§ 11.4 TANGENT PLANES & LINEAR APPROXIMATIONS

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3. $z = 1 + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

WHERE $f_x = \dfrac{y}{2\sqrt{xy}} = \dfrac{1}{2}\sqrt{\dfrac{y}{x}}$ & $f_y = \dfrac{x}{2\sqrt{xy}} = \dfrac{1}{2}\sqrt{\dfrac{x}{y}}$

$f_x(1,1) = \dfrac{1}{2}$ $\qquad$ $f_y(1,1) = \dfrac{1}{2}$

$$\therefore \boxed{z = 1 + \tfrac{1}{2}(x-1) + \tfrac{1}{2}(y-1) \quad \text{OR} \quad z = \tfrac{1}{2}(x+y)}$$

4. $f_x = e^{xy} + xye^{xy}$ | $f_y = x^2e^{xy}$

$f_x(2,0) = 1$ | $f_y(2,0) = 4$

$$\boxed{z = 2 + 1(x-2) + 4(y-0) \quad \text{OR} \quad z = x + 4y}$$

5. $f_x = \sin(x+y) + x\cos(x+y)$ | $f_y = x\cos(x+y)$

$f_x(-1,1) = -1$ | $f_y(-1,1) = -1$

$$\boxed{z = 0 - 1(x+1) - 1(y-1) \quad \text{OR} \quad z = -x - y}$$

6. $f_x = \dfrac{1}{x - 2y}$ | $f_y = \dfrac{-2}{x - 2y}$

$f_x(3,1) = 1$ | $f_y(3,1) = -2$

$$\boxed{\begin{array}{l} z = 0 + 1(x-3) - 2(y-1) \\ \text{or} \quad z = x - 2y - 1 \end{array}}$$

11. $f(x,y) = 1 + x\ln(xy-5)$, $(2,3)$

$f_x = \ln(xy-5) + \dfrac{xy}{xy-5}$

CONTINUOUS ON ITS DOMAIN &

$(2,3)$ IS IN ITS DOMAIN

$\Rightarrow$ CONTINUOUS AT $(2,3)$

$f_y = \dfrac{x^2}{xy-5}$ DITTO

$\therefore$ f IS DIFFERENTIABLE AT $(2,3)$

$f_x(2,3) = 6$, $f_y(2,3) = 4$, $f(2,3) = 1$

$$\therefore \boxed{\begin{array}{l} z = 1 + 6(x-2) + 4(y-3) \\ \text{or} \quad z = 6x + 4y - 23 \end{array}}$$

17. $f(x,y) \approx f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5)$

$$f(2.2, 4.9) \approx 6 + 1(2.2-2) - 1(4.9-5)$$

$$\approx 6 + .2 + .1$$

$$\approx \boxed{6.3}$$

19. $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$f_x = \frac{x}{\sqrt{x^2+y^2+z^2}}, \quad f_y = \frac{y}{\sqrt{x^2+y^2+z^2}}, \quad f_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$f_x(3,2,6) = \frac{3}{7}, \quad f_y(3,2,6) = \frac{2}{7}, \quad f_z(3,2,6) = \frac{6}{7}$$

$$f(3,2,6) = 7$$

$$\therefore\ f(x,y,z) \approx 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$$

$$f(3.02, 1.97, 5.99) \approx 7 + \frac{3}{7}(.02) + \frac{2}{7}(-.03) + \frac{6}{7}(-.01)$$

$$\approx 7 + \frac{.06 - .06 - .06}{7}$$

$$\approx 7 - \frac{.06}{7} = \frac{4900 - 6}{700} = \boxed{\frac{4894}{700}}$$

20. $du = f_x\,dx + f_y\,dy$

$$f_x = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+3y^2}}, \quad f_y = \frac{3\cancel{6}y}{\cancel{2}\sqrt{x^2+3y^2}}$$

$$\boxed{du = \frac{x}{\sqrt{x^2+3y^2}}\,dx + \frac{3y}{\sqrt{x^2+3y^2}}\,dy}$$

22. $dT = f_u\,du + f_v\,dv + f_w\,dw$

$$f_u = \frac{-v^2 w}{(1+uvw)^2}, \quad f_v = \frac{(1+uvw) - uvw}{(1+uvw)^2} = \frac{1}{(1+uvw)^2},$$

$$f_w = \frac{-uv^2}{(1+uvw)^2}$$

$$\boxed{dT = \frac{-v^2 w}{(1+uvw)^2}\,du + \frac{1}{(1+uvw)^2}\,dv - \frac{uv^2}{(1+uvw)^2}\,dw}$$

