Exam 2 Practice

Name: *ANSWER KEY*

1. Find the slope of the line through the points $P(6, 3)$ and $Q(2, 0)$.

$$\boxed{\text{SLOPE } m = \frac{y_2 - y_1}{x_2 - x_1}} = \frac{0-3}{2-6} = \frac{-3}{-4} = \frac{3}{4}$$

1. $3/4$

2. Find an equation of the line through the point $(1, -4)$ that is parallel to the line $x + 2y + 6 = 0$.

SAME SLOPE

$2y = -x - 6$

$y = -\frac{1}{2}x - 3$

SLOPE $m = -\frac{1}{2}$

POINT-SLOPE FORMULA

$y - y_1 = m(x - x_1)$

$y - (-4) = -\frac{1}{2}(x - 1)$

2. $y + 4 = -\frac{1}{2}(x - 1)$

$\left(\text{OR } y = -\frac{1}{2}x - \frac{7}{2}\right)$

3. Find all real solutions to $0 = x^2 - 4x + 1$.

$-1 = x^2 - 4x$

$-1 + 4 = x^2 - 4x + 4$ COMPLETE THE SQUARE

$3 = (x-2)^2$

$\pm\sqrt{3} = x - 2$

$2 \pm \sqrt{3} = x$

3. $2 \pm \sqrt{3}$

(OR USE QUADRATIC FORMULA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{4^2 - 4}}{2} = \ldots$$)

4. Find all real solutions to $\dfrac{4x}{x^2+4} = 1$.

CROSS MULTIPLY: $4x = x^2 + 4$

$0 = x^2 - 4x + 4$

$0 = (x-2)^2$

$x - 2 = 0$

$x = 2$

4. 2

5. Use interval notation to state the solutions to the inequality $4x + 7 < \dfrac{3}{2}$.

$4x < \dfrac{3}{2} - 7$

$4x < -\dfrac{11}{2}$

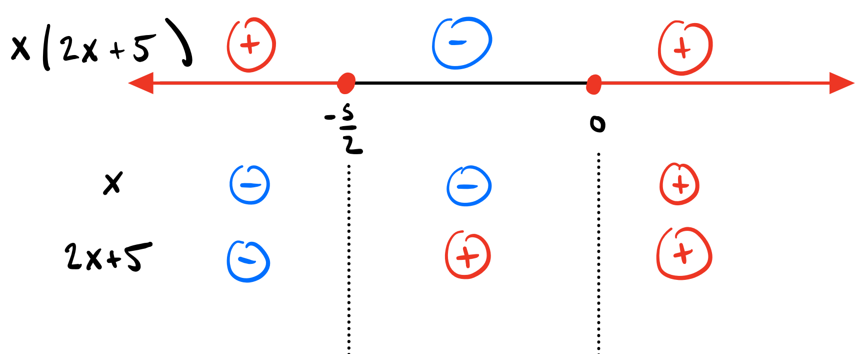
$x < -\dfrac{11}{8}$

5. $\left(-\infty, -\dfrac{11}{8}\right)$

6. Use interval notation to state the solutions to the inequality $2x^2 + 5x \geq 0$.

$\underbrace{x(2x+5)} \geq 0$

EQUALS 0 WHEN $x = 0$ OR $-\dfrac{5}{2}$



6. $\left(-\infty, -\dfrac{5}{2}\right] \cup [0, \infty)$

7. Evaluate $f(2) + f(-1)$ when $f(x) = x^3 - 3x$.

$f(2) = (2)^3 - 3(2) = 8 - 6 = 2$

$f(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2$

$f(2) + f(-1) = 2 + 2 = 4$

7. 4

8. Use interval notation to state the domain of the function $f(x) = \sqrt{1 - 5x}$.

EXPRESSIONS UNDER $\sqrt{}$ MUST BE ≥ 0 :

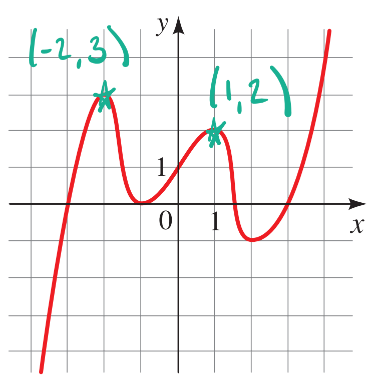
$1 - 5x \geq 0$

$1 \geq 5x$

$\frac{1}{5} > x$ i.e. $x < \frac{1}{5}$

8. $(-\infty, \frac{1}{5})$

9. Use the grah below to find the value of x at each local maximum.



9. -2 & 1

10. Determine the net change of $r(t) = 6 - \dfrac{t}{6}$ from $t = 6$ to $t = 12$.

$$r(b) - r(a) = r(12) - r(6) = \left[6 - \frac{12}{6}\right] - \left[6 - \frac{6}{6}\right] = 4 - 5 = -1$$

$(a < b)$

10. -1

11. Find $f^{-1}(10)$ when $f(x) = 6x + 7$.

$10 = 6x + 7$ FOR WHAT VALUE OF x DOES $f(x) = 10$?

