Exam 1

Answer all 20 questions for a total of 100 points. Write your solutions in the space provided, simplify all fractions and radical expressions, and put a box around your final answers. Good luck!

1. (5 points) Express the inequality in interval notation.

$$x \leq -5$$

LEFT NOT INCLUDED

RIGHT INCLUDED

$\boxed{(-\infty, -5]}$

2. (5 points) Perform the operations and simplify as one fraction.

$$3 + \frac{2}{10} - \frac{4}{15}$$

LCD : 30

$$3 \cdot \frac{30}{30} + \frac{2}{10} \cdot \frac{3}{3} - \frac{4}{15} \cdot \frac{2}{2}$$

$$\frac{90 + 6 - 8}{30} = \frac{88}{30} = \boxed{\frac{44}{15}}$$

3. (5 points) Evaluate the expression numerically.

$$\frac{1 - \frac{1}{8}}{\frac{3}{4} + \frac{1}{6}}$$

LCD $(4,6,8) = 24$

$$\frac{24\left(1 - \frac{1}{8}\right)}{24\left(\frac{3}{4} + \frac{1}{6}\right)} = \frac{24 - 3}{18 + 4} = \boxed{\frac{21}{22}}$$

(ALT: $\frac{1 - \frac{1}{8}}{\frac{3}{4} + \frac{1}{6}} = \frac{\frac{8}{8} - \frac{1}{8}}{\frac{9}{12} + \frac{2}{12}} = \frac{\frac{7}{8}}{\frac{11}{12}} = \frac{7}{8} \div \frac{11}{12} = \frac{7}{8} \times \frac{12}{11} = \frac{7 \cdot \cancel{4} \cdot 3}{\cancel{4} \cdot 2 \cdot 11} = \frac{21}{12}$ ✓)

4. (5 points) Simplify the expression completely.

$$(-3x^4)^3(2x^4)$$

$$\underbrace{(-3)^3}(x^4)^3 \cdot 2x^4 = -27x^{12} \cdot 2x^4 = \boxed{-54x^{16}}$$

(ALT: $(-1)^3 3^3 = -27$)

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5. (5 points) Simplify the expression completely and eliminate any negative exponents.

$$\left(\frac{8a^{-2}}{a^3}\right)^{-1}$$

$$= \left(\frac{8}{a^2 a^3}\right)^{-1} = \left(\frac{8}{a^5}\right)^{-1} = \boxed{\frac{a^5}{8}}$$

(NOTE THAT 8 HAS NO EXPONENT.
$8a^{-2} \neq (8a)^{-2}$

6. (5 points) Evaluate the expression numerically.

$$4^{3/2} - \frac{\sqrt{72}}{\sqrt{18}}$$

$$4^{3/2} = \left\{ \begin{array}{l} (4^{1/2})^3 = 2^3 \\ (4^3)^{1/2} = 64^{1/2} \end{array} \right\} = 8$$

$$\frac{\sqrt{72}}{\sqrt{18}} = \left\{ \begin{array}{l} \dfrac{\sqrt{36}\,\cancel{\sqrt{2}}}{\sqrt{9}\,\cancel{\sqrt{2}}} = \dfrac{6}{3} \\ \sqrt{\dfrac{72}{18}} = \sqrt{4} \end{array} \right\} = 2$$

$$8 - 2 = \boxed{6}$$

7. (5 points) Perform the indicated operations and simplify.

$$3(2x+1)(x-5) - 4(x^2 - 2x + 1)$$

$$(6x+3)(x-5) \quad - 4(x^2 - 2x + 1)$$

F.O.I.L. DISTRIBUTE

$$= 6x^2 - 30x + 3x - 15 - 4x^2 + 8x - 4$$

$$= \boxed{2x^2 - 19x - 19}$$

8. (5 points) Perform the indicated operations and simplify.

$$(2x+3)^2$$

$$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = \boxed{4x^2 + 12x + 9}$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

(ALT: $(2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$ ✓)
F. O. I. L.

