Name: * Answer Key * 7/25/2023

Solve each problem in the space provided and put a box around your final answer. Simplify all fractions and radical expressions. If you need more room, you can continue your work on the backs of the pages. Good luck!

1. (5 points) Find the slope of the line that passes through the points $(-1, -2)$ and $(5, 4)$.

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{5-(-1)} = \frac{6}{6} = 1$$

2. (5 points) Give an equation for the line through the point $\left(\frac{1}{2}, -\frac{2}{3}\right)$ with slope 6.

(x_1, y_1) m

$$y - y_1 = m(x - x_1)$$

$-3 - \frac{2}{3}$

$$y - \left(-\frac{2}{3}\right) = 6\left(x - \frac{1}{2}\right) \quad \text{or} \quad \boxed{y + \frac{2}{3} = 6\left(x - \frac{1}{2}\right)}$$

$$\text{or} \quad \boxed{y = 6x - \frac{11}{3}}$$

3. (5 points) Give an equation for the line that passes through the point $(4, -7)$ that is
   (a) horizontal;

$$\boxed{y = -7}$$

   (b) vertical.

$$\boxed{x = 4}$$

Hint: These are both *very* simple equations. Remember that horizontal lines have slope 0, and vertical lines have *undefined* slope.

4. (5 points) Find all real solutions of the equation $x^2 - 8x + 1 = 0$.

$x^2 - 8x + 16 = -1 + 16$ (COMPLETING THE SQR.)

HALF, SQR.

$(x-4)^2 = 15$

$x - 4 = \pm\sqrt{15}$

$\boxed{x = 4 \pm \sqrt{15}}$

QUADRATIC FORMULA

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{8 \pm \sqrt{64 - 4}}{2} = \dfrac{8 \pm \sqrt{60}}{2}$

(NOTE: $\sqrt{60} = \sqrt{4}\sqrt{15} = 2\sqrt{15}$)

$= \dfrac{8 \pm 2\sqrt{15}}{2} = \dfrac{\cancel{2}(4 \pm \sqrt{15})}{\cancel{2}}$

5. (5 points) Find all real solutions of the equation $\dfrac{3x^2 - 30}{x^2 - 5x} = 2$.

$\dfrac{3x^2 - 30}{\cancel{x^2 - 5x}} \cancel{(x^2 - 5x)} = 2(x^2 - 5x)$

$3x^2 - 30 = 2x^2 - 10x$

$x^2 + 10x + 25 = 30 + 25$

$(x+5)^2 = 55$

$x + 5 = \pm\sqrt{55}$

$\boxed{x = -5 \pm \sqrt{55}}$

6. (5 points) Find all real solutions of the equation $\sqrt{5x + 9} + 3 = x$.

$\left(\sqrt{5x+9}\right)^2 = (x - 3)^2$

$5x + 9 = x^2 - 6x + 9$

$0 = x^2 - 11x$

$0 = x(x - 11)$

$x = 0, 11$

CHECK:

$\cancel{x=0}$ $\quad \sqrt{5 \cdot 0 + 9} + 3 = 0$

✓ $x = 11$ $\quad \sqrt{5 \cdot 11 + 9} + 3 = 11$

$\sqrt{64} + 3 = 11$

$8 + 3 = 11$

7. (5 points) Use interval notation to state the solutions to the inequality $\frac{1}{3} - \frac{x}{2} \geq \frac{1}{6} + x$.

$$6\left(\frac{1}{3} - \frac{x}{2}\right) \geq 6\left(\frac{1}{6} + x\right)$$

$$2 - 3x \geq 1 + 6x$$

$$1 \geq 9x$$

$$\frac{1}{9} \geq x \quad \text{or} \quad x \leq \frac{1}{9}$$

$$\boxed{\left(-\infty, \frac{1}{9}\right]}$$

8. (5 points) Use interval notation to state the solutions to the inequality $x^2 - 2x > 8$.

$$x^2 - 2x - 8 > 0$$

$$(x-4)(x+2) > 0$$

$$x - 4 = 0 \qquad x + 2 = 0$$

$$x = 4 \qquad x = -2$$

		-2		4	
$x+2$	$-$		$+$		$+$
$x-4$	$-$		$-$		$+$
$(x+2)(x-4)$	$+$		$-$		$+$

$$\boxed{(-\infty, -2) \cup (4, \infty)}$$

9. (5 points) Let $f(x) = \frac{x^2 + 4}{5}$. Evaluate $f(a-2)$.

