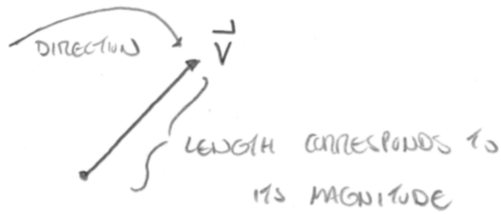
# §10.2 VECTORS

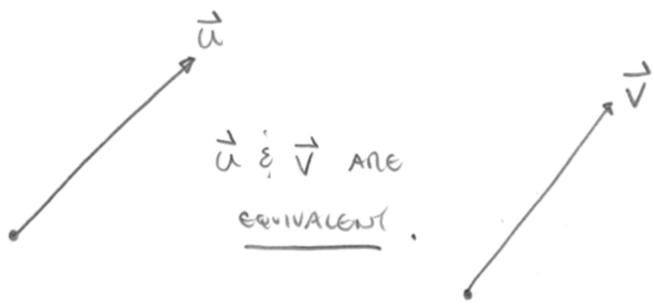
A VECTOR IS A QUANTITY WITH BOTH MAGNITUDE (SIZE) & DIRECTION.

e.g. DISPLACEMENT, VELOCITY, FORCE, ETC..

DENOTE BY BOLDFACE TYPE OR $\rightarrow$ ABOVE THE LETTER (e.g. VECTOR **v** OR $\vec{v}$).





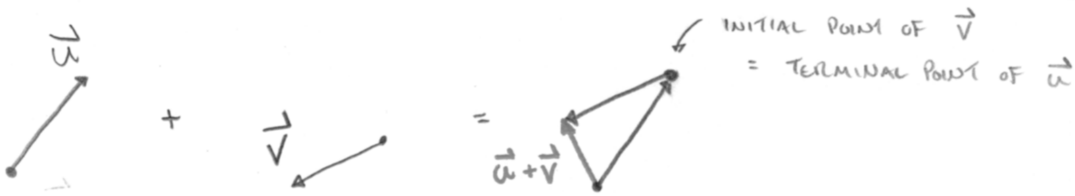


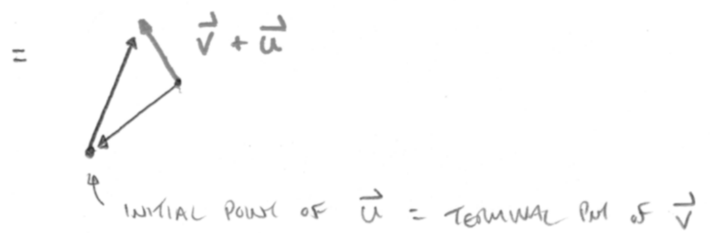

$\vec{u}$ & $\vec{v}$ ARE <u>EQUIVALENT</u>.

<u>NOTE</u>: 0 VECTOR DOES NOT HAVE SPECIFIC DIRECTION.

## THE GEOMETRY

ADDING VECTORS:



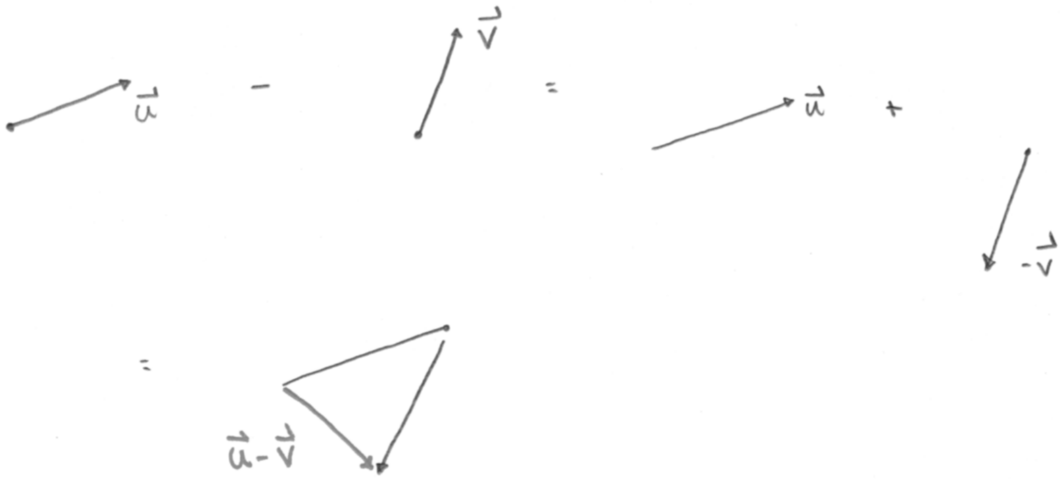




REAL #, NOT A VECTOR.

