

Name: *ANSWER KEY*

8/1/2023

Answer all 25 questions in the space provided and put a box around your final answers.
Simplify all fractions and radical expressions. Good luck!

1. (4 points) Perform the indicated operation(s) and simplify as much as possible.

$$\frac{1}{2} - \frac{1}{7} = \frac{7}{14} - \frac{2}{14} = \frac{5}{14}$$

$$1 - \frac{2}{7} = \frac{7}{7} - \frac{2}{7} = \frac{5}{7}$$

$$\therefore \frac{\frac{1}{2} - \frac{1}{7}}{1 - \frac{2}{7}} = \frac{\frac{5}{14}}{\frac{5}{7}} = \frac{\cancel{5}}{14} \times \frac{7}{\cancel{5}} = \frac{7}{14} = \boxed{\frac{1}{2}}$$

Alt: $\frac{\frac{1}{2} - \frac{1}{7}}{1 - \frac{2}{7}} \times \frac{14}{14} = \frac{7-2}{14-4} = \frac{5}{10} = \boxed{\frac{1}{2}}$

2. (4 points) Simplify the expression and eliminate any negative exponents.

$$\left(\frac{2a^{-1}b}{a^3b^{-2}} \right)^{-4}$$

$$\left(\left(\frac{2b^3}{a^4} \right)^{-1} \right)^4 = \left(\frac{a^4}{2b^3} \right)^4 = \boxed{\frac{a^{16}}{16b^{12}}}$$

3. (4 points) Find all real solutions to the equation.

$$x(x+6) = 1$$

$$x^2 + 6x = 1$$

$$x+3 = \pm\sqrt{10}$$

$$x^2 + 6x + 9 = 10$$

$$x = -3 \pm \sqrt{10}$$

$$(x+3)^2 = 10$$

4. (4 points) Factor the expression $(16a^4 - 81b^4)$ completely.

$$= (4a^2 + 9b^2)(4a^2 - 9b^2)$$

$$= (4a^2 + 9b^2)(2a + 3b)(2a - 3b)$$

5. (4 points) Perform the multiplication $(4a^2 - 9b^2)^2$.

$$(4a^2 - 9b^2)(4a^2 - 9b^2)$$

$$16a^4 - 72a^2b^2 + 81b^4$$

6. (4 points) Perform the indicated operation(s) and simplify as much as possible.

$$\frac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \frac{3x + 12}{x - 1}$$

$$= \frac{(x+3)\cancel{(x-1)}}{(x+4)\cancel{(x+4)}} \cdot \frac{3\cancel{(x+4)}}{\cancel{x-1}}$$

$$= \frac{3(x+3)}{x+4} = \frac{3x+9}{x+4}$$

7. (4 points) Evaluate $25^{3/2}$.

$$= (25^{1/2})^3 = 5^3 = \boxed{125}$$

8. (4 points) Find all real solutions to the equation.

$$\frac{1}{2}x + \frac{5}{2} = 7 - \frac{1}{4}x$$

$$4 \cdot \frac{1}{2}x + 4 \cdot \frac{5}{2} = 4 \cdot 7 - 4 \cdot \frac{1}{4}x \quad (\text{LCD} = 4)$$

$$2x + 10 = 28 - x$$

$$3x = 18$$

$$\boxed{x = 6}$$

9. (4 points) Perform the indicated operation(s) and simplify completely as one rational expression.

$$\frac{1}{x^2 - 2x} - \frac{1}{x^2 + 2x}$$

$$\frac{1}{x(x-2)} - \frac{1}{x(x+2)} = \frac{x+2}{x(x+2)(x-2)} - \frac{-x+2}{x(x+2)(x-2)}$$

$$= \boxed{\frac{4}{x(x+2)(x-2)}}$$

10. Give an equation of the line through $(3, -1)$ that is ...

(a) (2 points) horizontal.

$$y = -1$$

(b) (2 points) vertical.

$$x = 3$$

11. (4 points) Give an equation of the line through $(-6, 8)$ that is parallel to the line $5x + 4y = 3$.

$$(x_1, y_1) = (-6, 8)$$

$$y = -\frac{5}{4}x + \frac{3}{4} \quad \text{HAS SLOPE } m = -\frac{5}{4}$$

$$y - y_1 = m(x - x_1) \longrightarrow$$

$$y - 8 = -\frac{5}{4}(x + 6)$$

or

$$y = -\frac{5}{4}x + \frac{1}{2}$$

12. (4 points) Find all real solutions to the equation.

