JOHN ADAMSKI
10/23/2014

1. $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{dx}{dt} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dt}$

$$\boxed{\frac{dz}{dt} = (2x+y)\cos(t) + (2y+x)e^t}$$

3. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x} \cdot \dfrac{dx}{dt} + \dfrac{\partial w}{\partial y} \cdot \dfrac{dy}{dt} + \dfrac{\partial w}{\partial z} \cdot \dfrac{dz}{dt}$

$$\boxed{\frac{dw}{dt} = \left(e^{y/z}\right)(2t) + \left(\frac{x}{z} e^{y/z}\right)(-1) + \left(-\frac{xy}{z^2} e^{y/z}\right)(2)}$$

5. $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial s} = \boxed{\left(2xy^3\right)\cos(t) + \left(3x^2y^2\right)\sin(t)}$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial t} = \boxed{\left(2xy^3\right)\left(-s\sin(t)\right) + \left(3x^2y^2\right)\left(s\cos(t)\right)}$

7. $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial r} \cdot \dfrac{\partial r}{\partial s} + \dfrac{\partial z}{\partial \theta} \cdot \dfrac{\partial \theta}{\partial s} = \boxed{\left(e^r\cos\theta\right)t - \left(e^r\sin\theta\right)\dfrac{s}{\sqrt{s^2+t^2}}}$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial r} \cdot \dfrac{\partial r}{\partial t} + \dfrac{\partial z}{\partial \theta} \cdot \dfrac{\partial \theta}{\partial s} = \boxed{\left(e^r\cos\theta\right)s - \left(e^r\sin\theta\right)\dfrac{t}{\sqrt{s^2+t^2}}}$

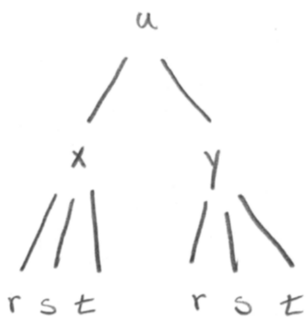
9. NOTE: WHEN $t = 3$, $x = g(3) = 2$, $y = h(3) = 7$

$$\left.\frac{dz}{dt}\right|_{t=3} = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=7}} \cdot \left.\frac{dg}{dt}\right|_{t=3} + \left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=7}} \cdot \left.\frac{dh}{dt}\right|_{t=3}$$

$$= f_x(2,7)\, g'(3) + f_y(2,7)\, h'3)$$

$$= (6)(5) + (-8)(7) = 30 - 56 = \boxed{-26}$$

13.

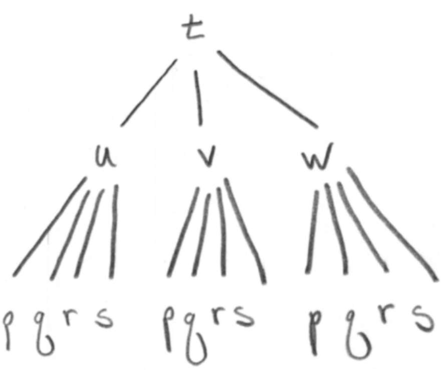


or

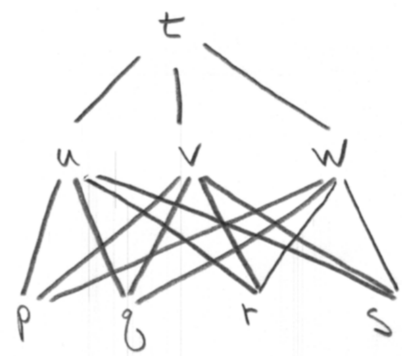
$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r}, \quad \frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

16.



or



$$\frac{\partial t}{\partial p} = \frac{\partial t}{\partial u}\cdot\frac{\partial u}{\partial p} + \frac{\partial t}{\partial v}\cdot\frac{\partial v}{\partial p} + \frac{\partial t}{\partial w}\cdot\frac{\partial w}{\partial p}, \quad \frac{\partial t}{\partial q} = \frac{\partial t}{\partial u}\cdot\frac{\partial u}{\partial q} + \frac{\partial t}{\partial v}\cdot\frac{\partial v}{\partial q} + \frac{\partial t}{\partial w}\cdot\frac{\partial w}{\partial q}$$

$$\frac{\partial t}{\partial r} = \frac{\partial t}{\partial u}\cdot\frac{\partial u}{\partial r} + \frac{\partial t}{\partial v}\cdot\frac{\partial v}{\partial r} + \frac{\partial t}{\partial w}\cdot\frac{\partial w}{\partial r}, \quad \frac{\partial t}{\partial s} = \frac{\partial t}{\partial u}\cdot\frac{\partial u}{\partial s} + \frac{\partial t}{\partial v}\cdot\frac{\partial v}{\partial s} + \frac{\partial t}{\partial w}\cdot\frac{\partial w}{\partial s}$$