$$f(a-2) = \frac{(a-2)^2 + 4}{5} = \frac{a^2 - 4a + 4 + 4}{5}$$

$$= \boxed{\frac{a^2 - 4a + 8}{5}}$$

10. (5 points) Find the domain of the function $f(x) = 3 - \sqrt{x+1}$

$x + 1 \geq 0$

$x \geq -1$ or $-1 \leq x$

$\boxed{[-1, \infty)}$

11. (5 points) Find the domain of the function $g(x) = \dfrac{\sqrt{x}}{x-6}$

$\left.\begin{array}{l} x \geq 0 \\ x \neq 6 \end{array}\right\}$ BOTH!

$\boxed{[0,6) \cup (6,\infty)}$

12. (5 points) Let $v(t) = 2t^2 + t$. Find the net change in f from $t = 2$ to $t = 4$.

$f(4) - f(2) = \left[2(4)^2 + (4)\right] - \left[2(2)^2 + (2)\right]$

$= 36 - 10$

$= \boxed{26}$

13. (5 points) Let $f(x) = \dfrac{x}{3} - 2$. Find $f^{-1}(x)$.

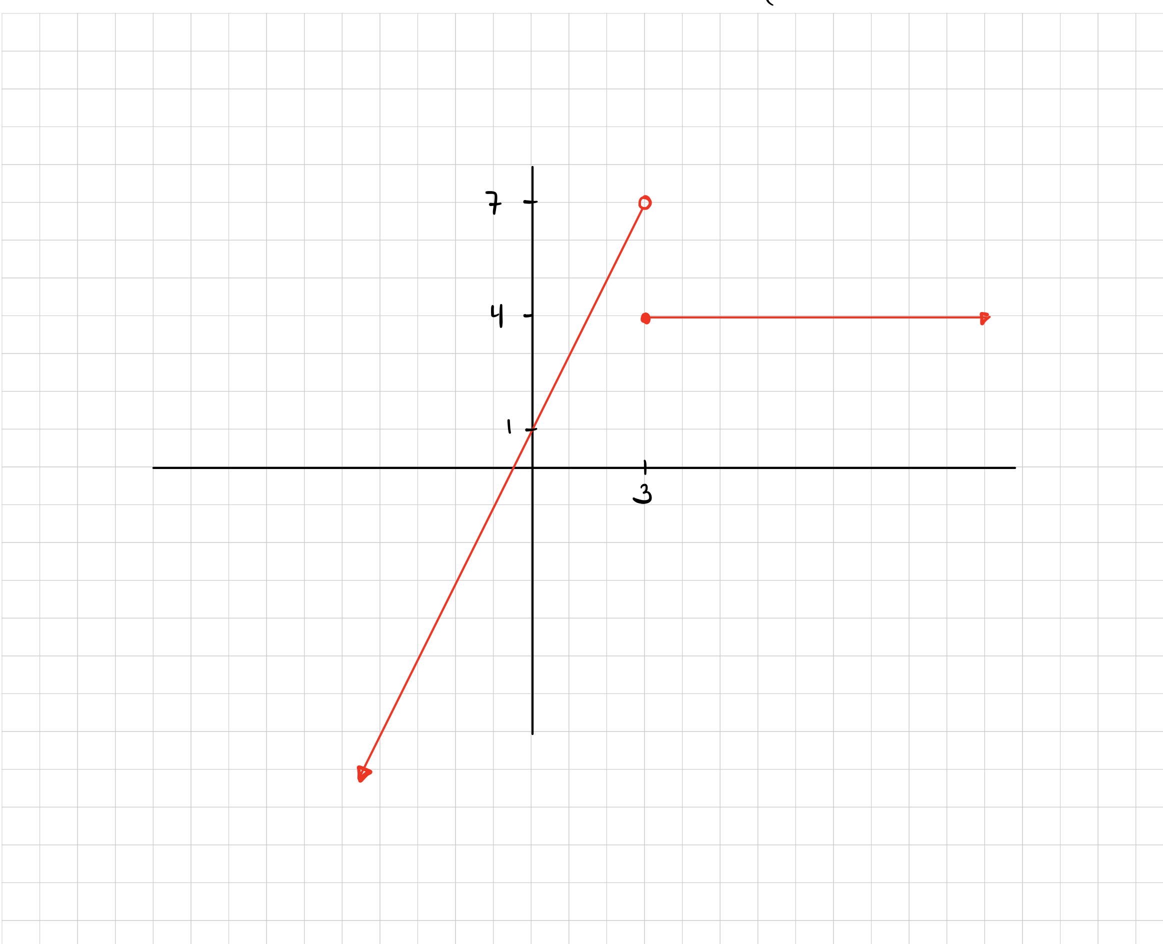
$$y = \frac{x}{3} - 2$$

$$y + 2 = \frac{x}{3}$$

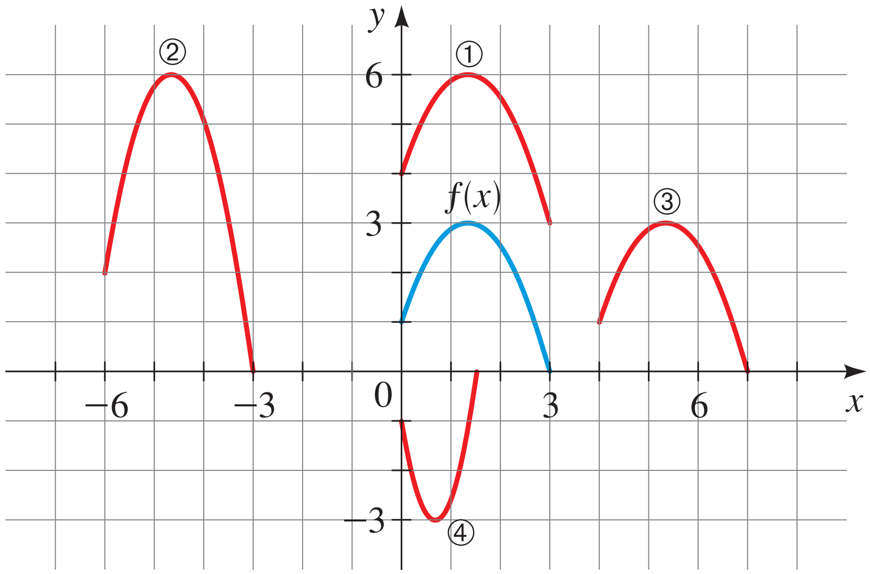
$$\underbrace{3(y+2)}_{f^{-1}(y)} = x$$

$$\boxed{\begin{aligned} f^{-1}(x) &= 3(x+2) \\ &= 3x + 6 \end{aligned}}$$

14. (5 points) Sketch a graph of the piecewise defined function $f(x) = \begin{cases} 1 + 2x & \text{if } x < 3 \\ 4 & \text{if } x \geq 3 \end{cases}$.



15. (5 points) The graph of $y = f(x)$ is given. Match each equation with its graph (# 1-4).



(a) $y = f(x-4)$ 3

(b) $y = f(x)+3$ 1

(c) $y = 2f(x+6)$ 2

(d) $y = -f(2x)$ 4

16. (5 points) True or false: $f(x) = \dfrac{2}{3x+5}$ is the inverse of $g(x) = \dfrac{2-5x}{3x}$.