$$x - \sqrt{2x + 31} = 2$$

$$(x - 2)^2 = (\sqrt{2x + 31})^2$$

$$x^2 - 4x + 4 = 2x + 31$$

$$x^2 - 6x - 27 = 0$$

$$(x - 9)(x + 3) = 0$$

$$x = -3, 9$$

CHECK: $x = -3 :$

$$-3 - \sqrt{25} = 2 \quad \text{No } \ddot{\text{a}}$$

$$x = 9 :$$

$$9 - \sqrt{49} = 2 \quad \text{Yes } \ddot{\text{u}}$$

$$x = 9$$

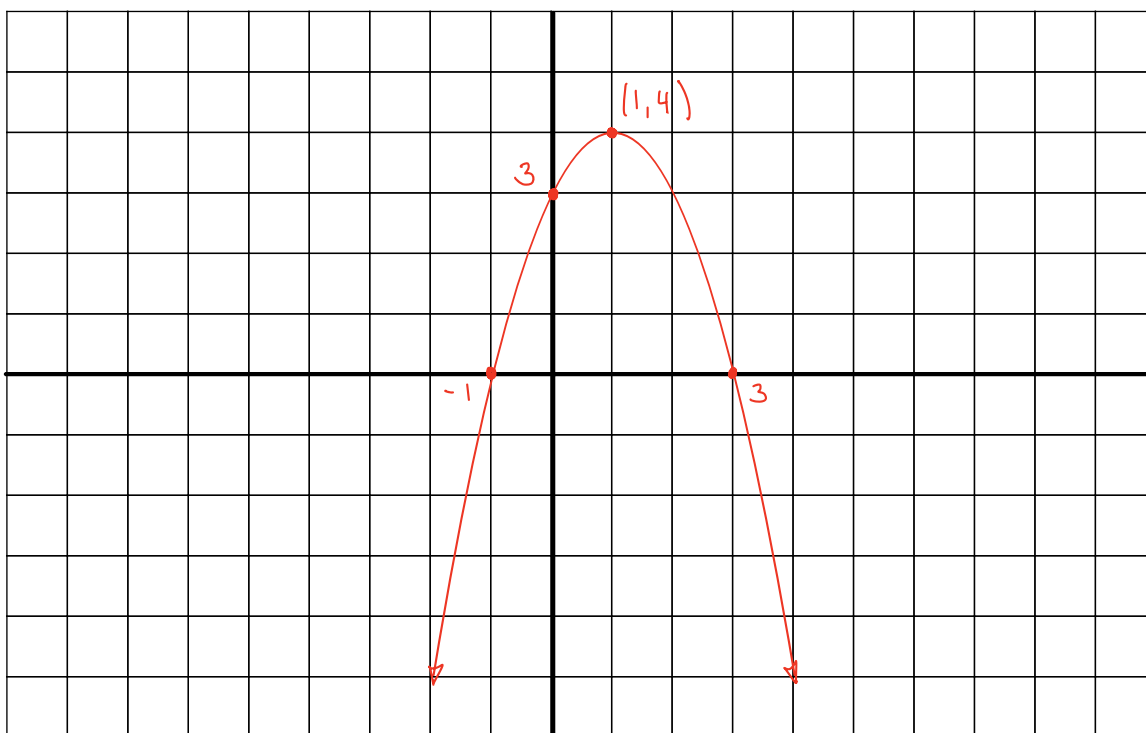
13. (4 points) Let $f(x) = x^2 + 3x$. Evaluate and simplify the expression $f(x+h)$.

$$\begin{aligned} f(x+h) &= (x+h)^2 + 3(x+h) \\ &= x^2 + 2xh + h^2 + 3x + 3h \end{aligned}$$

14. (4 points) Sketch the graph

$$y = 4 - (x-1)^2$$

not by plotting points but by starting with the graph of a standard function and applying transformations. Label any/all x -intercepts, y -intercepts, and/or asymptotes.



