§10.2 VECTORS

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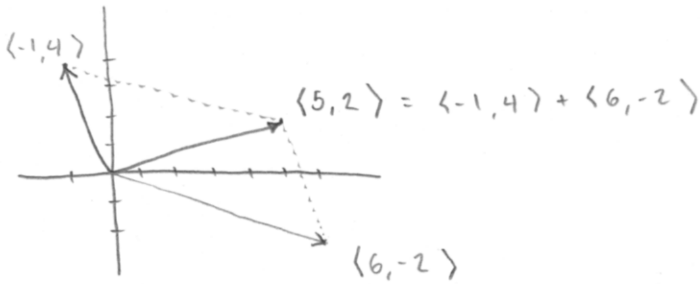
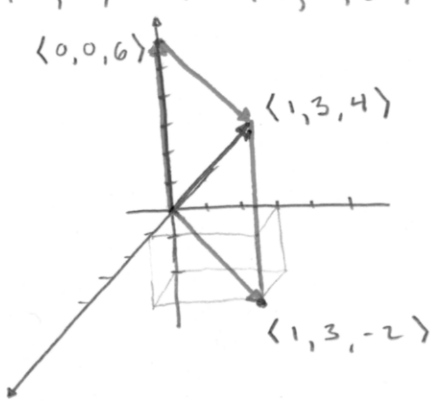
2. (a) $\overrightarrow{AC}$ (b) $\overrightarrow{CB}$ (c) $\overrightarrow{DA}$ (d) $\overrightarrow{DB}$

9. $\langle -1, 4\rangle + \langle 6, -2\rangle = \langle -1+6, 4-2\rangle = \boxed{\langle 5, 2\rangle}$



12. $\langle 1, 3, -2\rangle + \langle 0, 0, 6\rangle = \langle 1+0, 3+0, -2+6\rangle = \boxed{\langle 1, 3, 4\rangle}$



14. $\vec{a} + \vec{b} = (4+1)\hat{i} + (1-2)\hat{j} = \boxed{5\hat{i} - \hat{j}}$

$2\vec{a} + 3\vec{b} = [2(4)+3(1)]\hat{i} + [2(1)+3(-2)]\hat{j} = \boxed{11\hat{i} - 4\hat{j}}$

$|\vec{a}| = \sqrt{4^2 + 2^2} = \sqrt{20} = \boxed{2\sqrt{5}}$

$|\vec{a} - \vec{b}| = |\langle 4-1, 1-(-2)\rangle| = \sqrt{3^2 + 3^2} = \boxed{3\sqrt{2}}$

15. $\vec{a} + \vec{b} = \langle -1, 1, 2\rangle = \boxed{-\hat{i} + \hat{j} + 2\hat{k}}$

$2\vec{a} + 3\vec{b} = \langle 2-6, 4-3, -6+15\rangle = \boxed{4\hat{i} + \hat{j} + 9\hat{k}}$

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \boxed{\sqrt{14}}$

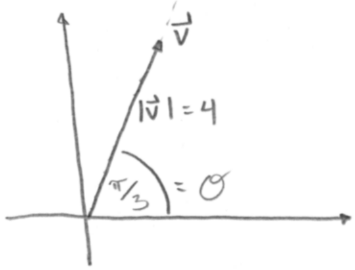
$|\vec{a} - \vec{b}| = |\langle 1-(-2), 2-(-1), -3-5\rangle| = \sqrt{3^2 + 3^2 + 8^2} = \boxed{\sqrt{82}}$

17. $\left| 8\hat{i} - \hat{j} + 4k \right| = \sqrt{8^2 + 1^2 + 4^2} = \sqrt{81} = 9$

$\therefore \underbrace{\frac{1}{9}\left(8\hat{i} - \hat{j} + 4\hat{k}\right)} = \boxed{\frac{8}{9}\hat{i} - \frac{1}{9}\hat{j} + \frac{4}{9}\hat{k}}$

$\frac{1}{|\vec{v}|}\vec{v}$: UNIT VECTOR

21.