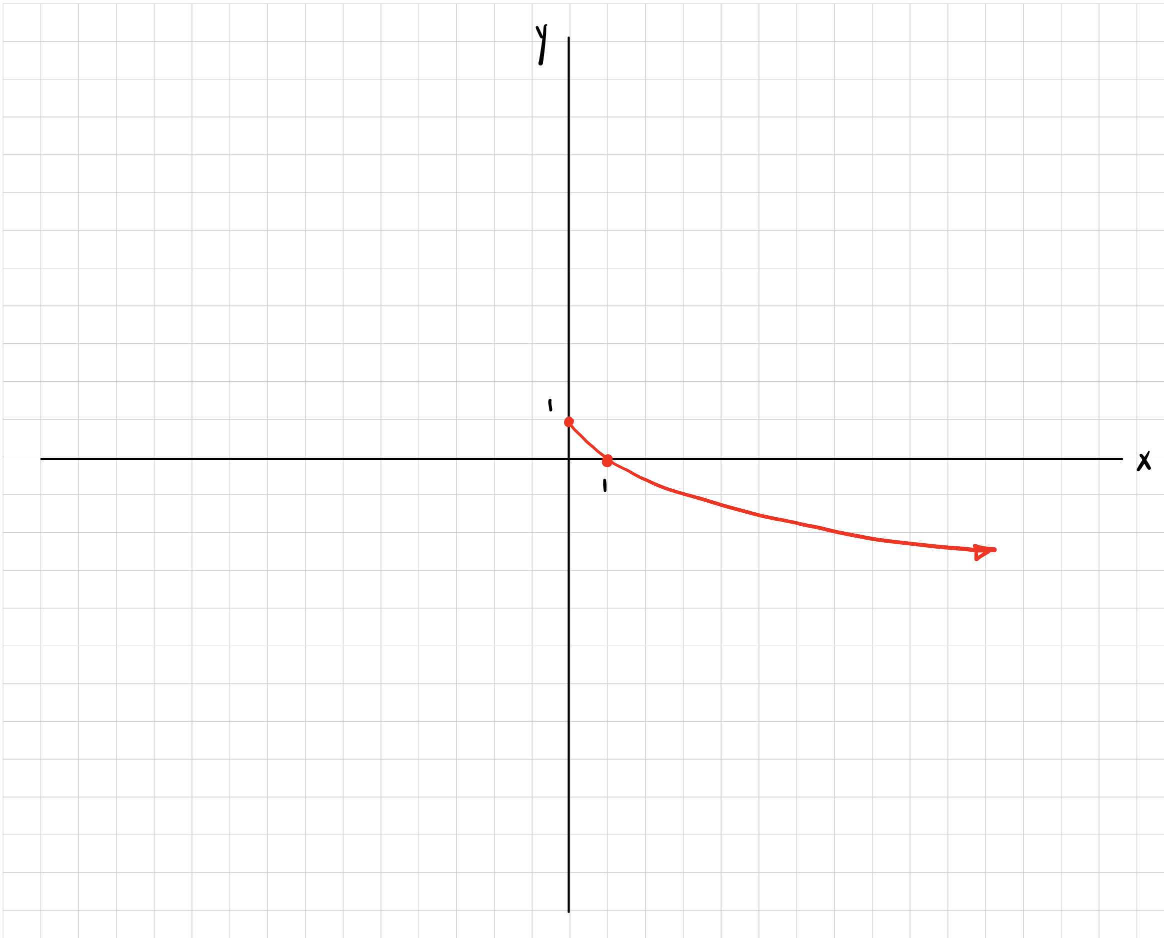
$$f(g(x)) = \frac{2}{\cancel{3}\left(\frac{2-5x}{\cancel{3}x}\right)+5} = \frac{2}{\frac{2-5x}{x}+5} \cdot \frac{x}{x}$$

$$= \frac{2x}{\frac{2-5x}{\cancel{x}}\cancel{x} + 5x} = \frac{2x}{2 - \cancel{5x} + \cancel{5x}}$$

$$= \frac{\cancel{2}x}{\cancel{2}} = x \ \checkmark$$

$\boxed{\text{TRUE}}$

17. (5 points) Sketch the graph of $y = 1 - \sqrt{x}$ not by plotting points, but by starting with the graph of a standard function and applying transformations.



18. (5 points) Let $f(x) = 3x - 4$ and $g(x) = x^2 + 2x + 5$. Evaluate the following.