15. (4 points) Evaluate $\log_2\left(\frac{1}{16}\right)$.

$$\log_2 \frac{1}{16} = -4 \quad \text{BECAUSE} \quad 2^{-4} = \frac{1}{16}$$

16. Let $f(x) = 2x - 5$ and let $g(x) = 3 - x^2$. Evaluate and simplify the following expressions.

(a) (2 points) $f(g(4))$

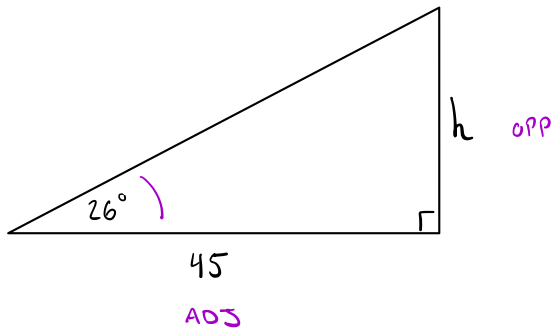
$$g(4) = 3 - 4^2 = 3 - 16 = -13$$

$$f(g(4)) = f(-13) = 2(-13) - 5 = -26 - 5 = \boxed{-31}$$

(b) (2 points) $g(f(x))$

$$\begin{aligned} g(f(x)) &= g(2x - 5) = 3 - (2x - 5)^2 \\ &= 3 - (4x^2 - 20x + 25) = \boxed{-4x^2 + 20x - 22} \end{aligned}$$

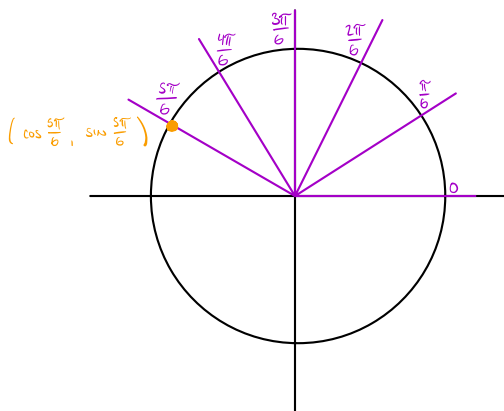
17. (4 points) The angle of elevation to the top of a building is found to be 26° at a distance of 45 meters from the building. Find the height of the building (you may leave your answer in terms of sin, cos, tan, etc).



$$\tan 26^\circ = \frac{h}{45}$$

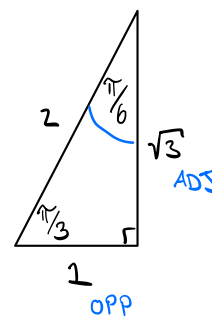
$$h = \boxed{45 \tan 26^\circ}$$

18. (4 points) Find the exact value of $\cos\left(\frac{5\pi}{6}\right)$.



$$\theta = \frac{5\pi}{6} \Rightarrow \text{REFERENCE ANGLE } \bar{\theta} = \frac{\pi}{6}$$

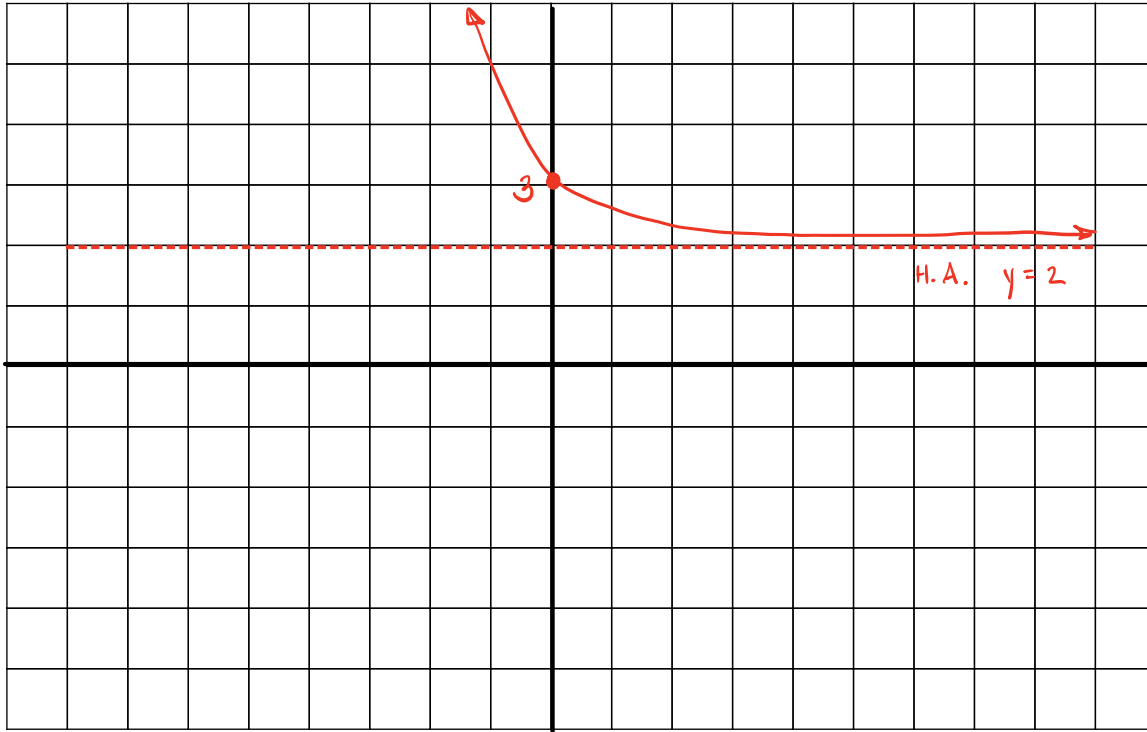
$$\therefore \cos \frac{5\pi}{6} = \ominus \cos \frac{\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$$