9. (5 points) Factor the expression completely.

$$2(x-3)^2 + 2x(x-3)$$

LCD: $2(x-3)$

$$= 2(x-3)\left[(x-3) + x\right]$$

$$= \boxed{2(x-3)(2x-3)}$$

10. (5 points) Factor the expression completely.

$$x^5 + 5x^4 - 36x^3$$

$$\underbrace{x^3}_{\text{GCF}}(x^2 + 5x - 36) = \boxed{x^3(x+9)(x-4)}$$

11. (5 points) Factor the expression completely.

$$36x^2 - 49$$

$$= (6x)^2 - 7^2 = \boxed{(6x+7)(6x-7)}$$

$$A^2 - B^2 = (A+B)(A-B)$$

12. (5 points) Perform the indicated operation and simplify.

$$\frac{8x-3}{2x-1} - 4 \cdot \frac{2x-1}{2x-1}$$

$$= \frac{\cancel{8x} - 3 - \cancel{8x} + 4}{2x-1} = \boxed{\frac{1}{2x-1}}$$

To add/subtract fractions, you need to get common denominators. That's what we need to do here.

As opposed to when SOLVING EQUATIONS, when you can get rid of denominators by multiplying BOTH SIDES OF THE EQUATION by a common denominator.

13. (5 points) Perform the indicated operation and simplify.

$$\frac{x^2-4}{x^2-1}\cdot\frac{x^2+3x-4}{x^2+6x+8}$$

$$= \frac{\cancel{(x+2)}(x-2)}{(x+1)\cancel{(x-1)}}\cdot\frac{\cancel{(x+4)}\cancel{(x-1)}}{\cancel{(x+4)}\cancel{(x+2)}} = \boxed{\frac{x-2}{x+1}}$$

14. (5 points) Solve the equation

$$\frac{2x+2}{3}-\frac{9x-6}{4}=\frac{2x-1}{6}$$

LCD = 12

$$\frac{2x+2}{3}\cdot 12 - \frac{9x-6}{4}\cdot 12 = \frac{2x-1}{6}\cdot 12$$

Since this is an EQUATION, we can MULTIPLY BOTH SIDES OF THE EQUATION by anything we want, e.g. the LCD.

$$4(2x+2) \quad -3(9x-6) = 2(2x-1)$$

$$8x+8-27x+18 = 4x-2$$

$$-19x+26 = 4x-2$$

$$+19x+2 \qquad +19x+2$$

$$28 = 23x$$

$$\frac{28}{23} = \frac{\cancel{23}x}{\cancel{23}}$$

$$\boxed{x = \frac{28}{23}}$$

15. (5 points) Solve the equation.

$$\cancel{(x-3)}(x+4)\frac{6}{\cancel{x-3}} = \frac{5}{\cancel{x+4}}(x-3)\cancel{(x+4)}$$

$$6(x+4) = 5(x-3)$$

$$6x+24 = 5x-15$$

$$-5x-24 \qquad -5x-24$$

$$\boxed{x = -39}$$

16. (5 points) Solve the equation.

$$\frac{1}{5+x} - \frac{1}{5-x} = \frac{2x-8}{\underbrace{25-x^2}_{(5+x)(5-x)}}$$

LCD = $(x+5)(x-5)$

EQUATION $\Rightarrow$ MULTIPLY ALL TERMS BY LCD.

$$\frac{1}{\cancel{5+x}} \cdot (\cancel{5+x})(5-x) - \frac{1}{\cancel{5-x}} \cdot (5+x)(\cancel{5-x}) = \frac{2x-8}{(\cancel{5+x})(\cancel{5-x})} \cdot (\cancel{5+x})(\cancel{5-x})$$

$$5-x-(5+x) = 2x-8$$

$$-2x = 2x-8$$

$$-4x = -8$$

$$\boxed{x = 2}$$

17. (5 points) Find the distance between the points $(1, -4)$ and $(-2, 5)$.

