§10.5 EQ'S OF LINES & PLANES

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1. (a) TRUE (b) FALSE (c) TRUE (d) FALSE (e) FALSE
(f) TRUE (g) FALSE (h) TRUE (i) TRUE (j) FALSE (k) TRUE

3. $\vec{r} = \vec{r}_0 + t\vec{v} \longrightarrow$

$$\boxed{\langle x, y, z \rangle = \langle 2, 2.4, 3.5 \rangle + t\langle 3, 2, -1 \rangle}$$

$$\boxed{\langle x, y, z \rangle = \langle 2+3t, 2.4+2t, 3.5-t \rangle}$$

5. $\vec{r} = \vec{r}_0 + t\vec{v}$ WHERE $\vec{r}_0 = \langle 1, 0, 6 \rangle$, $\vec{v} = \vec{n} = \langle 1, 3, 1 \rangle$

$$\boxed{\langle x, y, z \rangle = \langle 1+t, 3t, 6+t \rangle} \boxed{= \langle 1, 0, 6 \rangle + t\langle 1, 3, 1 \rangle}$$

PARAMETRIC VECTOR

7. $\vec{r} = \vec{r}_0 + t\vec{v}$ WHERE $\vec{r}_0 = \langle 0, \frac{1}{2}, 1 \rangle$ & $\vec{v} = \langle 2, 1, -3 \rangle - \langle 0, \frac{1}{2}, 1 \rangle$
$= \langle 2, \frac{1}{2}, -4 \rangle$

$\langle x, y, z \rangle = \langle 2t, \frac{1}{2} + \frac{1}{2}t, 1-4t \rangle$

$$\begin{aligned} x &= 2t \\ y &= \tfrac{1}{2} + \tfrac{1}{2}t \\ z &= 1 - 4t \end{aligned} \longleftrightarrow \frac{x}{2} = 2\left(y - \frac{1}{2}\right) = \frac{1-z}{4}$$

10. $\vec{r} = \vec{r}_0 + t\vec{v}$ WHERE $\vec{r}_0$ IS ON BOTH PLANES - e.g. $\vec{r}_0 = \langle 1, 0, 0 \rangle$
& $\vec{v}$ IS $\perp$ TO BOTH $\langle 1, 2, 3 \rangle$ & $\langle 1, -1, 1 \rangle$

e.g. $\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \langle 5, 2, -3 \rangle$

$\langle x, y, z\rangle = \langle 1+5t, 2t, -3t\rangle$

$$x = 1+5t \qquad \frac{x-1}{5} = \frac{y}{2} = -\frac{z}{3}$$
$$y = 2t \longleftrightarrow$$
$$z = -3t$$

15. $\vec{r} = (1-t)\vec{r}_0 + t\vec{r}_1, \quad 0 \le t \le 1$

$$\boxed{\langle x, y, z\rangle = (1-t)\langle 2, -1, 4\rangle + t\langle 4, 6, 1\rangle}, \quad 0 \le t \le 1$$

$$= \langle 2+2t, -1+7t, 4-3t\rangle, \quad 0 \le t \le 1$$

17. NOT PARALLEL BECAUSE $\langle 2, -1, 3\rangle \times \langle 4, -2, 5\rangle \neq 0$

INTERSECT? $3+2t = 1+4s$ $\boxed{1}$

$1+3t = 4+5s$ $\boxed{2}$

$3\boxed{1} - 2\boxed{2} \rightarrow 7 = -5+2s$

$12 = 2s$

$\underline{\underline{6 = s}} \longrightarrow 3+2t = 1+4(6) = 25$

$2t = 22$

$\underline{\underline{t = 11}}$

POINT OF INTERSECTION? $\langle 1+4(6), 3-2(6), 4+5(6)\rangle$

$= \langle 25, -9, 34\rangle$

CHECK: $\stackrel{?}{=} \langle 3+2(11), 4-11, 1+3(11)\rangle = \langle 25, -7, 34\rangle$

$\Rightarrow$ NO SOLUTION. SKEW.

