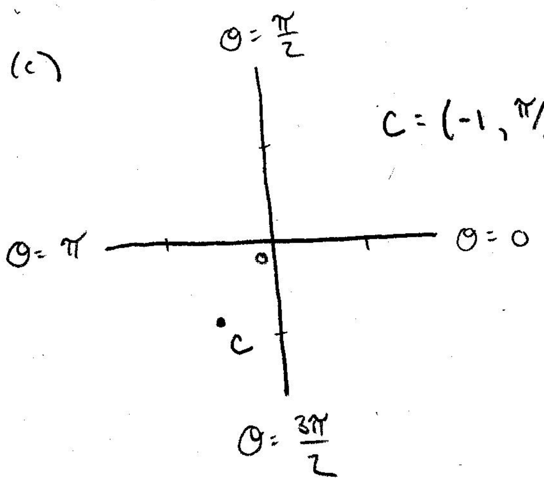
$b = (2\sqrt{2}, \frac{3\pi}{4})$

$x = (2\sqrt{2})\cos(\frac{3\pi}{4}) = (2\sqrt{2})(\frac{-\sqrt{2}}{2}) = -2$

$y = (2\sqrt{2})\sin(\frac{3\pi}{4}) = (2\sqrt{2})(\frac{\sqrt{2}}{2}) = 2$

$\rightarrow \boxed{(-2, 2)}$

(c)



$c = (-1, \pi/3)$

$x = (-1)\cos(\frac{\pi}{3}) = (-1)(\frac{1}{2}) = -\frac{1}{2}$

$y = (-1)\sin(\frac{\pi}{3}) = (-1)\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\rightarrow \boxed{(-\frac{1}{2}, -\frac{\sqrt{3}}{2})}$

5 (i) $r^2 = x^2 + y^2 = 1^2 + 1^2 = 2 \Rightarrow r = \pm\sqrt{2}$

$\tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4} + n\pi$

$\boxed{\left(\sqrt{2}, \frac{\pi}{4}\right)}$

(ii) $\boxed{\left(-\sqrt{2}, \frac{5\pi}{4}\right)}$

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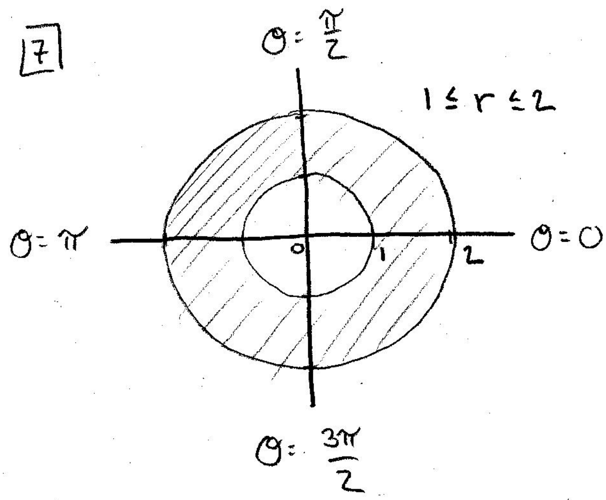

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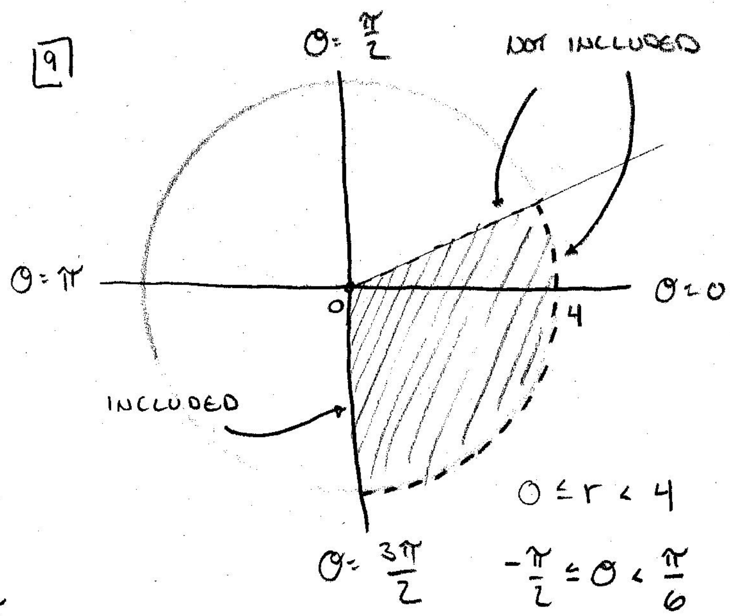

