§10.3 Dot Product

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3. $\langle -2, \frac{1}{3} \rangle \cdot \langle -5, 12 \rangle = (-2)(-5) + \frac{1}{3} \cdot 12 = \boxed{14}$

7. $(2\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j} + \hat{k}) = \langle 2, 1, 0 \rangle \cdot \langle 1, -1, 1 \rangle = 2 - 1 + 0 = \boxed{1}$

9. $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = (6)(5) \cos \frac{2\pi}{3} = 30 \cdot (-\frac{1}{2}) = \boxed{-15}$

14. $2a + 1.5b + c$

Money made on soft drinks

Money made on hot dogs

Money made on hamburgers

Total money made by street vendor

15. $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\langle 4, 3 \rangle \cdot \langle 2, -1 \rangle = \sqrt{4^2 + 3^2} \sqrt{2^2 + 1^2} \cos\theta$

$8 - 3 = 5\sqrt{5} \cos\theta$

$\cos\theta = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}} \longrightarrow \boxed{\theta = \cos^{-1} \frac{1}{\sqrt{5}}} \approx 63°$

17. $\langle 4, -3, 1 \rangle \cdot \langle 2, 0, -1 \rangle = \sqrt{4^2 + 3^2 + 1^2} \sqrt{2^2 + 0^2 + 1^2} \cos\theta$

$7 = \sqrt{26}\sqrt{5} \cos\theta$

$\cos\theta = \frac{7}{\sqrt{130}} \longrightarrow \boxed{\theta = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right)} \approx 52°$

19. (a) $\vec{a}\cdot\vec{b} = -30 - 24 + 14 = -40 \neq 0 \Rightarrow$ NOT ORTHOGONAL.

NOTE THAT $\vec{a} \parallel \vec{b} \iff \vec{b} = k\vec{a}$ FOR SOME SCALAR K

i.e. $\langle 6, -8, 2 \rangle = K \langle -5, 3, 7 \rangle$

$$\left.\begin{aligned} \Rightarrow\ 6 &= -5K \longrightarrow K = -\tfrac{6}{5} \\ -8 &= 3K \longrightarrow K = -\tfrac{8}{3} \\ 2 &= 7K \longrightarrow K = \tfrac{2}{7} \end{aligned}\right\}$$

IMPOSSIBLE. HENCE $\vec{a}$ IS NOT PARALLEL TO $\vec{b}$ EITHER.

$\therefore$ $\boxed{\text{NEITHER}}$

(b) $\vec{a}\cdot\vec{b} = 4(-3) + 6(2) = 0 \rightarrow \boxed{\text{ORTHOGONAL}}$

(c) $\vec{a}\cdot\vec{b} = \langle -1, 2, 5 \rangle \cdot \langle 3, 4, -1 \rangle = -3 + 8 - 5 = 0 \rightarrow$ ORTHOGONAL

(d) NOTE: $\vec{b} = -\frac{3}{2}\vec{a} \longrightarrow \boxed{\text{PARALLEL}}$

$\left(\langle -3, -9, 6 \rangle = -\frac{3}{2} \langle 2, 6, -4 \rangle \right)$

22. $\langle 2, 1, -1 \rangle \cdot \langle 1, x, 0 \rangle = \sqrt{2^2 + 1^2 + 1^2}\,\sqrt{1^2 + x^2}\,\cos\frac{\pi}{4}$

$2 + x = \sqrt{6}\,\sqrt{1 + x^2} \cdot \frac{\sqrt{2}}{2} = \sqrt{3}\,\sqrt{1 + x^2}$

$$\longrightarrow (2 + x)^2 = 3(1 + x^2)$$

$$4 + 4x + x^2 = 3 + 3x^2$$

$$0 = 2x^2 - 4x - 1$$ USE QUAD. FORMULA.