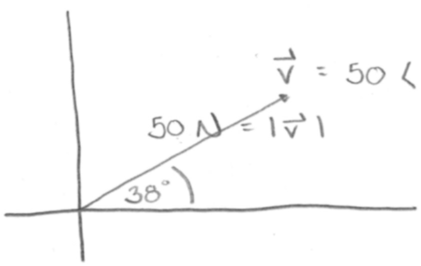


$\vec{v} = |v| \langle \cos\theta , \sin\theta \rangle$

$= 4 \left\langle \frac{1}{2} , \frac{\sqrt{3}}{2} \right\rangle = \boxed{\langle 2 , 2\sqrt{3} \rangle}$

OR $2\hat{i} + 2\sqrt{3}\hat{j}$

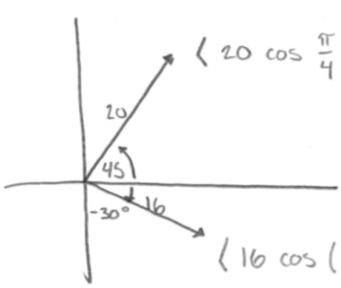
22.



$\vec{v} = 50 \langle \cos 38^\circ , \sin 38^\circ \rangle$

$$\boxed{\begin{array}{ll} \text{HORIZONTAL COMPONENT :} & 50\cos 38^\circ \\ \text{VERTICAL COMPONENT :} & 50\sin 38^\circ \end{array}}$$

24.



$\left\langle 20\cos\frac{\pi}{4} , 20\sin\frac{\pi}{4} \right\rangle = \langle 10\sqrt{2} , 10\sqrt{2} \rangle$

ADD THESE TO GET RESULTANT FORCE!

$\left\langle 16\cos\left(-\frac{\pi}{6}\right) , 16\sin\left(-\frac{\pi}{6}\right) \right\rangle = \langle 8\sqrt{3} , -8 \rangle$

$\longrightarrow \langle 10\sqrt{2} + 8\sqrt{3} , 10\sqrt{2} - 8 \rangle$ IS RESULTANT FORCE

MAGNITUDE $= \sqrt{(10\sqrt{2} + 8\sqrt{3})^2 + (10\sqrt{2} - 8)^2} = \sqrt{200 + 160\sqrt{6} + 192 + 200 - 160\sqrt{2} + 64}$

$= \boxed{\sqrt{656 + 160(\sqrt{6} - \sqrt{2})}} \ (\approx 28.6)$

SINCE $10\sqrt{2} + 8\sqrt{3} = \sqrt{656 + 160(\sqrt{6} - \sqrt{2})}\cos\theta$

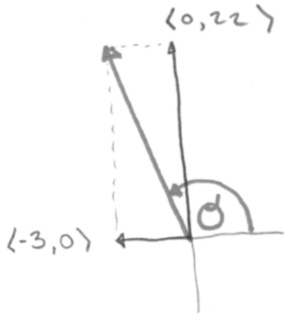
$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{10\sqrt{2} + 8\sqrt{3}}{\sqrt{656 + 160(\sqrt{6} - \sqrt{2})}}\right)} \ (\approx 12.37^\circ)$

27. FIND WOMAN'S VELOCITY

$\Downarrow$

VECTOR W/ HORIZONTAL COMPONENT -3 (WEST IS LEFT)

VERTICAL COMPONENT 22 (NORTH IS UP)



WOMAN'S VELOCITY IS $\langle -3, 22\rangle$

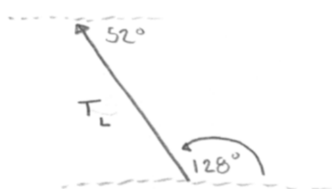
$$\boxed{\text{MAGNITUDE} = \sqrt{3^2+22^2} = \sqrt{493}}$$

NOTE: $-3 = \sqrt{493}\cos\theta$ $\longrightarrow$ $\boxed{\theta = \cos^{-1}\left(\frac{-3}{\sqrt{493}}\right)}$ $\approx 97.7^\circ$

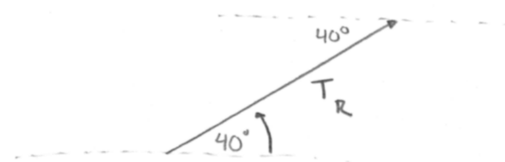
$22 = \sqrt{493}\sin\theta$

28.