$0 \le r < 4$

$-\frac{\pi}{2} \le \theta < \frac{\pi}{6}$

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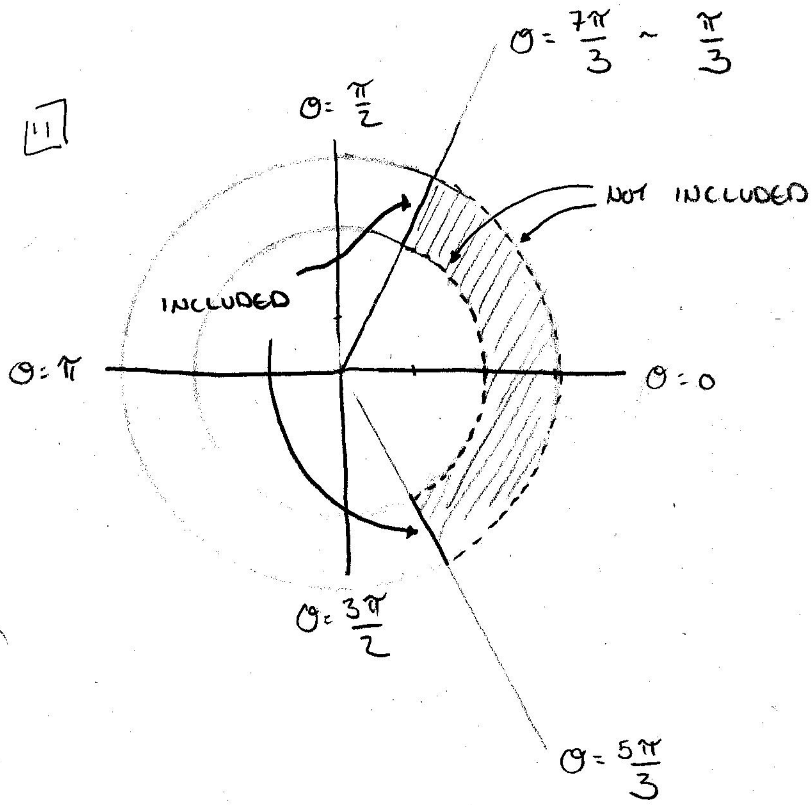

$2 < r < 3$

$\frac{5\pi}{3} \le \theta \le \frac{7\pi}{3}$

13 $r = 3\sin\theta \Rightarrow r^2 = \underline{3r\sin\theta} = 3y$.

Since $x^2 + y^2 = r^2$ we have $x^2 + y^2 = 3y$.

$\Rightarrow x^2 + y^2 - 3y = 0 \Rightarrow \boxed{x^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2}$

(circle w/ radius $\frac{3}{2}$, center $\left(0, \frac{3}{2}\right)$)

15 $r = \csc\theta \Rightarrow r\sin\theta = 1 \Rightarrow \boxed{y = 1}$

22 (a) Cartesian: $\boxed{(x-2)^2 + (y-3)^2 = 5^2}$

(b) Polar: $\boxed{r = 4}$

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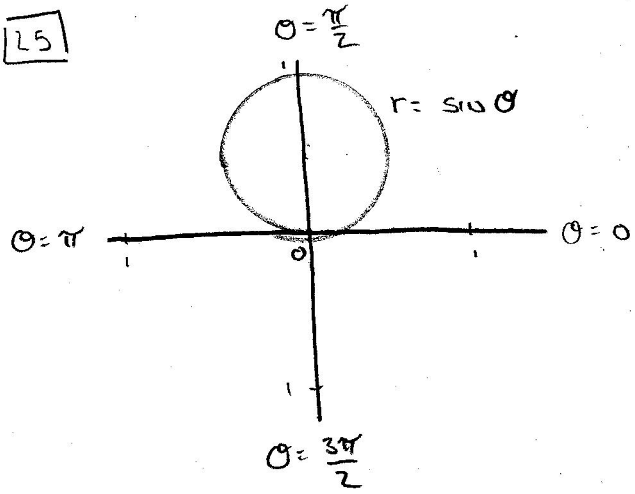

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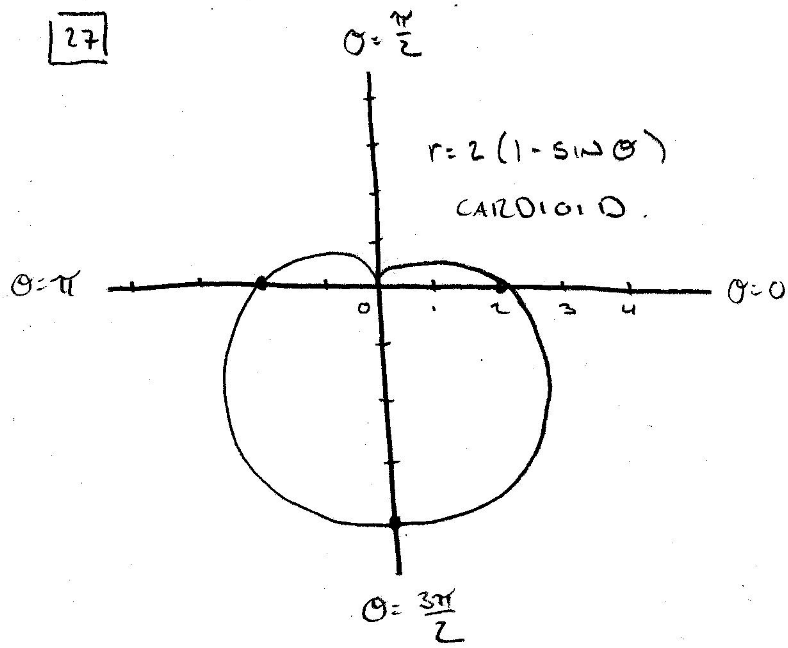