$x = \frac{4 \pm \sqrt{16 + 8}}{4} = \boxed{1 \pm \frac{\sqrt{6}}{2}}$

23. NOTICE THAT $\langle 1,-1,-1\rangle \cdot \langle 1,1,0\rangle = \langle 1,-1,-1\rangle \cdot \langle 1,0,1\rangle = 0$

SO $\langle 1,-1,-1\rangle$ IS ORTHOGONAL TO BOTH.

TO MAKE IT A UNIT VECTOR, $\longrightarrow \frac{1}{\sqrt{3}}\langle 1,-1,-1\rangle = \boxed{\left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\rangle}$

(NOTE THAT $\frac{1}{\sqrt{3}}\langle -1,1,1\rangle$ IS GOOD TOO.)

25. LINES: $y = 2x - 3 \longrightarrow$ SLOPE $2 \longrightarrow \langle 1,2\rangle$

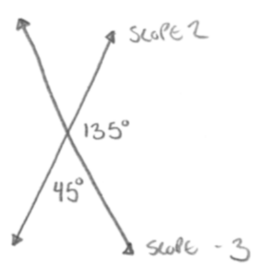
$y = -3x + 7 \longrightarrow$ SLOPE $-3 \longrightarrow \langle 1,-3\rangle$

$$\langle 1,2\rangle \cdot \langle 1,-3\rangle = |\langle 1,2\rangle|\,|\langle 1,-3\rangle| \cos\theta$$

$$-5 = \sqrt{5}\sqrt{10}\cos\theta = 5\sqrt{2}\cos\theta$$

$$-\frac{1}{\sqrt{2}} = \cos\theta \longrightarrow \theta = \frac{3\pi}{4} \text{ or } 135^\circ$$

PICTURE:



SO THE ACUTE ANGLE IS $\boxed{45^\circ}$

28. $y = \sin x$ & $y = \cos x$ INTERSECT WHEN $x = \frac{\pi}{4}$ $\left(\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\right)$

$y' = \cos x$ $\qquad y' = -\sin x$

$y' = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\qquad y' = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

NOTE: 2 VECTORS WITH THOSE SLOPES WOULD BE $\left\langle 1, \frac{\sqrt{2}}{2}\right\rangle$ & $\left\langle 1, -\frac{\sqrt{2}}{2}\right\rangle$

$$\left\langle 1, \frac{\sqrt{2}}{2}\right\rangle \left\langle 1, -\frac{\sqrt{2}}{2}\right\rangle = \left|\left\langle 1, \frac{\sqrt{2}}{2}\right\rangle\right| \left|\left\langle 1, -\frac{\sqrt{2}}{2}\right\rangle\right| \cos\theta$$

$$\frac{1}{2} = \frac{3}{2}\cos\theta \rightarrow \cos\theta = \frac{1}{3} \rightarrow \boxed{\theta = \cos^{-1}\frac{1}{3}} \approx 70.5^\circ$$

29. $\text{comp}_{\vec{a}} \vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \dfrac{\langle -5, 12\rangle \cdot \langle 4, 6\rangle}{\sqrt{\langle -5,12\rangle\cdot\langle -5,12\rangle}} = \dfrac{-20+72}{\sqrt{25+144}} = \dfrac{52}{13} = \boxed{4}$

$\text{proj}_{\vec{a}} \vec{b} = \left(\text{comp}_{\vec{a}} \vec{b}\right) \dfrac{\vec{a}}{|\vec{a}|} = 4 \cdot \dfrac{1}{13} \langle -5, 12\rangle = \boxed{\left\langle -\dfrac{20}{13}, \dfrac{48}{13} \right\rangle}$

31. $\text{comp}_{\vec{a}} \vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \dfrac{\langle 3,6,-2\rangle\cdot\langle 1,2,3\rangle}{\sqrt{3^2+6^2+2^2}} = \dfrac{3+12-6}{\sqrt{49}} = \boxed{\dfrac{9}{7}}$