LEFT WIRE



RIGHT WIRE



$T_L = |T_L|\langle\cos(128^\circ), \sin(128^\circ)\rangle$ $\qquad$ $T_R = |T_R|\langle\cos(40^\circ), \sin(40^\circ)\rangle$

THUS, $|T_L|\cos(128^\circ) + |T_R|\cos(40^\circ) = 0$

$|T_L|\sin(128^\circ) + |T_R|\sin(40^\circ) = (5\,kg)(9.8\,m/s^2) = 49\,N$

$\therefore \quad |T_L|\left[\sin(40^\circ)\cos(128^\circ) - \cos(40^\circ)\sin(128^\circ)\right] = -49\cos(40^\circ)$

$$\boxed{|T_L| = \frac{-49\cos(40^\circ)}{\sin(40^\circ)\cos(128^\circ) - \cos(40^\circ)\sin(128^\circ)}} \approx 37.6\,N$$

$|T_R|\left[\sin(128^\circ)\cos(40^\circ) - \cos(128^\circ)\sin(40^\circ)\right] = -49\cos(128^\circ)$

$$\boxed{|T_R| = \frac{-49\cos(128^\circ)}{\sin(128^\circ)\cos(40^\circ) - \cos(128^\circ)\sin(40^\circ)}} \approx 30.2\,N$$

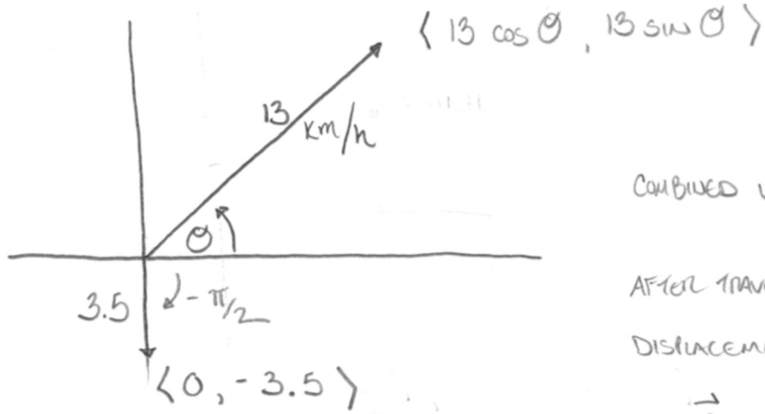
30. THE VERTICAL COMPONENT OF THE TENSION AT EACH END IS

$25 \sin(37°)$. SO, COMBINED, THE VERTICAL COMPONENT

OF THE FORCE ON THE CHAIN IS $\boxed{50 \sin(37°).}$

THIS MUST BE EQUAL TO THE FORCE DUE TO GRAVITY (i.e. WEIGHT).

31.



COMBINED VELOCITY: $\vec{v} = \langle 13\cos\theta, 13\sin\theta - 3.5\rangle$

AFTER TRAVELING FOR t HOURS, THE BOAT HAS DISPLACEMENT

$$\vec{s} = t\vec{v} = \langle t13\cos\theta, t(13\sin\theta - 3.5)\rangle$$

$$= \langle 3, 2\rangle$$

TOTAL DISPLACEMENT WHEN DESTINATION IS REACHED.