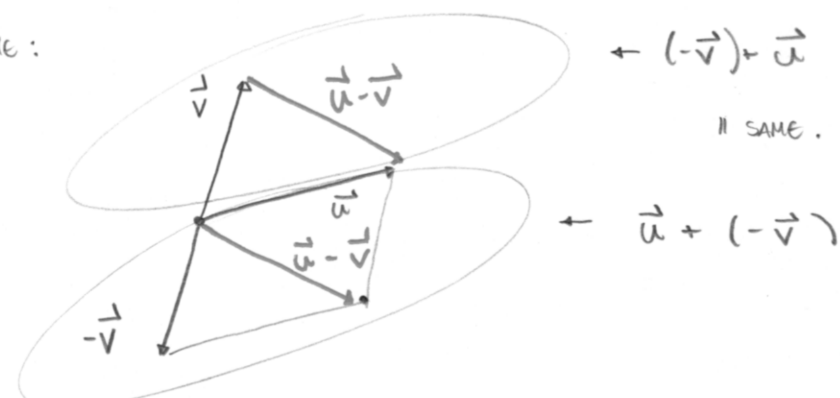
MULTIPLYING BY SCALAR: LET $c$ BE A SCALAR & $\vec{v}$ BE A VECTOR. THEN $c\vec{v}$ IS THE VECTOR WHOSE MAGNITUDE (LENGTH) IS $|c|$ TIMES THE MAGNITUDE (LENGTH) OF $\vec{v}$ & WHOSE DIRECTION IS THE SAME AS $\vec{v}$ IS $c > 0$, OPPOSITE $\vec{v}$ IF $c < 0$, AND $c\vec{v} = 0$ IF EITHER $\vec{v} = 0$ OR $c = 0$.

e.g.


SUBTRACTION: $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$




NOTE:

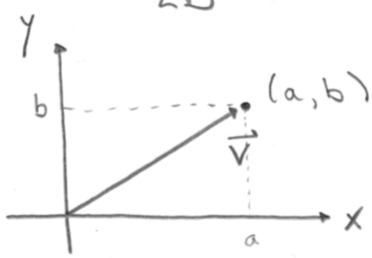



## THE ALGEBRA

GIVEN A VECTOR, WE CAN INTRODUCE A COORDINATE SYSTEM IN ORDER TO DESCRIBE IT PRECISELY BY CALLING ITS INITIAL POINT THE ORIGIN & CALLING ITS TERMINAL POINT BY ITS COORDINATES, CALLED THE COMPONENTS OF THE VECTOR.

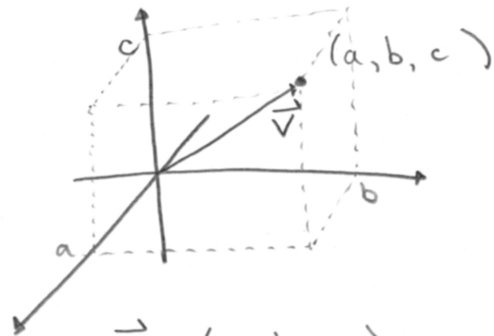
e.g.