24. $f_x = ze^{-y^2-z^2}, \quad f_y = -2xyz\,e^{-y^2-z^2}, \quad f_z = xe^{-y^2-z^2} - 2xz^2 e^{-y^2-z^2}$

$$\boxed{dL = ze^{-y^2-z^2}\,dx - 2xyze^{-y^2-z^2}\,dy + (1-2z^2)\,xe^{-y^2-z^2}\,dz}$$

25. $dz = 10x\,dx + 2y\,dy$

$= 10(1)(1.05-1) + 2(2)(2.1-2)$

$= .5 + .4 = \boxed{.9}$

NOTE: $dx = \Delta x$, $dy = \Delta y$ (NOT IN GENERAL, BUT FOR NOW... SURE)

$\Delta z = f(1.05, 2.1) - f(1,2)$

$= 5(1.05)^2 + (2.1)^2 - 5(1)^2 - (2)^2$

$= 5.5125 + 4.41 - 5 - 4 = \boxed{.9225}$

27. AREA $A(\ell, w) = \ell w$

$A_\ell = w, \quad A_w = \ell \quad \Rightarrow \quad dA = w\,d\ell + \ell\,dw$

$\ell = 30, \quad w = 24 \quad \Rightarrow \quad dA = 24\,d\ell + 30\,dw$

$dA \le 24(.1) + 30(.1) = 2.4 + 3 = \boxed{5.4 \text{ cm}^2}$

29. VOLUME $V = \pi r^2 h$

$V_r = 2\pi r h, \quad V_h = \pi r^2 \quad \Rightarrow \quad dV = 2\pi r h\,dr + \pi r^2\,dh$

$r = 4, \quad h = 12, \quad dr = dh = .04$

$dV = 2\pi(4)(12)(.04) + \pi(4)^2(.04)$

$= 3.84\pi + .64\pi = \boxed{4.48\pi \text{ cm}^3}$

$\approx 14.07 \text{ cm}^3$

30. $PV = 8.31T \longrightarrow P = 8.31 \dfrac{T}{V}$

$$P_V = -8.31 \frac{T}{V^2} \quad , \quad P_T = 8.31 \frac{1}{V}$$

$$dP = -8.31 \frac{T}{V^2}\, dV + 8.31 \frac{1}{V}\, dT$$

$$dP = -8.31 \frac{310}{12^2} \cdot 0.3 + 8.31 \frac{1}{12} (-5)$$

$$= -5.366875 - 3.4625 = \boxed{-8.829375 \text{ KILOPASCALS}}$$

33. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$$\frac{\partial R}{\partial R_i} : \quad \frac{-1}{R^2} \frac{\partial R}{\partial R_i} = \frac{-1}{R_i^2} \quad \Rightarrow \quad \frac{\partial R}{\partial R_i} = \frac{R^2}{R_i^2}$$

$$\boxed{dR = \underbrace{\sum_{i=1}^{3} \frac{\partial R}{\partial R_i}\, dR_i}_{\text{I'M SO FANCY!}} = \frac{R^2}{R_1^2}\, dR_1 + \frac{R^2}{R_2^2}\, dR_2 + \frac{R^2}{R_3^2}\, dR_3}$$

34. THE TANGENT VECTORS TO EACH CURVE AT $(2,1,3)$ LIE IN THE TANGENT PLANE. THE CROSS PRODUCT OF THESE TANGENT VECTORS WILL GIVE A NORMAL VECTOR TO THE PLANE.

BY INSPECTION, WE HAVE: $\vec{r}_1(0) = (2,1,3)$ AND

$$\vec{r}_2(1) = (2,1,3)$$

AND $\vec{r}_1\,'(t) = \langle 3, -2t, -4+2t \rangle$

$*\quad \vec{r}_1\,'(0) = \langle 3, 0, -4 \rangle$

$$\vec{r}_2\,'(u) = \langle 2u, 6u^2, 2 \rangle$$

$*\quad \vec{r}_2\,'(1) = \langle 2, 6, 2 \rangle$

NORMAL VECTOR: $\vec{r}_1\,'(0) \times \vec{r}_2\,'(1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -4 \\ 2 & 6 & 2 \end{vmatrix} = \langle 24, -14, 18 \rangle$

EQ OF PLANE: $\langle 24, -14, 18 \rangle \cdot \langle x-2, y-1, z-3 \rangle = 0$

$$24x - 48 - 14y + 14 + 18z - 54 = 0$$

OR $\quad 24x - 14y + 18z = 88$

OR $\quad \boxed{12x - 7y + 9z = 44}$