$3 = 6x$

$\frac{1}{2} = x$ CHECK: $f(\frac{1}{2}) = 10$. THUS $f^{-1}(10) = \frac{1}{2}$

11. $\frac{1}{2}$

12. Evaluate $g(f(2))$ when $f(x) = 5x - 2$ and $g(x) = 3 - x^2$.

$f(2) = 5(2) - 2 = 10 - 2 = 8$

$g(f(2)) = g(8) = 3 - (8)^2 = 3 - 64 = -61$

12. -61

13. Use the table below to evaluate $g(f(3))$.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

$$g(f(3)) = g(5) = 1$$

13. 1

14. True of false: $f(x) = 4 - 3x$ is the inverse of $g(x) = \frac{3-x}{4}$.

THIS IS TRUE IF & ONLY IF $f(g(x)) = x$ & $g(f(x)) = x$

$$f(g(x)) = 4 - 3\left(\frac{3-x}{4}\right) = \frac{16}{4} + \frac{-9+3x}{4} = \frac{7+3x}{4} \neq x$$

14. FALSE

15. Find a formula for the inverse of $f(x) = 2x^3 - 5$.

$$y = 2x^3 - 5$$

$$\frac{y+5}{2} = x^3$$

$$x = \underbrace{\left(\frac{y+5}{2}\right)^{1/3}}_{\text{THIS IS } f^{-1}(y)}$$

15. $f^{-1}(x) = \left(\frac{x+5}{2}\right)^{1/3}$

16. Find the vertex of the parabola $y = x^2 + 4x$.

STANDARD FORM $y = a(x-h)^2 + k$

VERTEX (h, k)

$y = x^2 + 4x + 4 - 4$

$y = (x+2)^2 - 4$

$-h = 2$ $k = -4$

$h = -2$

16. $(-2, -4)$

17. Use interval notation to state the range of the quadratic function $h(x) = -x^2 - 4x + 4$.

THE GRAPH IS A PARABOLA THAT OPENS DOWNWARD

IT HAS MAX VALUE $h\left(-\frac{b}{2a}\right) = h(-2) = -(-2)^2 - 4(-2) + 4$

$= -4 + 8 + 4 = 8$

NO MINIMUM

17. $(-\infty, 8]$

18. Find all x-intercepts of the graph $y = -x^3 + 3x^2$.

$0 = -x^2(x-3)$

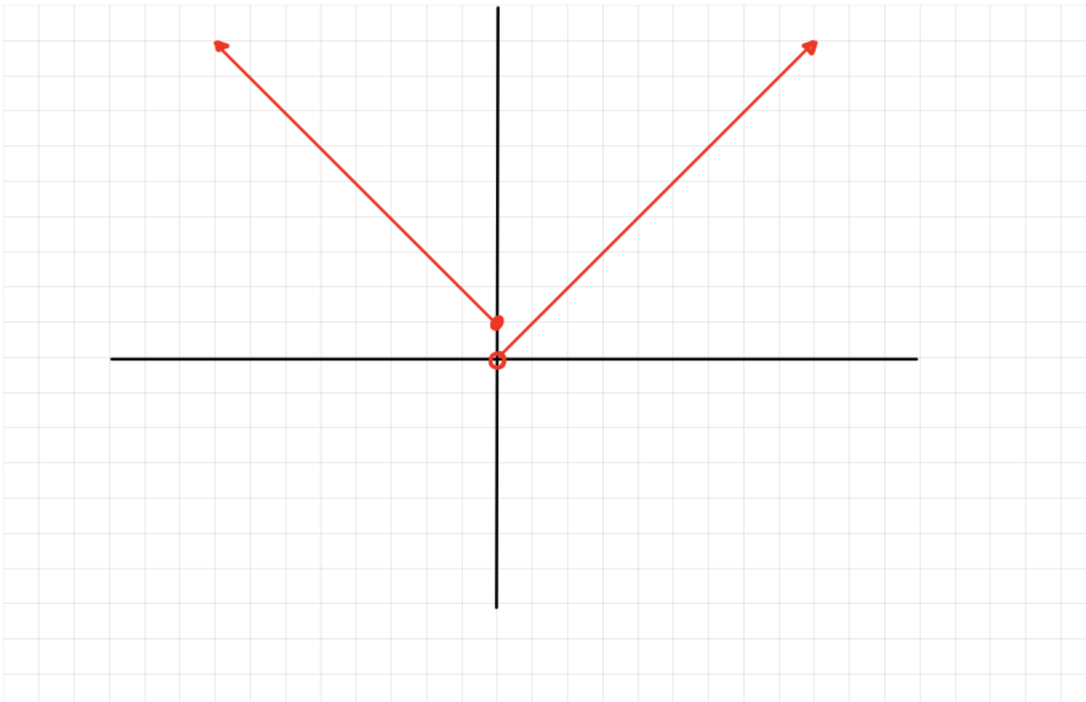
$-x^2 = 0$ OR $x - 3 = 0$

$x = 0$ $x = 3$

18. $x = 0, 3$

19. Sketch the graph of the function

$$f(x) = \begin{cases} 1 - x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$$



20. Sketch the graph $y = -\sqrt{x+2}$ not by plotting points but by starting with the graph of a standard function and applying transformations.