21. $\left(\langle x, y, z\rangle - \langle -1, \tfrac{1}{2}, 3\rangle\right) \cdot \langle 1, 4, 1\rangle = 0$

$(x+1) + 4\left(y - \tfrac{1}{2}\right) + (z-3) = 0$

$$\boxed{x + 4y + z = 4}$$

23. (SAME $\vec{n}$)

$\left(\langle x, y, z\rangle - \langle 1, -1, -1\rangle\right) \cdot \langle 5, -1, -1\rangle = 0$

$5(x-1) - (y+1) - (z+1) = 0$

$$\boxed{5x - y - z = 7}$$

25. Let $\vec{a} = \langle 0, 1, 1\rangle - \langle 1, 0, 1\rangle = \langle -1, 1, 0\rangle$

$\vec{b} = \langle 0, 1, 1\rangle - \langle 1, 1, 0\rangle = \langle -1, 0, 1\rangle$

$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \langle 1, 1, 1\rangle$$

$\Rightarrow \left(\langle x, y, z\rangle - \langle 0, 1, 1\rangle\right) \cdot \langle 1, 1, 1\rangle = 0$

$(x-0) + (y-1) + (z-1) = 0$

$$\boxed{x + y + z = 2}$$

27. WHEN $t=0$, WE GET ANOTHER POINT IN PLANE: $\langle 4,3,7\rangle$

LET $\vec{a} = \langle 4,3,7\rangle - \langle 6,0,-2\rangle = \langle -2,3,9\rangle$ ← VECTOR IN PLANE

$\vec{b} = \langle -2,5,4\rangle$

BECAUSE LINE: $\vec{r} = \vec{r}_0 + t\langle -2,5,4\rangle$ ← VECTOR IN PLANE

LET $\vec{n} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 9 \\ -2 & 5 & 4 \end{vmatrix} = \langle -33,-10,-4\rangle$

$\therefore \left(\langle x,y,z\rangle - \langle 6,0,-2\rangle\right)\cdot\langle -33,-10,-4\rangle = 0$

$-33(x-6) - 10y - 4(z+2) = 0$

$-33x - 10y - 4z = -190$ OR $\boxed{33x + 10y + 4z = 190}$

29. NOTE THAT $\langle 1,1,0\rangle$ LIES ON BOTH PLANES.

THE DIRECTION OF THE LINE OF INTERSECTION MUST BE $\perp$ TO BOTH NORMAL VECTORS.

e.g. $\vec{v} = \langle 1,1,-1\rangle \times \langle 2,-1,3\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix}$

$= \langle 2,-5,-3\rangle$

$\therefore \langle x,y,z\rangle = \langle 1,1,0\rangle + t\langle 2,-5,-3\rangle$

THIS IS OVERKILL ... I JUST NEED 2 PNTS ON THE LINE

$t=0 \rightarrow \langle 1,1,0\rangle$

$t=1 \rightarrow \langle 3,-4,-3\rangle$

2 VECTORS IN THE PLANE: $\vec{a} = \langle 1,1,0\rangle - \langle -1,2,1\rangle = \langle 2,-1,-1\rangle$

$\vec{b} = \langle 3,-4,-3\rangle - \langle -1,2,1\rangle = \langle 4,-6,-4\rangle$

$\Rightarrow \vec{n}$ TO PLANE $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -1 \\ 4 & -6 & -4 \end{vmatrix} = \langle -2, 4, -8\rangle$

$\therefore$ EQ OF PLANE: $\left(\langle x,y,z\rangle - \langle -1,2,1\rangle\right)\cdot\langle -2,4,-8\rangle = 0$

$-2(x+1) + 4(y-2) - 8(z-1) = 0$

$-2x + 4y - 8z = 2$ or $\boxed{x - 2y + 4z = -1}$

31. IF P IS THE PLANE $\perp$ TO $2x + y - 2z = 2$ & $x + 3z = 4$, THEN THE NORMAL VECTOR $\vec{n}$ TO P MUST BE $\perp$ TO $\langle 2,1,-2\rangle$ & $\langle 1,0,3\rangle$.

i.e. $\vec{n} = \langle 2,1,-2\rangle \times \langle 1,0,3\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$= \langle 3, -8, -1\rangle$

EQ OF P: $\left(\langle x,y,z\rangle - \langle 1,5,1\rangle\right)\cdot\langle 3,-8,-1\rangle = 0$

