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15. $$\int_{-3}^{3}\int_{0}^{\pi/2} y + y^2\cos x \, dx\, dy = \int_{-3}^{3} \left[xy + y^2 \sin x \right]_{x=0}^{x=\frac{\pi}{2}} dy$$

$$= \int_{-3}^{3} \frac{\pi}{2} y + y^2 \, dy = \left[\frac{\pi}{4} y^2 + \frac{1}{3} y^3 \right]_{-3}^{3} = \boxed{18}$$

19. $$\int_0^1\int_0^1 v(u+v^2)^4 \, du\, dv = \int_0^1 \frac{v}{5}(u+v^2)^5 \Big|_{u=0}^{u=1} dv$$

$$= \int_0^1 \frac{v}{5}\left[(1+v^2)^5 - v^{10} \right] dv = \frac{1}{5}\int_0^1 v(1+v^2)^5 - v^{11} \, dv$$

$$= \frac{1}{5}\left[\frac{1}{2}\cdot\frac{1}{6}(1+v^2)^6 - \frac{1}{12}v^{12} \right]_0^1 = \frac{1}{60}\left[(2^6 - 1) - (1-0) \right]$$

$$= \boxed{\frac{31}{30}}$$

LET $u = x^2 + 1$
$du = 2x\,dx$

21. $$\int_{-3}^{3} y^2 \int_0^1 \frac{x}{x^2+1} \, dx\, dy = \frac{1}{2}\int_{-3}^{3} y^2 \int_1^2 \frac{1}{u}\, du\, dy$$

$$= \frac{\ln(2)}{2}\int_{-3}^{3} y^2\, dy = \ln(2)\int_0^3 y^2\, dy = \frac{\ln 2}{3} y^3 \Big|_0^3 = \boxed{9 \ln 2}$$

25. $$\int_0^3\int_0^2 y e^{-xy}\,dx\,dy = \int_0^3 -e^{-xy}\Big|_0^2\,dy$$

$$= \int_0^3 1 - e^{-2y}\,dy = y + \tfrac{1}{2}e^{-2y}\Big|_0^3 = 3 + \tfrac{1}{2}(e^{-6} - 1)$$

$$= \boxed{\frac{5 + e^{-6}}{2}}$$

29. $$z = 2x + 3y + \frac{15}{2}$$

$$V = \int_{-1}^{1}\int_{-1}^{2} 2x + 3y + \frac{15}{2}\,dx\,dy = \int_{-1}^{1} x^2 + 3xy + \frac{15}{2}x\Big|_{-1}^{2}\,dy$$

$$= \int_{-1}^{1} 3 + 9y + \frac{45}{2}\,dy = 6 + 45 = \boxed{51}$$

32. $$\int_0^{\pi}\int_{-1}^{1} 1 + e^x \sin y\,dx\,dy = \int_0^{\pi}\Big[x + e^x \sin y\Big]_{x=-1}^{x=1}\,dy$$

$$= \int_0^{\pi} 2 + \left(e - \tfrac{1}{e}\right)\sin y\,dy = 2y + \left(\tfrac{1}{e} - e\right)\cos y\Big|_0^{\pi}$$

$$= \boxed{2\pi + 2\left(e - \tfrac{1}{e}\right)}$$

33. $$\int_0^2 \int_0^{\pi/4} x \sec^2 y \, dy \, dx = \int_0^2 x \tan y \Big|_{y=0}^{y=\frac{\pi}{4}} dx$$

$$= \int_0^2 x \, dx = \frac{1}{2} x^2 \Big|_0^2 = \boxed{2}$$

39. $A(R) = 2 \times 5 = 10$

$$f_{AVE} = \frac{1}{10} \int_{-1}^{1} \int_0^5 x^2 y \, dy \, dx = \frac{1}{20} \int_{-1}^{1} x^2 y^2 \Big|_{y=0}^{y=5} dx$$

$$= \frac{25}{20} \int_{-1}^{1} x^2 \, dy = \frac{25}{60} x^3 \Big|_{-1}^{1} = \frac{50}{60} = \boxed{\frac{5}{6}}$$

43. NOTE THAT FOR EACH (a,b) IN DOMAIN, $(-a,b)$ IS ALSO IN DOMAIN

& $f(a,b) = -f(-a,b)$, WHERE $f(x,y) = \dfrac{xy}{1+x^4}$

THAT IS $$\int_{-1}^{1} \int_0^1 \frac{xy}{1+x^4} \, dy \, dx = \underbrace{\int_{-1}^{0} \int_0^1 \frac{xy}{1+x^4} \, dy \, dx}_{} + \underbrace{\int_0^1 \int_0^1 \frac{xy}{1+x^4} \, dy \, dx}_{}$$

EXACT OPPOSITES

$$= \boxed{0}.$$