Name: *Answer Key* 4/7/2016

Math 202 Quiz 2

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Good luck!

1. Evaluate the following three integrals.

   (a) (8 points) $\int e^x \cos x \, dx$

Let $u = e^x$ $\quad v = \sin x$

$du = e^x\,dx$ $\quad dv = \cos x\,dx$

CONSISTANT CHOICE OF u!

Then $\int e^x \cos x\,dx = e^x \sin x - \underbrace{\int e^x \sin x\,dx}$

Let $u = e^x$ $\quad v = -\cos x$

$du = e^x\,dx$ $\quad dv = \sin x\,dx$

Then $\int e^x \cos x\,dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x\,dx \right]$

$= e^x(\sin x + \cos x) - \int e^x \cos x\,dx$

$2\int e^x \cos x\,dx = e^x(\sin x + \cos x)$

$\int e^x \cos x\,dx = \boxed{\frac{1}{2} e^x (\sin x + \cos x) + C}$

(b) (8 points) $\int_0^3 x^3\sqrt{9-x^2}\,dx$

LET $x = 3\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = 3\cos\theta\, d\theta$

NOTE THAT WHEN $x = 3 \longrightarrow \sin\theta = 1 \longrightarrow \theta = \frac{\pi}{2}$

$x = 0 \longrightarrow \sin\theta = 0 \longrightarrow \theta = 0$

$$\rightsquigarrow \int_0^{\pi/2} 3^3 \sin^3\theta \sqrt{9 - 9\sin^2\theta}\; 3\cos\theta\, d\theta$$

$$3^5 \int_0^{\pi/2} \sin^3\theta \cos^2\theta\, d\theta = 3^5 \int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta \sin\theta\, d\theta$$

LET $u = \cos\theta$

$du = -\sin\theta\, d\theta$

NOTE THAT WHEN $\theta = \pi/2 \longrightarrow u = 0$

$\theta = 0 \longrightarrow u = 1$

$$\rightsquigarrow -3^5 \int_1^0 (1-u^2)u^2\, du = 3^5 \int_0^1 u^2 - u^4\, du$$

$$= 3^5 \left[\frac{1}{3}u^3 - \frac{1}{5}u^5 \right]_0^1 = 3^5\left(\frac{1}{3} - \frac{1}{5}\right) = 3^5 \cdot \frac{2}{15}$$

$$= \frac{2 \cdot 3^4}{5} = \boxed{\frac{162}{5}}$$

(c) (8 points) $\int \frac{x^2+3x+3}{x(x+2)^2}\,dx$ PROPER RATIONAL FUNCTION

BY METHOD OF PARTIAL FRACTIONS :

$$\frac{x^2+3x+3}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$\Rightarrow \quad x^2+3x+3 = A(x+2)^2 + Bx(x+2) + Cx$$

LET $x = 0$: $3 = 4A \Rightarrow \underline{\underline{A = \frac{3}{4}}}$

$x = -2$: $1 = -2C \Rightarrow \underline{\underline{C = -\frac{1}{2}}}$

NOW LET $x = 1$ (ARBITRARY)

$A = \frac{3}{4}$

$C = -\frac{1}{2}$

$$\Rightarrow \quad 7 = \frac{3}{4}(9) + 3B - \frac{1}{2}$$

$$\frac{3}{4} = 3B \Rightarrow \underline{\underline{B = \frac{1}{4}}}$$

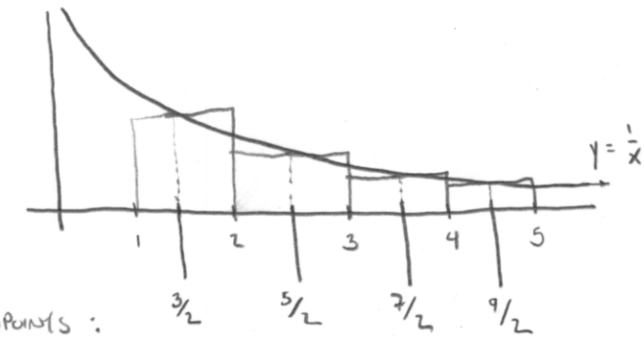
$$\therefore \int \frac{x^2+3x+3}{x(x+2)^2}\,dx = \int \frac{3/4}{x} + \frac{1/4}{x+2} - \frac{1/2}{(x+2)^2}\,dx$$