17. WHEN $s=4$, $t=2$, AND $u=1$ WE HAVE $x=7$, $y=8$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (4x^3+2xy)(1) + (x^2)(tu^2)$$

$$\rightarrow (4\cdot7^3 + 2\cdot7\cdot8) + 7^2(2\cdot1^2) =$$

$$= 1372 + 112 + 98 = \boxed{1582}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (4x^3+2xy)(2) + (x^2)(su^2)$$

$$\rightarrow (1484)(2) + (49)(4) = \boxed{3164}$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = (4x^3+2xy)(-1) + (x^2)(2stu)$$

$$= (1484)(-1) + (49)(16) = \boxed{-700}$$

19. $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$

WHEN $r=2$, $\theta = \frac{\pi}{2}$ WE HAVE $x=0$, $y=2$, $z=\pi$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} = (y+z)\cos\theta + (x+z)\sin\theta + (y+x)\theta$$

$$\rightarrow (2+\pi)\underbrace{\cos\frac{\pi}{2}}_{0} + (0+\pi)\underbrace{\sin\frac{\pi}{2}}_{1} + (2+0)\frac{\pi}{2} = \boxed{2\pi}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \theta} = (y+z)(-r\sin\theta) + (x+z)r\cos\theta + (y+x)r$$

$$\rightarrow (2+\pi)(-2)\underbrace{\sin\frac{\pi}{2}}_{1} + (0+\pi)(2)\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + (2+0)(2)$$

$$= -4 - 2\pi + 4 = \boxed{-2\pi}$$

21. $N = \dfrac{p+q}{p+r}$, $p = u + vw$, $q = v + uw$, $r = w + uv$

WHEN $u = 2$, $v = 3$, $w = 4$ WE HAVE

$p = 2 + 3 \cdot 4 = 14$

$q = 3 + 2 \cdot 4 = 11$

$r = 4 + 2 \cdot 3 = 10$

$$\frac{\partial N}{\partial u} = \frac{\partial N}{\partial p}\frac{\partial p}{\partial u} + \frac{\partial N}{\partial q}\frac{\partial q}{\partial u} + \frac{\partial N}{\partial r}\frac{\partial r}{\partial u}$$

$$= \frac{r-q}{(p+r)^2} + \frac{1}{p+r} w - \frac{p+q}{(p+r)^2} v = \frac{-1}{576} + \frac{4}{24} - \frac{75}{576}$$

$$= \frac{-1 + 96 - 75}{576} = \frac{20}{576} = \boxed{\frac{5}{144}}$$

$$\frac{\partial N}{\partial v} = \frac{\partial N}{\partial p}\frac{\partial p}{\partial v} + \frac{\partial N}{\partial q}\frac{\partial q}{\partial v} + \frac{\partial N}{\partial r}\frac{\partial r}{\partial v}$$

$$= \frac{r-q}{(p+r)^2} w + \frac{1}{p+r} - \frac{p+q}{(p+r)^2} u = \frac{-4}{576} + \frac{1}{24} - \frac{50}{576}$$

$$= \frac{-4 + 24 - 50}{576} = \frac{-30}{576} = \boxed{\frac{-5}{96}}$$

$$\frac{\partial N}{\partial w} = \frac{\partial N}{\partial p}\frac{\partial p}{\partial w} + \frac{\partial N}{\partial q}\frac{\partial q}{\partial w} + \frac{\partial N}{\partial r}\frac{\partial r}{\partial w}$$

$$= \frac{r-q}{(p+r)^2} v + \frac{1}{p+r} u - \frac{p+q}{(p+r)^2} = \frac{-3}{576} + \frac{2}{24} - \frac{25}{576}$$