SYSTEM OF EQ'S:

$$\Rightarrow \quad 13t\cos\theta = 3 \longrightarrow$$

$$t(13\sin\theta - 3.5) = 2 \longrightarrow \quad \frac{13\cos\theta}{13\sin\theta - 3.5} = \frac{3}{2}$$

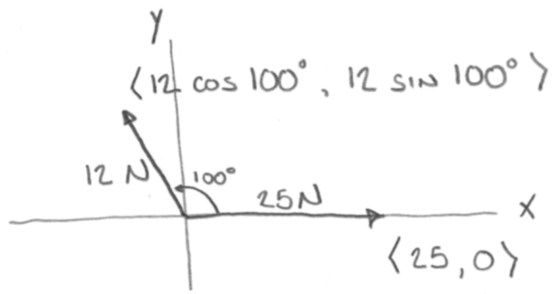
YOU WILL WANT TO USE A CALCULATOR TO SOLVE:

$$\boxed{\theta \approx 46.6°}$$

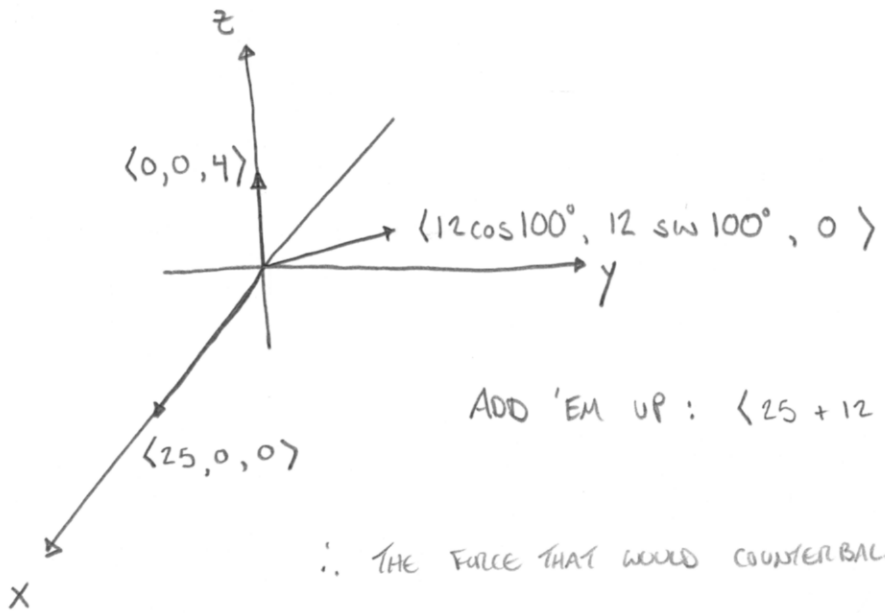
THEN $t \approx \dfrac{3}{13\cos(46.6°)} \approx .3359$ HOURS

$\approx \boxed{\text{20 MIN 9 SEC.}}$

32. LET'S PUT THE 1ST 2 FORCES IN THE XY-PLANE :



NOW DRAW THEM IN 3D, ALONG WITH THE THIRD FORCE



ADD 'EM UP : $\langle 25 + 12 \cos 100°, 12 \sin 100°, 4 \rangle$

$\therefore$ THE FORCE THAT WOULD COUNTERBALANCE THIS IS ITS OPPOSITE : $\langle -25 - 12 \cos 100°, -12 \sin 100°, -4 \rangle$

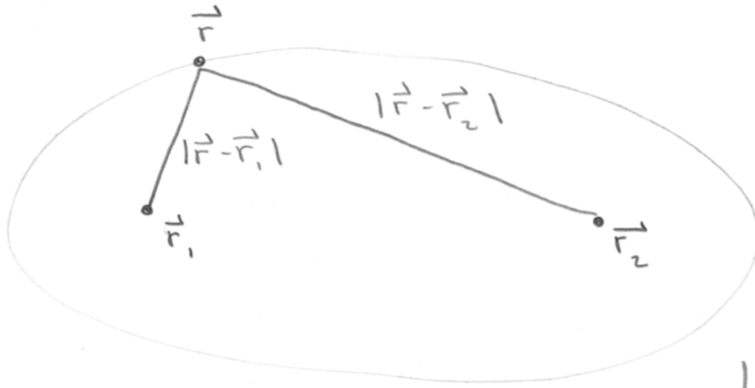
37. $|\vec{r} - \vec{r}_0| = 1$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = 1$$

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = 1$$

SPHERE OF RADIUS 1 CENTERED AT $\vec{r}_0$

38.



$\boxed{\text{ELLIPSE.}}$