$$= \boxed{\frac{3}{4}\ln|x| + \frac{1}{4}\ln|x+2| + \frac{1}{2(x+2)} + C}$$

2. Approximate $\ln(5) = \int_1^5 \frac{1}{x}\,dx$ in two different ways.

You can leave your answers as an arithmetic expression.

(a) (5 points) Midpoint rule with $n = 4$ (i.e. M_n)

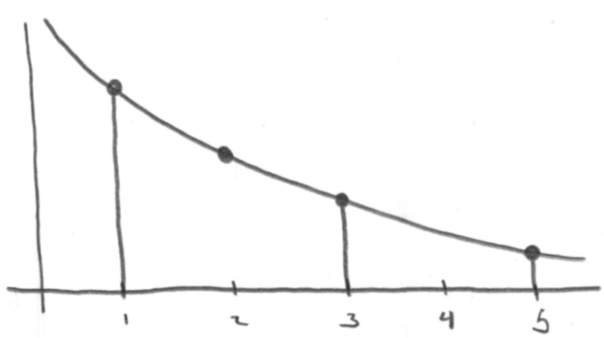


MIDPOINTS :

$$M_4 = \Delta x \left(f\left(\tfrac{3}{2}\right) + f\left(\tfrac{5}{2}\right) + f\left(\tfrac{7}{2}\right) + f\left(\tfrac{9}{2}\right) \right), \quad \Delta x = 1$$

$$= \boxed{\frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9}} = \frac{496}{315} \approx 1.5746$$

(b) (5 points) Simpson's Rule with $n = 4$ (i.e. S_n)



(FYI, $\ln 5 \approx 1.6094$)

$$S_4 = \frac{\Delta x}{3} \left(f(1) + 4f(2) + 2f(3) + 4f(4) + f(5) \right), \quad \Delta x = 1$$

$$= \boxed{\frac{1}{3}\left(1 + 2 + \frac{2}{3} + 1 + \frac{1}{5} \right)} = \frac{73}{45} \approx 1.6222$$

3. (8 points) Evaluate the following integral or show that it diverges.

$$\int_0^\infty xe^{-x^2}\,dx$$

LET $u = x^2$ NOTE THAT WHEN $x \to \infty$, $u \to \infty$

$du = 2x\,dx$ $x = 0$, $u = 0$

$$\rightsquigarrow \frac{1}{2}\int_0^\infty e^{-u}\,du = \frac{1}{2}\lim_{t\to\infty}\int_0^t e^{-u}\,du$$

$$= \frac{1}{2}\lim_{t\to\infty} -e^{u}\Big|_0^t$$

$$= \frac{1}{2}\lim_{t\to\infty} \underbrace{-e^{t}}_{\downarrow\, 0} + \underbrace{e^{0}}_{= 1} = \boxed{\frac{1}{2}}$$

4. (8 points) Without attempting to evaluate it, determine whether the following integral converges or diverges. Justify your answer.

$$\int_1^\infty \frac{1}{\sqrt{x^5+2}}\,dx$$

NOTE THAT WHEN $x \geq 1$ WE HAVE

$$x^5 + 2 \geq x^5$$

$$\sqrt{x^5+2} \geq \sqrt{x^5}$$

$$\frac{1}{\sqrt{x^5+2}} \leq \frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}}$$

$$\therefore \quad 0 \leq \int_1^\infty \frac{1}{\sqrt{x^5+2}}\,dx \leq \underbrace{\int_1^\infty \frac{1}{x^{5/2}}\,dx}_{\text{CONVERGES BY } p\text{-TEST}}$$

$\therefore$ BY COMPARISON THM, $\int_1^\infty \frac{1}{\sqrt{x^5+2}}\,dx$ ALSO CONVERGES.