(a) $f(g(3))$

$g(3) = 3^2 + 2(3) + 5 = 9 + 6 + 5 = 20$

$f(g(3)) = f(20) = 3(20) - 4 = 60 - 4 = \boxed{56}$

(b) $f(f(0))$

$f(0) = 3(0) - 4 = -4$

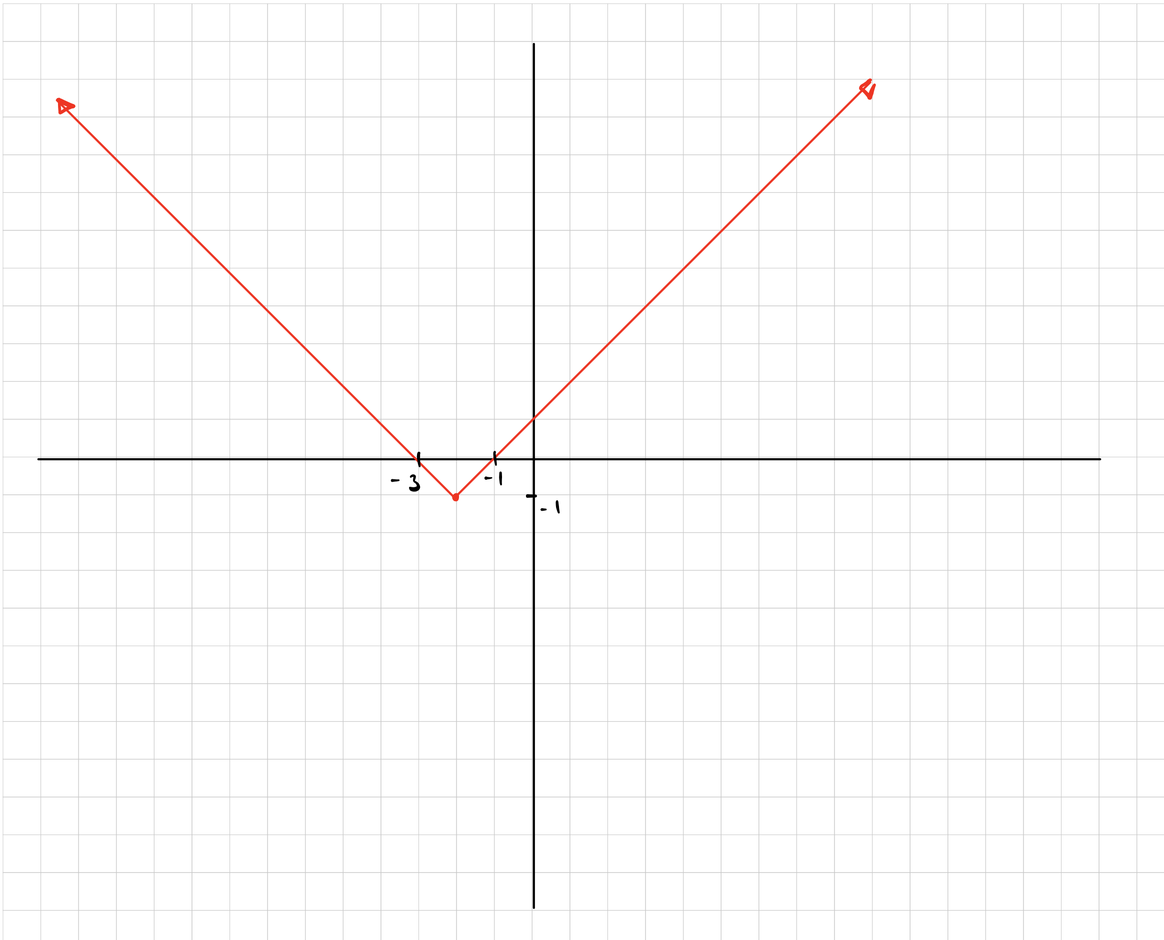
$f(f(0)) = f(-4) = 3(-4) - 4 = -12 - 4 = \boxed{-16}$

(c) $g(f(x))$ $= g(3x-4) = \boxed{(3x-4)^2 + 2(3x-4) + 5}$

$= 9x^2 - 24x + 16 + 6x - 8 + 5$

$= \boxed{9x^2 - 18x + 13}$

19. (5 points) Sketch the graph of $y = |x+2| - 1$ not by plotting points, but by starting with the graph of a standard function and applying transformations.



20. (5 points) Find the x- and y-intercepts of the graph $y = 4 - \sqrt{x+9}$.
(You do not need to sktch the graph.)

X-INT: $0 = 4 - \sqrt{x+9}$

$\sqrt{x+9} = 4$

$x + 9 = 16$

$x = 7$

$\boxed{(7,0)}$

y-INT: $y = 4 - \sqrt{0+9}$

$y = 4 - \sqrt{9}$

$y = 4 - 3$

$y = 1$

$\boxed{(0,1)}$