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§12.2 DOUBLE INTEGRALS OVER GENERAL DOMAINS

5. $$\int_0^1\int_0^{s^2} \cos(s^3)\,dt\,ds = \int_0^1 \left[t\cos(s^3)\right]_{t=0}^{t=s^2} ds$$

$$= \int_0^1 s^2\cos(s^3)\,ds$$ LET $u = s^3$, $du = 3s^2\,ds \longrightarrow \frac{1}{3}du = s^2\,ds$

$$= \frac{1}{3}\int_0^1 \cos(u)\,du = \frac{1}{3}\sin(u)\Big|_{u=0}^{u=1} = \boxed{\frac{1}{3}\sin(1)}$$

7. $$\int_{-1}^1\int_{-y-2}^{y} y^2\,dx\,dy = \int_{-1}^1 \underbrace{\left[xy^2\right]_{x=-y-2}^{x=y}}_{y^3+(y+2)y^2 = 2y^3+2y^2} dy$$

$$= \int_{-1}^1 2y^3+2y^2\,dy = 2\left[\frac{1}{4}y^4+\frac{1}{3}y^3\right]_{-1}^{1} = \boxed{\frac{4}{3}}$$

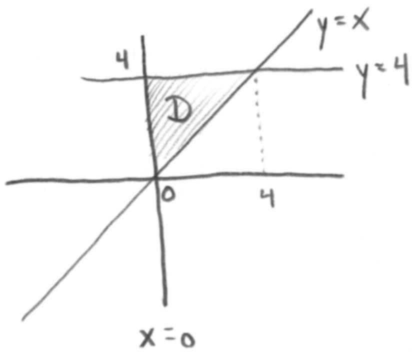
9. $$\int_0^\pi\int_0^{\sin x} x\,dy\,dx = \int_0^\pi xy\Big|_{y=0}^{y=\sin x} dx = \int_0^\pi x\sin x\,dx$$

$u = x$ $\quad v = -\cos x$
$du = dx$ $\quad dv = \sin x\,dx$

$$= -x\cos x\Big|_0^\pi + \int_0^\pi \cos x\,dx$$

$$= \boxed{\pi}$$

14.



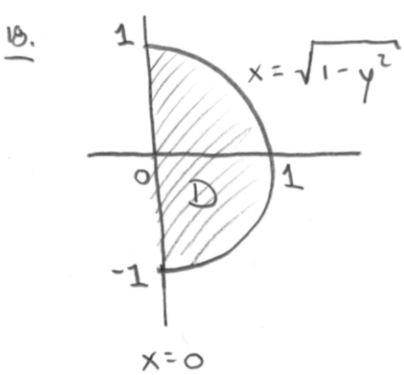
TYPE I: $\displaystyle\int_0^4 \int_x^4 y^2 e^{xy}\, dy\, dX$ ← THIS INNER INTEGRAL WOULD REQUIRE MULTIPLE APPLICATIONS OF INTEGRATION BY PARTS.

TYPE II: $\displaystyle\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy$ ← EASIER

$$= \int_0^4 y e^{xy} \Big|_{x=0}^{x=y} dy = \int_0^4 y e^{y^2} - y \, dy = \frac{1}{2} e^{y^2} - \frac{1}{2} y^2 \Big|_0^4$$

$$= \frac{1}{2} e^{16} - 8 - \frac{1}{2} = \boxed{\frac{1}{2}\left(e^{16} - 17\right)}$$

18.



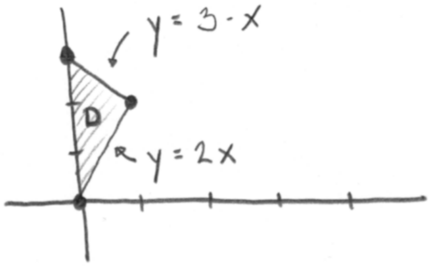
TYPE II $\displaystyle\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} x y^2 \, dx\, dy$

$$= \int_{-1}^{1} \frac{1}{2} x^2 y^2 \Big|_{x=0}^{x=\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{1} \frac{1}{2}(1-y^2) y^2 \, dy = \frac{1}{2} \int_{-1}^{1} y^2 - y^4 \, dy = \frac{1}{2}\left[\frac{1}{3} y^3 - \frac{1}{5} y^5\right]_{-1}^{1}$$

$$= \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$$

20.



$$\int_0^1 \int_{2x}^{3-x} 2xy \, dy \, dx$$

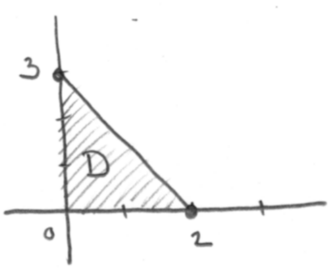
$$= \int_0^1 xy^2 \Big|_{y=2x}^{y=3-x} dx$$

$$= \int_0^1 x(3-x)^2 - 4x^3 \, dx = \int_0^1 -3x^3 - 6x^2 + 9x \, dx$$

$$= -\frac{3}{4}x^4 - 2x^3 + \frac{9}{2}x^2 \Big|_0^1 = \frac{18 - 3 - 8}{4} = \boxed{\frac{7}{4}}$$