$3(x-1) - 8(y-5) - (z-1) = 0$

$\boxed{3x - 8y - z = -38}$

33. SUBSTITUTE: $(3-t)-(2+t)+2(5t)=9$

$8t+1=9$

$t=1$

POINT $=(3-1,\ 2+1,\ 5(1))=\boxed{(2,3,5)}$

35. SINCE THE NORMAL VECTORS $\langle 1,1,1\rangle$ & $\langle 1,-1,1\rangle$ ARE NOT $\parallel$, THE PLANES ARE NOT $\parallel$. ANGLE BETWEEN PLANES = ANGLE BETWEEN NORMAL VECTORS:

$$\langle 1,1,1\rangle\cdot\langle 1,-1,1\rangle = |\langle 1,1,1\rangle||\langle 1,-1,1\rangle|\cos\theta$$

$$1 = 3\cos\theta \rightarrow \boxed{\theta=\cos^{-1}\left(\tfrac{1}{3}\right)}\quad (\neq 90^\circ \text{ so not } \perp.)$$

39. (a) NOTE: POINT $\underbrace{(1,0,0)}_{\vec{r}_0}$ IS ON BOTH PLANES.

LINE: $\vec{r}=\vec{r}_0+t\vec{v}$ WHERE $\vec{v}=\underbrace{\langle 1,1,1\rangle\times\langle 1,2,2\rangle}_{\text{NORMAL VECTORS OF THE 2 PLANES}}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 2\end{vmatrix}=\langle 0,-1,1\rangle$$

LINE: $\boxed{\langle x,y,z\rangle=\langle 1,-t,t\rangle}$

(b) ANGLE BETWEEN NORMAL VECTORS: $\langle 1,1,1\rangle\cdot\langle 1,2,2\rangle=|\langle 1,1,1\rangle||\langle 1,2,2\rangle|\cos\theta$

$$\rightarrow 5=3\sqrt{3}\cos\theta \rightarrow \boxed{\theta=\cos^{-1}\left(\frac{5}{3\sqrt{3}}\right)}$$

41. 3 POINTS ON PLANE: $A(a,0,0)$, $B(0,b,0)$, $C(0,0,c)$

2 VECTORS IN PLANE: $\overrightarrow{AB}\ \langle -a, b, 0\rangle$, $\overrightarrow{AC}\ \langle -a, 0, c\rangle$

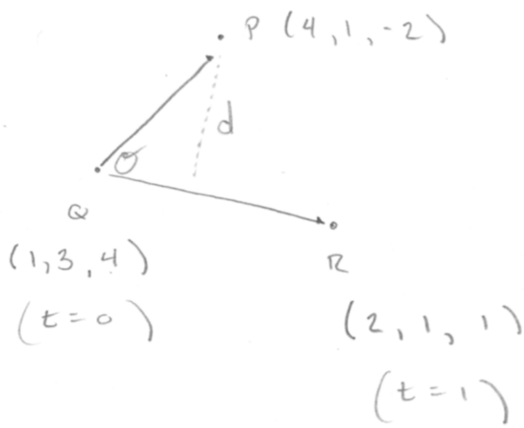
$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = \langle bc, ac, ab \rangle$$

$\therefore$ PLANE: $$\big(\langle x, y, z\rangle - \langle a, 0, 0\rangle\big)\cdot \langle bc, ac, ab\rangle = 0$$

$$bc(x-a) + acy + abz = 0$$

$$\boxed{bcx + acy + abz = abc}$$

47.



$$d = \frac{|\overrightarrow{QP} \times \overrightarrow{QR}|}{|\overrightarrow{QR}|} = \frac{\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -6 \\ 1 & -2 & -3 \end{matrix} \right\|}{|\langle 1, -2, -3\rangle|}$$

$$= \frac{\sqrt{(-6)^2 + (3)^2 + (-4)^2}}{\sqrt{(1)^2 + (-2)^2 + (-3)^2}}$$

$$= \frac{\sqrt{61}}{\sqrt{14}} = \boxed{\sqrt{\frac{61}{14}}}$$

49.

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

WHERE POINT = (x_1, y_1, z_1)

NORMAL VECTOR TO PLANE

$\vec{n} = \langle a, b, c \rangle$

EQ OF PLANE: $ax + by + cz + d = 0$

$$D = \frac{|3(1) + 2(-2) + 6(4) + 5|}{\sqrt{3^2+2^2+6^2}} = \frac{28}{\sqrt{49}} = \boxed{4}$$

51. NOTE: $(0,0,4)$ IS IN PLANE $2x - 3y + z - 4 = 0$

$$D = \frac{|4(0) - 6(0) + 2(4) - 3|}{\sqrt{4^2+6^2+2^2}} = \frac{5}{\sqrt{56}} = \boxed{\frac{5}{2\sqrt{14}}}$$