19. (4 points) Sketch the graph

$$y = 3^{-x} + 2$$

not by plotting points but by starting with the graph of a standard function and applying transformations. Label any/all x -intercepts, y -intercepts, and/or asymptotes.



20. (a) (2 points) Convert 72° to radians.

$$72 \text{ DEG} \times \frac{\pi \text{ RAD}}{180 \text{ DEG}} = \frac{72\pi}{180} \text{ RAD} = \frac{36 \cdot 2\pi}{36 \cdot 5} \text{ RAD}$$

$$= \frac{2\pi}{5} \text{ RAD}$$

(b) (2 points) Convert $\frac{4\pi}{9}$ rad to degrees.

$$\frac{4\pi}{9} \text{ RAD} \times \frac{180 \text{ DEG}}{\pi \text{ RAD}} = \frac{4 \times 180 \text{ DEG}}{9} = \frac{4 \times 9 \times 20 \text{ DEG}}{9}$$

$$= 80^\circ$$

21. (4 points) Let $f(x) = \frac{2}{5x+3}$. Find $f^{-1}(x)$.

$$y = \frac{2}{5x+3}$$

$$5xy = 2 - 3y$$

$$y(5x+3) = 2$$

$$x = \frac{2-3y}{5y}$$

$$\Rightarrow f^{-1}(x) = \frac{2-3x}{5x}$$

$$5xy + 3y = 2$$

22. (4 points) Find all real solutions to the equation.

$$x(2x-1) + (2x-1)^2 = 0$$

$$(2x-1) \left[x + (2x-1) \right] = 0$$

$$(2x-1)(3x-1) = 0 \quad \Rightarrow \quad 2x-1=0 \quad \text{or} \quad 3x-1=0$$

$$x = \frac{1}{2}$$

$$x = \frac{1}{3}$$

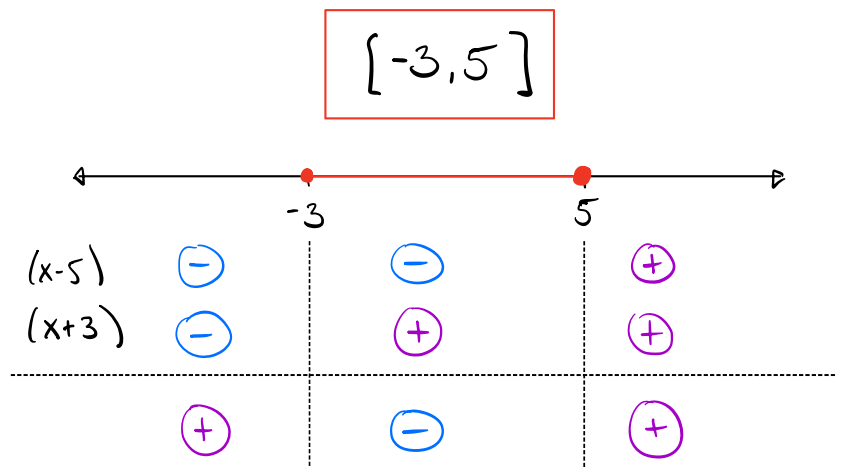
23. (4 points) Use interval notation to state the solutions to the inequality.