2D




$\vec{v} = \langle a, b \rangle$

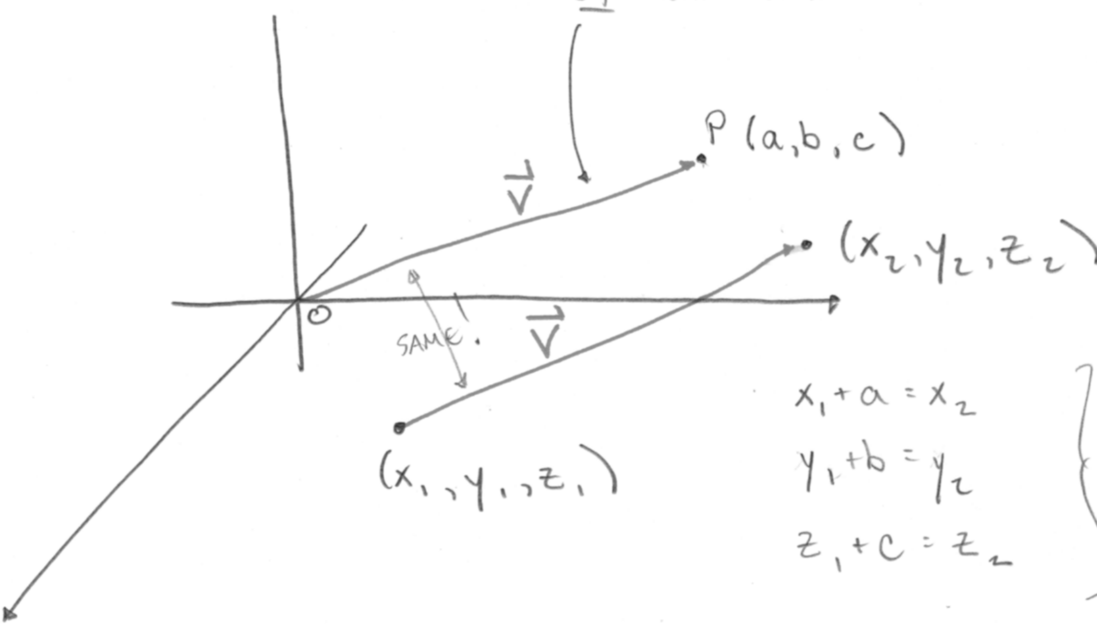
3D




$\vec{v} = \langle a, b, c \rangle$

NOTATION!

<u>Def</u>: POSITION VECTOR (INITIAL PNT ORIGIN)




$$\left.\begin{aligned} x_1 + a &= x_2 \\ y_1 + b &= y_2 \\ z_1 + c &= z_2 \end{aligned}\right\} \rightarrow \begin{aligned} x_2 - x_1 &= a \\ y_2 - y_1 &= b \\ z_2 - z_1 &= c \end{aligned}$$

GIVEN $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$, THE VECTOR $\vec{v}$ WITH REPRESENTATION $\overrightarrow{AB}$ IS GIVEN BY

$$\vec{v} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

e.g. FIND VECTOR REPRESENTED BY THE DIRECTED LINE SEGMENT FROM $A(3, -7, 5)$ TO $B(5, 3, -6)$.

THE MAGNITUDE (LENGTH) OF A VECTOR $\vec{v} = \langle x, y \rangle$ OR $\vec{v} = \langle x, y, z \rangle$ IS DENOTED $|\vec{v}|$ OR $\|\vec{v}\|$ AND IS GIVEN BY

$$|\vec{v}| = \sqrt{x^2 + y^2} \quad \text{OR} \quad |\vec{v}| = \sqrt{x^2 + y^2 + z^2}$$

NOTE THAT THIS FOLLOWS FROM THE DISTANCE FORMULA.

NOTE THAT

$|\vec{v}| \geq 0$ & $|\vec{v}| = 0 \Leftrightarrow \vec{v} = \vec{0}$

IF $\vec{a} = \langle a_1, a_2 \rangle$ & $\vec{b} = \langle b_1, b_2 \rangle$

THEN $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$

$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$

$c\vec{a} = \langle ca_1, ca_2 \rangle$

& FOR 3D AS WELL.

e.g. $\langle 3, -4 \rangle - 5\langle -6, 8 \rangle = \langle 3 - 5(-6), -4 - 5(8) \rangle = \langle 33, -44 \rangle$

PROPERTIES OF VECTORS

IF $\vec{a}, \vec{b}, \vec{c}$ ARE $n$-DIMENSIONAL VECTORS & $c, d$ ARE SCALARS THEN

(i) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (COMM.)

(ii) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (ASS.)

(iii) $\vec{a} + \vec{0} = \vec{a}$ (ADD ID.)

(iv) $\vec{a} - \vec{a} = \vec{0}$ (ADD INVERSE.)

(v) $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$ (DIST.)

(vi) $(\vec{c} + d)\vec{a} = c\vec{a} + d\vec{a}$ (DIST.)

(vii) $(cd)\vec{a} + c(d\vec{a})$ (ASSOC.)

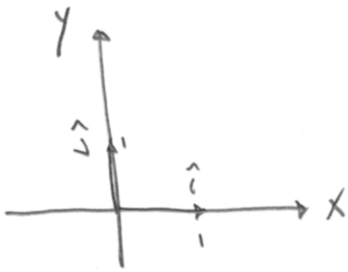
(viii) $1\vec{a} = \vec{a}$ (MULT. ID.)