$$= \frac{-3 + 48 - 25}{576} = \frac{20}{576} = \boxed{\frac{5}{144}}$$

23. $F(x,y) = \tan^{-1}(x^2y) - x - xy^2 = 0$

$$F_x = \frac{1}{1+(x^2y)^2} \cdot 2xy - 1 - y^2$$

$$F_y = \frac{1}{1+(x^2y)^2} \cdot x^2 - 2xy$$

$$\therefore \frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{\dfrac{2xy}{1+x^4y^2} - 1 - y^2}{\dfrac{x^2}{1+x^4y^2} - 2xy}$$

$$= \boxed{-\frac{2xy - (1+y^2)(1+x^4y^2)}{x^2 - 2xy(1+x^4y^2)}}$$

25. $F(x,y,z) = x^2 + 2y^2 + 3z^2 - 1 = 0$

$F_x = 2x$, $F_y = 4y$, $F_z = 6z$

$$\therefore \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-2x}{6z} = \boxed{-\frac{x}{3z}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{-4y}{6z} = \boxed{\frac{-2y}{3z}}$$

27. $F(x,y,z) = e^z - xyz = 0$

$F_x = -yz \quad , \quad F_y = -xz \quad , \quad F_z = e^z - xy$

$$\therefore \quad \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \boxed{\frac{yz}{e^z - xy}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \boxed{\frac{xz}{e^z - xy}}$$

31. $C = C(T, D) \quad , \quad T = T(t) \quad , \quad D = D(t)$

OR

$$\frac{dC}{dt} = \frac{\partial C}{\partial T} \cdot \frac{dT}{dt} + \frac{\partial C}{\partial D} \cdot \frac{dD}{dt}$$

THIS IS MY ESTIMATE BASED ON THE GRAPH

$$\Rightarrow \left.\frac{dC}{dt}\right|_{t=20} = (4.6 - 0.11\,T + 0.00087\,T^2)(-0.1) + 0.016(0.5)$$

NOTE THAT WHEN $t = 20$, $T \approx 12.7$ (ANOTHER ESTIMATE BASED ON THE GRAPH)

$$= (4.6 - 0.11(12.7) + 0.00087(12.7)^2)(-0.1) + 0.016(0.5)$$

$$\approx \boxed{-0.3263 \ \frac{m/s}{MIN}}$$

"METERS PER SECOND PER MINUTE"

36. $V_o = 34 \text{ m/s}$, $\dfrac{dV_o}{dt} = 1.2 \text{ m/s}^2$

$V_s = 40 \text{ m/s}$, $\dfrac{dV_s}{dt} = 1.4 \text{ m/s}^2$

$f_s = 460 \text{ Hz}$

THE PERCEIVED FREQUENCY IS SIMPLY

$$f_o = \left(\frac{c + V_o}{c - V_s}\right) f_s = \left(\frac{332 + 34}{332 - 40}\right)(460) = \frac{366 \cdot 460}{328}$$

$$\approx \boxed{513 \text{ Hz}}$$

HOW FAST IS IT CHANGING?

$$\frac{df_o}{dt} = \frac{\partial f_o}{\partial V_o} \cdot \frac{dV_o}{dt} + \frac{\partial f_o}{\partial V_s} \cdot \frac{dV_s}{dt}$$

$$= \frac{f_s}{c - V_s} \cdot 1.2 + \frac{c + V_o}{(c - V_s)^2} \cdot 1.4$$

$$= \left(\frac{460}{332 - 40}\right)(1.2) + \left(\frac{332 + 34}{(332 - 40)^2}\right)(1.4)$$

$$\approx \boxed{1.896 \text{ Hz/s}}$$

37. $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$

(a) $$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = f_x \cos\theta + f_y \sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = -f_x r\sin\theta + f_y r\cos\theta$$

(b) $$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(f_x\cos\theta + f_y\sin\theta\right)^2 + \frac{\left(-f_x r\sin\theta + f_y r\cos\theta\right)^2}{r^2}$$

$$= f_x^2\cos^2\theta + 2f_xf_y\cos\theta\sin\theta + f_y^2\sin^2\theta$$
$$+ f_x^2\sin^2\theta - 2f_xf_y\cos\theta\sin\theta + f_y^2\cos^2\theta$$

$$= f_x^2 + f_y^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

39. $z = f(u)$, $u = x - y$

```
  z
  |
  u
 / \
x   y
```

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{dz}{du}\cdot\frac{\partial u}{\partial x}, \quad \frac{\partial z}{\partial y} = \frac{dz}{du}\cdot\frac{\partial u}{\partial y}$$

$$\therefore \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{dz}{du}(1) + \frac{dz}{du}(-1) = 0$$