$$x^2 \leq 2x + 15$$

$$x^2 - 2x - 15 \leq 0$$

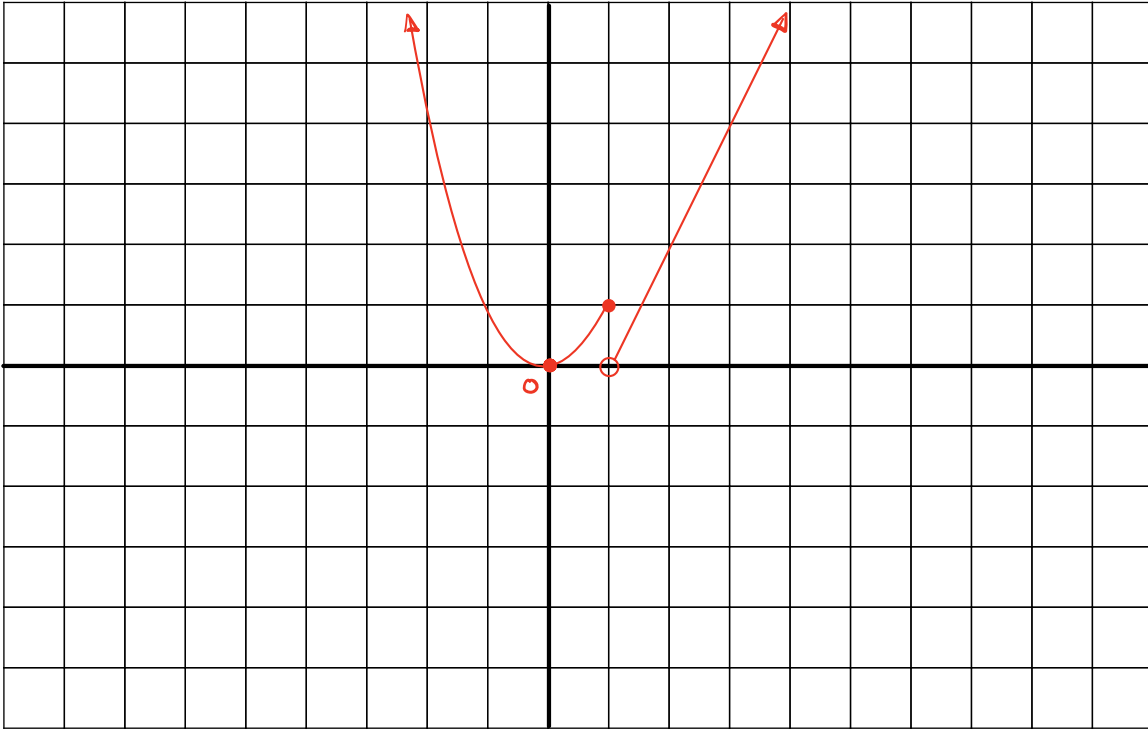
$$(x-5)(x+3) \leq 0$$

$$\text{Zeros: } x = -3, 5$$

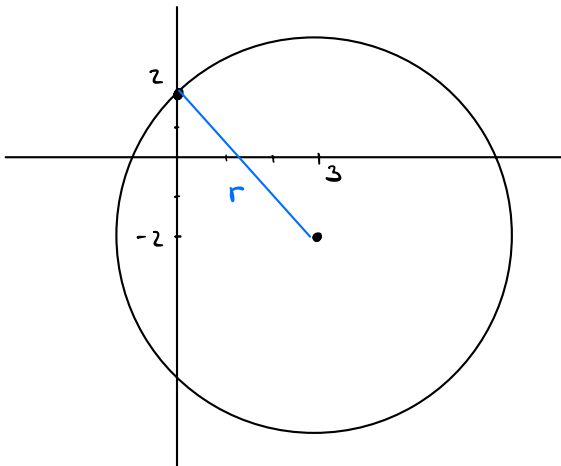


24. (4 points) Sketch the graph of the piecewise defined function.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x - 2 & \text{if } x > 1 \end{cases}$$



25. (4 points) Give an equation for the circle that has center $(3, -2)$ and passes through the point $(0, 2)$.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\downarrow$$

$$r = \sqrt{(0 - 3)^2 + (2 + 2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\downarrow$$

$$(x - 3)^2 + (y + 2)^2 = 25$$