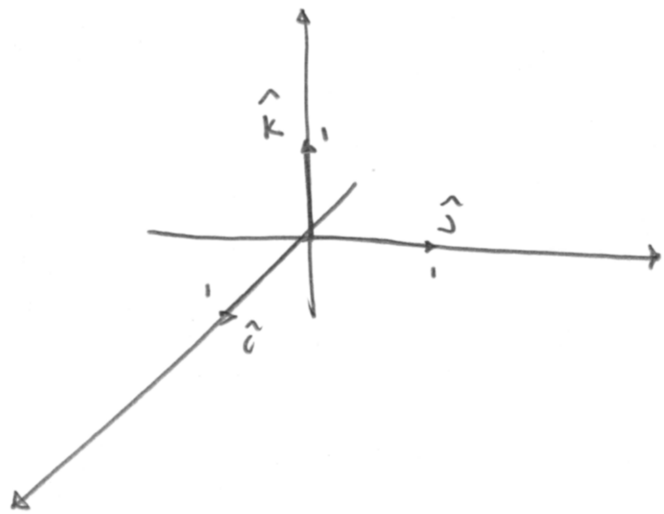
STANDARD BASIS VECTORS:

$$\hat{i} = \langle 1, 0, 0 \rangle \quad , \quad \hat{j} = \langle 0, 1, 0 \rangle \quad , \quad \hat{k} = \langle 0, 0, 1 \rangle$$

POINT IN DIR OF X-AXIS, Y-AXIS, Z-AXIS.

ALL LENGTH 1 (UNIT VECTORS).

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THUS $\vec{v} = \langle x, y, z \rangle = \langle x, 0, 0 \rangle + \langle 0, y, 0 \rangle + \langle 0, 0, z \rangle$

$$= x\langle 1, 0, 0 \rangle + y\langle 0, 1, 0 \rangle + z\langle 0, 0, 1 \rangle$$

$$= x\hat{i} + y\hat{j} + z\hat{k}$$

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(IN 2D) $\vec{v} = \langle x, y \rangle = x\hat{i} + y\hat{j}$

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e.g. IF $\vec{a} = 7\hat{i} + 2\hat{j} + 0\hat{k}$ & $\vec{a} = 2\hat{i} - 4\hat{j} - 11\hat{k}$

THEN $3\vec{a} + 4\vec{b} = 29\hat{i} - 10\hat{j} - 20\hat{k}$.

THE <u>UNIT VECTOR</u> THAT POINTS IN THE SAME DIRECTION AS $\vec{v}$ IS $\dfrac{1}{|\vec{v}|}\vec{v}$.

TO SEE THIS, FIRST NOTE THAT FOR SCALAR $c$, WE HAVE

$$\begin{aligned}|c\vec{v}| &= |\langle cx, cy, cz\rangle| \\ &= \sqrt{(cx)^2+(cy)^2+(cz)^2} \\ &= \sqrt{c^2(x^2+y^2+z^2)} = |c|\sqrt{x^2+y^2+z^2} \\ &= |c||\vec{v}|\end{aligned}$$

$$\therefore \left|\frac{1}{|\vec{v}|}\vec{v}\right| = \left|\frac{1}{|\vec{v}|}\right| |\vec{v}| = \frac{1}{|\vec{v}|}|\vec{v}| = 1. \checkmark$$

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e.g. FIND UNIT VECTOR IN SAME DIRECTION AS $\langle -1, 12, -12\rangle$.

$\left(\langle -\frac{1}{17}, \frac{12}{17}, -\frac{12}{17}\rangle\right)$

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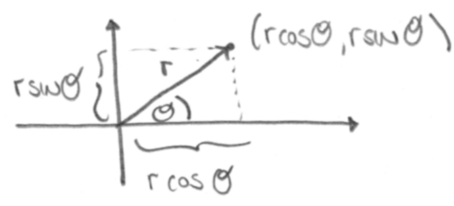
RECALL FROM POLAR COORDINATES:





NOW IF $\vec{v} = \langle x, y\rangle$ THEN

$$x = |\vec{v}|\cos\theta$$

$$y = |\vec{v}|\sin\theta$$

WHERE $\theta$ IS ANGLE BETWEEN $\hat{i}$ & $\vec{v}$.

[EXAMPLE LIKE #28, OR EX 7 P. 554]