25. PLANE INTERSECTS XY - PLANE WHEN $z = 0$

$$\rightarrow \quad 3x + 2y = 6 \quad \rightarrow \quad y = -\frac{3}{2}x + 3$$



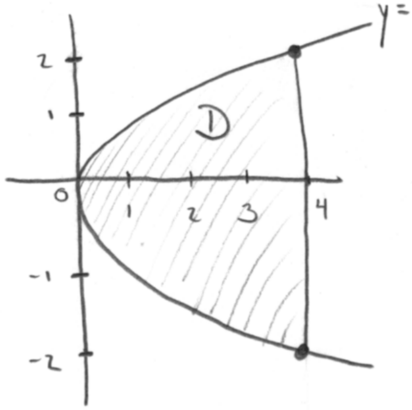
INTEGRATE $\underbrace{z = 6 - 3x - 2y}_{\text{PLANE}}$ OVER D.

$$\int_0^2 \int_0^{3-\frac{3}{2}x} 6 - 3x - 2y \, dy \, dx$$

$$= \int_0^2 6y - 3xy - y^2 \Big|_0^{3-\frac{3}{2}x} dx = \int_0^2 6\left(3 - \frac{3}{2}x\right) - 3x\left(3 - \frac{3}{2}x\right) - \left(3 - \frac{3}{2}x\right)^2 dx$$

$$= \int_0^2 \frac{9}{4}x^2 - 9x + 9 \, dx = \frac{3}{4}x^3 - \frac{9}{2}x^2 + 9x \Big|_0^2 = 6 - 18 + 18 = \boxed{6}$$

27.



$$x^2 \le y \le 4$$

$$-2 \le x \le 2$$

VOLUME UNDER $z = x^2$

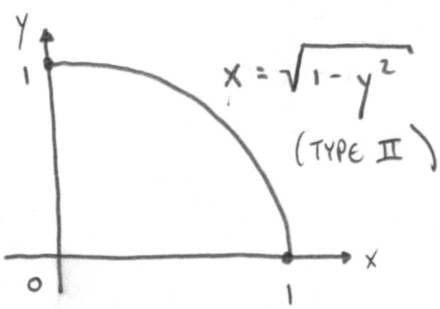
ABOVE $z = 0$

$$V = \int_{-2}^{2} \int_{x^2}^{4} x^2 \, dy \, dx$$

$$= \int_{-2}^{2} x^2 y \Big|_{y=x^2}^{y=4} dx = \int_{-2}^{2} 4x^2 - x^4 \, dx$$

$$= \frac{4}{3}x^3 - \frac{1}{5}x^5 \Big|_{-2}^{2} = \frac{64}{3} - \frac{64}{5} = \frac{2}{15} \cdot 64 = \boxed{\frac{128}{15}}$$

29.



$$\int_0^1 \int_0^{\sqrt{1-y^2}} y \, dx \, dy$$

$$= \int_0^1 y\sqrt{1-y^2} \, dy$$

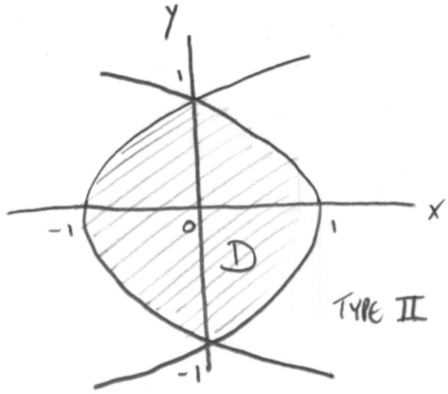
LET $u = 1 - y^2$

$du = -2y \, dy$

$-\frac{1}{2} du = y \, dy$

$$= \frac{1}{2} \int_0^1 \sqrt{u} \, du = \frac{1}{3} u^{3/2} \Big|_0^1 = \boxed{\frac{1}{3}}$$

31.



TYPE II

PLANES: $z = 2 - x - y$ (BOT)

$z = 10 + 2x + 2y$ (TOP)

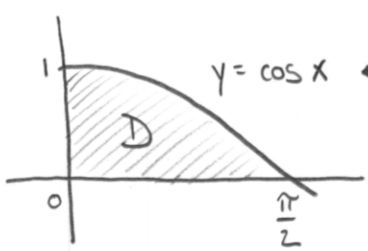
$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} (10+2x+2y)-(2-x-y)\,dy\,dx$$

$$= \int_{-1}^{1}\int_{x^2-1}^{1-x^2} 8+3x+3y \;dy\,dx = \int_{-1}^{1} 8y + 3xy + \frac{3}{2}y^2 \Big|_{y=x^2-1}^{y=1-x^2} dx$$

$$= \int_{-1}^{1} (8+3x)\big((1-x^2)-(x^2-1)\big) + \frac{3}{2}\Big(\underbrace{(1-x^2)^2-(x^2-1)^2}_{0}\Big)\,dx$$

$$= \int_{-1}^{1} 16 - 16x^2 \underbrace{+6x - 6x^3}_{0}\,dx = 16x - \frac{16}{3}x^3 \Big|_{-1}^{1} = 32 - \frac{32}{3} = \boxed{\frac{64}{3}}$$