$x_1 \ y_1 \qquad x_2 \ y_2$

DISTANCE FORMULA: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$$\therefore \ d = \sqrt{(-2-1)^2 + (5-(-4))^2}$$

$$= \sqrt{(-3)^2 + 9^2} = \sqrt{9+81}$$

$$= \sqrt{90} = \sqrt{9}\sqrt{10} = \boxed{3\sqrt{10}}$$

18. (5 points) Find the midpoint of the line segment connecting $(1, -4)$ and $(-2, 5)$.

$x_1 \ y_1 \qquad x_2 \ y_2$

AVERAGE OF X-COORD. AVERAGE OF Y-COORD.

MIDPOINT OF (x_1, y_1) & (x_2, y_2) IS $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$$\therefore \left(\frac{1-2}{2}, \frac{-4+5}{2}\right) = \boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$$

19. (5 points) Determine which of the given points are on the graph of the equation.

$$\sqrt{x-3} = (y+2)^2$$

(a) $(4,-3)$ (b) $(4,-6)$ ✗ (c) $(19,-1)$ ✗ (d) $(19,0)$

(a) $\sqrt{4-3} = (-3+2)^2 \iff \sqrt{1} = 1^2$ ✓

(b) $\sqrt{4-3} = (-6+2)^2 \iff \sqrt{1} = (-4)^2$ ✗

(c) $\sqrt{19-3} = (-1+2)^2 \iff \sqrt{16} = 1^2$ ✗

(d) $\sqrt{19-3} = (0+2)^2 \iff \sqrt{16} = 2^2$ ✓

20. (5 points) Give an equation of the circle with center $(5,-3)$ that passes through the point $(1,1)$.

RADIUS r = DISTANCE FROM CENTER TO ANY POINT ON CIRCLE

= DISTANCE FROM $(5,-3)$ TO $(1,1)$

$= \sqrt{(5-1)^2 + (-3-1)^2} = \sqrt{16+16} = \sqrt{32}$

$r = \sqrt{32} \Rightarrow r^2 = 32$

EQUATION OF CIRCLE WITH CENTER (h,k) & RADIUS r

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\boxed{(x-5)^2 + (y+3)^2 = 32}$$

FRACTIONS

EQUATION?

YES, EQUATION

MULTIPLY EVERY TERM ON BOTH SIDES OF THE EQUATION BY LCD.

BOOM. EQUIVALENT EQUATION (WITH SAME SOLUTIONS) WITH NO DENOMINATORS.

NO

ADD/SUBTRACT

GET COMMON DENOMINATORS (LCD) BY MULTIPLYING TOPS & BOTTOMS OF EACH FRACTION BY EXTRA FACTORS TO TURN DENOMINATORS INTO LCD.

MULTIPLY/DIVIDE

FACTOR ALL NUMERATORS & DENOMINATORS. CANCEL FACTORS THAT APPEAR IN BOTH A NUMERATOR & A DENOMINATOR (SAME OR DIFFERENT FRACTIONS, DOESN'T MATTER).

TO FIND LCD:

1. FACTOR ALL DENOMINATORS
2. LCD = PRODUCT OF EACH DISTINCT FACTOR THAT APPEARS IN THE DENOMINATORS, RAISED TO THE HIGHEST EXPONENT THAT APPEARS.

e.g. $\frac{1}{2x(x+1)^2} + \frac{1}{3x^2(x+1)}$ LCD $= 2 \cdot 3 \cdot x^2 \cdot (x+1)^2 = 6x^2(x+1)^2$

$$= \frac{1}{2x(x+1)^2} \cdot \frac{3x}{3x} + \frac{1}{3x^2(x+1)} \cdot \frac{2(x+1)}{2(x+1)} = \frac{3x}{6x^2(x+1)^2} + \frac{2(x+1)}{6x^2(x+1)^2}$$

$$= \frac{3x + 2(x+1)}{6x^2(x+1)^2} = \frac{3x + 2x + 2}{6x^2(x+1)^2} = \frac{5x+2}{6x^2(x+1)^2}$$