§10.3 THE DOT PRODUCT

2 WAYS TO MULT. VECTORS. ONE WAY: DOT PRODUCT.

Def: GIVEN VECTORS $\vec{a} = \langle x_1, y_1, z_1 \rangle$ & $\vec{b} = \langle x_2, y_2, z_2 \rangle$

$$\text{"}\vec{a} \text{ DOT } \vec{b}\text{"} = \vec{a} \cdot \vec{b} := \underbrace{x_1 x_2 + y_1 y_2 + z_1 z_2}_{\text{SCALAR!}}$$

ALSO 2D DEF

DOT PRODUCT = SCALAR PRODUCT = INNER PRODUCT

e.g. $\langle 3, -2 \rangle \cdot \langle 5, 4 \rangle = 3 \cdot 5 + (-2) \cdot 4 = 7.$

e.g. $(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (-7\hat{i} - \hat{j} + \hat{k}) = -6$

PROPERTIES OF DOT PRODUCT

LET $\vec{a}, \vec{b}, \vec{c}$ BE VECTORS. LET c BE SCALAR.

(1) $\vec{a} \cdot \vec{a} = |a|^2$

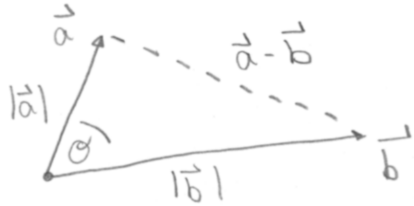
(2) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

(3) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

(4) $(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (c\vec{b})$

(5) $\vec{0} \cdot \vec{a} = 0$

GEOMETRIC DEF: OF DOT PRODUCT: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$



LAW OF COSINES: $|a - b|^2 = |\vec{a}|^2 + |b|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$$\downarrow$$

$$(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} - 2|\vec{a}||\vec{b}|\cos\theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta \quad \square$$

e.g. FIND THE ANGLE BETWEEN $\langle 1, 0 \rangle$ AND $\langle 1, 1 \rangle$

$\langle 1, 0, 0 \rangle$ AND $\langle 1, 1, 1 \rangle$

COR. IF θ IS ANGLE BETWEEN $\vec{a}$ & $\vec{b}$, THEN

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$$

e.g. FIND ACUTE ANGLE OF INTERSECTION FOR $y = 3x + 1$, $y = -x + 3$.

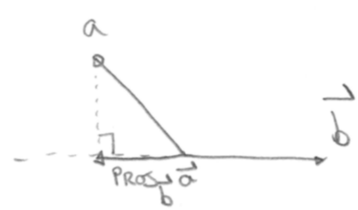
($\perp$, $\parallel$?)

TWO VECTORS ARE ORTHOGONAL IFF THEIR DOT PRODUCT IS ZERO
(PERP.) (i.e. $\theta = \frac{\pi}{2}$)

TWO VECTORS ARE PARALLEL IFF THEIR DOT PRODUCT IS $\pm |\vec{a}||\vec{b}|$

(i.e. $\theta = 0$ or π)

PROJECTIONS



MAGNITUDE

$|\vec{b}|\cos\theta = \text{COMP}_{\vec{a}}\vec{b}$

DIR $\dfrac{\vec{a}}{|\vec{a}|}$ ← UNIT VECTOR.

COMPONENT OF $\vec{b}$ ALONG $\vec{a}$

SCALAR PROJECTION

SCALAR PROJECTION OF $\vec{b}$ ONTO $\vec{a}$: $\quad \operatorname{comp}_{\vec{a}} \vec{b} = |\vec{b}| \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a|}$

VECTOR PROJECTION OF $\vec{b}$ ONTO $\vec{a}$: $\quad \operatorname{proj}_{\vec{a}} \vec{b} = |b| \cos\theta \cdot \dfrac{\vec{a}}{|\vec{a}|}$

$$= \left(\frac{\vec{a}\cdot\vec{b}}{|a|}\right)\frac{\vec{a}}{|a|} = \frac{\vec{a}\cdot\vec{b}}{|a|^2}\,\vec{a}$$

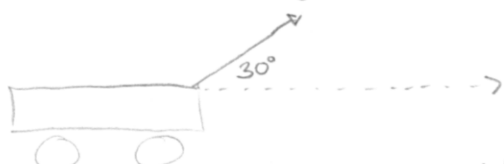
---

e.g. FIND SCALAR & VECTOR PROJECTION OF $\langle 3, -4, 2\rangle$ ONTO $\langle -1, 4, 8\rangle$.

$W = Fd$ WHEN FORCE IS IN THE DIRECTION OF DISPLACEMENT,

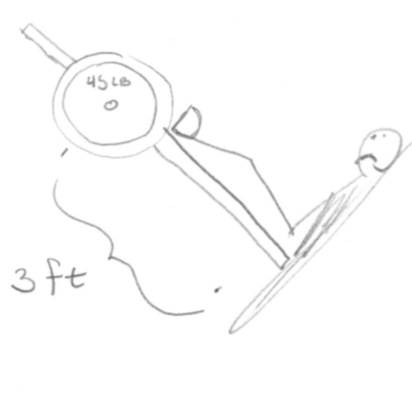
OTHERWISE, $\quad W = \underbrace{\left(|\vec{F}|\cos\theta\right)}_{\text{COMPONENT OF } \vec{F} \text{ ALONG } \vec{D}} |\vec{D}| = \vec{F}\cdot\vec{D}$

e.g.



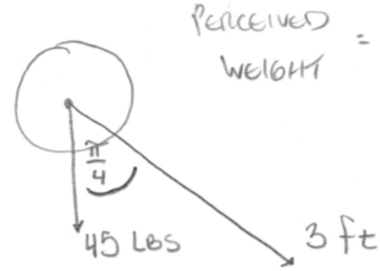
$W = (60\text{ N})(100\text{ m}) \cos\frac{\pi}{6} = 3000\sqrt{3}$ J

e.g.

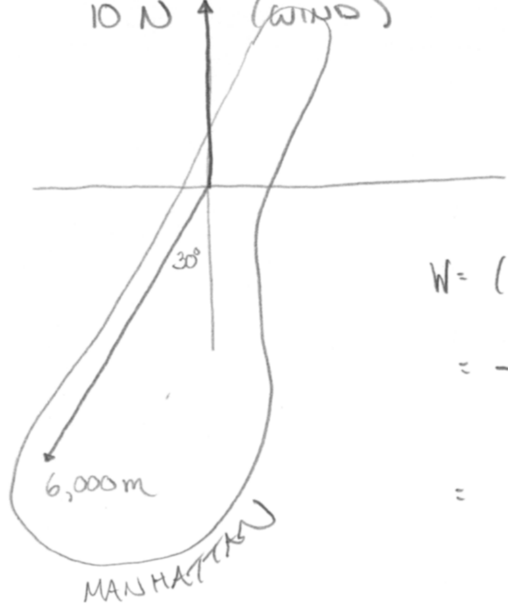


$W = (45\text{ LBS})(3\text{ ft}) \cos\frac{\pi}{4} = \dfrac{135\sqrt{2}}{2}$ J

PERCEIVED WEIGHT $= \dfrac{45}{\sqrt{2}} \approx$ 31.8 LBS.



e.g.



BIKE RIDERS TAKE NOTE!

$$W = (10\text{ N})(6000\text{ m})\cos 150^\circ$$

$$= -(6000)\cos 30^\circ$$

$$= -3000\sqrt{3}\text{ Joules}$$