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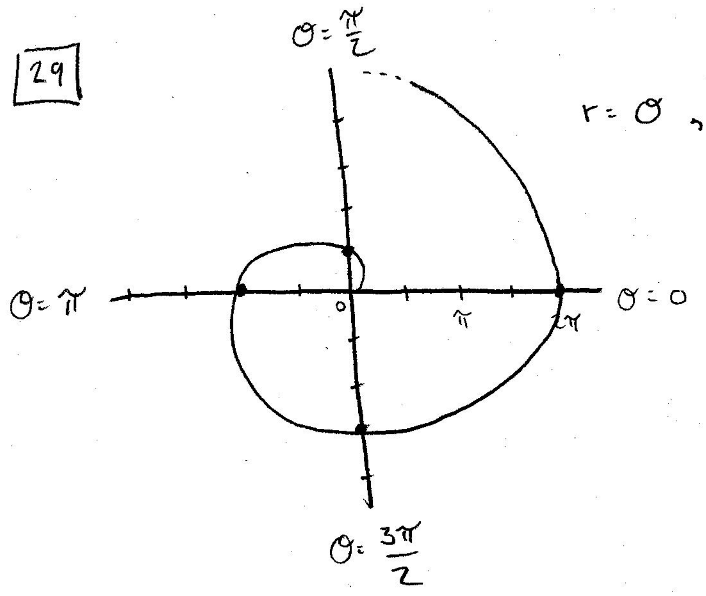

$r = \theta \ , \quad \theta \geq 0$

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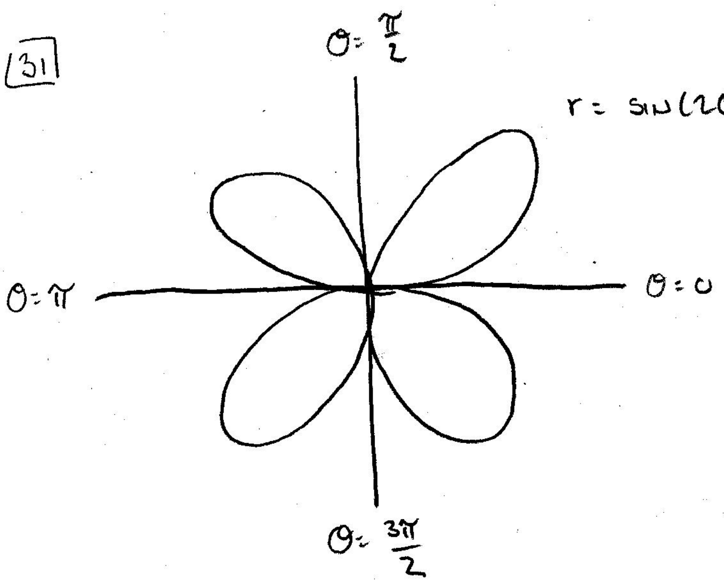

$r = \sin(2\theta)$

47 $r = 2\sin\theta \ , \quad \theta = \pi/6$

$y = r\sin\theta = 2\sin^2\theta$

$x = r\cos\theta = 2\sin\theta\cos\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4\sin\theta\cos\theta}{2\cos^2\theta - 2\sin^2\theta} = \frac{\cancel{2}\sin(2\theta)}{\cancel{2}\cos(2\theta)}$$

$$\therefore \ @ \ \theta = \pi/6 : \quad \frac{dy}{dx} = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$$

AND AT $\theta = \pi/6$, $\quad x = 2\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

$$y = 2\sin^2\left(\frac{\pi}{6}\right) = (2)\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\therefore \quad y - \frac{1}{2} = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right) \quad \text{OR} \quad \boxed{y = \sqrt{3}\,x - 1}$$

(ACTUALLY, THE QUESTION DIDN'T ASK FOR THIS EQUATION OF TANGENT LINE.)

49 $\quad r = \frac{1}{\theta}, \quad \theta = \pi$

$$y = r\sin\theta = \frac{\sin\theta}{\theta}$$

$$x = r\cos\theta = \frac{\cos\theta}{\theta}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{\theta\cos\theta - \sin\theta}{\theta^2}}{\frac{-\theta\sin\theta - \cos\theta}{\theta^2}}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\sin\theta - \theta\cos\theta}{\theta\sin\theta + \cos\theta}$$

$$@\ \theta = \pi \longrightarrow \frac{dy}{dx} = \frac{0 - \pi(-1)}{0 + (-1)} = \boxed{-\pi}$$