$\text{proj}_{\vec{a}} \vec{b} = \left(\text{comp}_{\vec{a}} \vec{b}\right) \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{9}{7} \cdot \dfrac{1}{7} \langle 3,6,-2\rangle = \boxed{\left\langle \dfrac{27}{7}, \dfrac{54}{7}, \dfrac{-18}{7} \right\rangle}$

33. $\text{orth}_{\vec{a}} \vec{b} \cdot \vec{a} = (\vec{b} - \text{proj}_{\vec{a}} \vec{b}) \cdot \vec{a} = \vec{b}\cdot\vec{a} - (\text{proj}_{\vec{a}} \vec{b})\cdot\vec{a}$

$$= \vec{b}\cdot\vec{a} - \left(\frac{\vec{a}\cdot\vec{b}}{|a|^2}\vec{a}\right)\cdot\vec{a}$$

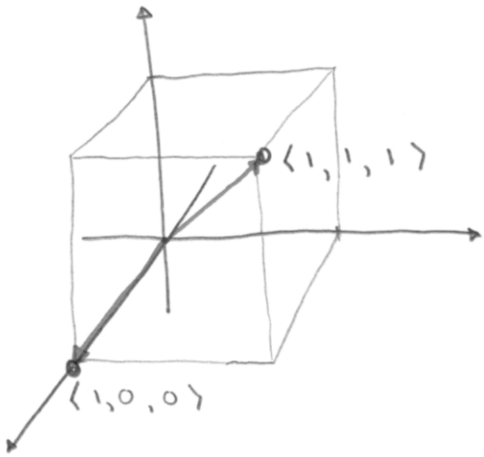
$$= \vec{b}\cdot\vec{a} - \vec{a}\cdot\vec{b}\underbrace{\left(\frac{\vec{a}\cdot\vec{a}}{|a|^2}\right)}_{1} = 0 \quad \checkmark$$

37. Force $\vec{F} = \langle 8, -6, 9\rangle$

Displacement $\vec{D} = \langle 6-0, 12-10, 20-8\rangle = \langle 6, 2, 12\rangle$

Work $W = \vec{F}\cdot\vec{D} = 48 - 12 + 108 = \boxed{144 \text{ Joules}}$

43.

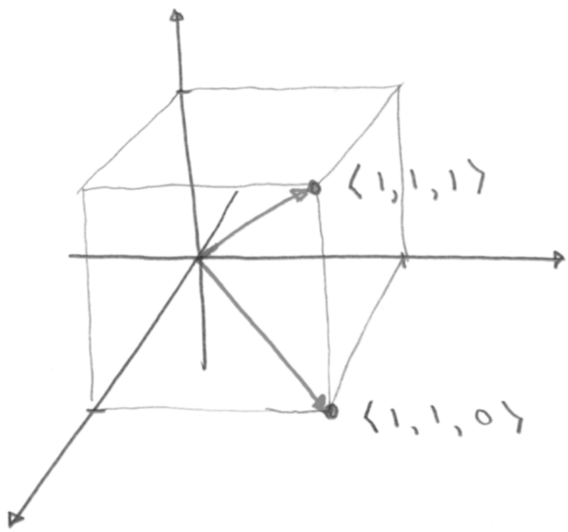


$$\langle 1,0,0\rangle \cdot \langle 1,1,1\rangle = |\langle 1,0,0\rangle|\,|\langle 1,1,1\rangle| \cos\theta$$

$$1 = \sqrt{3} \cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{3}} \longrightarrow \boxed{\theta = \cos^{-1}\frac{1}{\sqrt{3}}} \approx 54.7^\circ$$

44.



$$\langle 1,1,0\rangle \cdot \langle 1,1,1\rangle = |\langle 1,1,0\rangle|\,|\langle 1,1,1\rangle| \cos\theta$$

$$2 = \sqrt{6} \cos\theta$$

$$\cos\theta = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

$$\boxed{\theta = \cos^{-1}\sqrt{\frac{2}{3}}} \approx 35.3^\circ$$