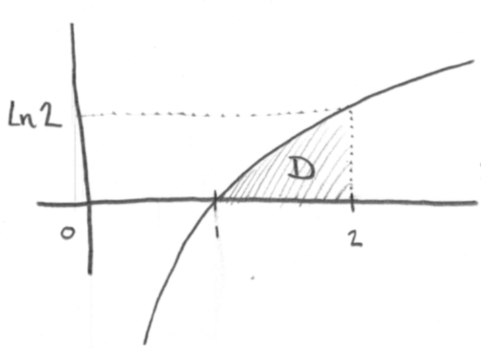
39.



$y = \cos x \longleftrightarrow x = \cos^{-1} y$

$$\int_0^1 \int_0^{\cos^{-1} y} f(x,y)\,dx\,dy$$

41.



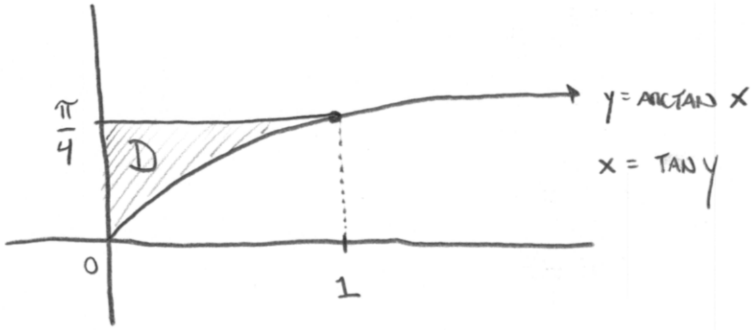
$y = \ln x$

$\updownarrow$

$x = e^y$

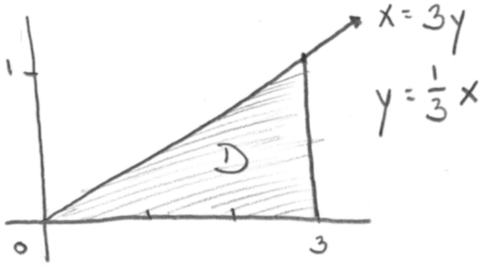
$$\int_0^{\ln 2}\int_{e^y}^{2} f(x,y)\,dx\,dy$$

42.



$$\int_0^{\pi/4}\int_0^{\text{TAN } y} f(x,y)\,dx\,dy$$

43.



$$\int_0^3\int_0^{\frac{1}{3}x} e^{x^2}\,dy\,dx$$

$$= \int_0^3 \frac{1}{3}x\,e^{x^2}\,dx$$

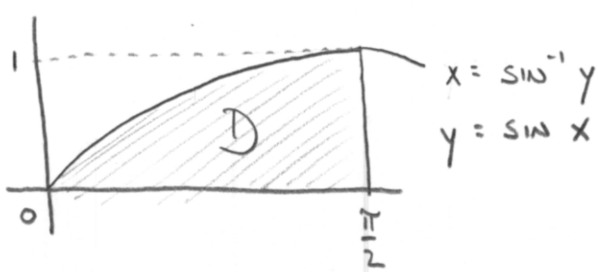
LET $u = x^2$

$du = 2x\,dx$

$\frac{1}{6}du = \frac{1}{3}x\,dx$

$$\rightsquigarrow \frac{1}{6}\int_0^9 e^u\,du = \boxed{\frac{1}{6}\left(e^9 - 1\right)}$$

47.



$$\int_0^{\frac{\pi}{2}}\int_0^{\text{SIN } x} \cos x\,\sqrt{1+\cos^2 x}\;dy\,dx$$

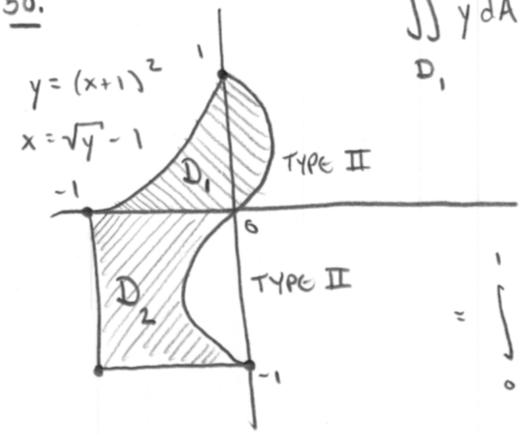
$$= \int_0^{\pi/2} \text{SIN } x\ \cos x\,\sqrt{1+\cos^2 x}\;dx$$

LET $u = 1 + \cos^2 x$

$du = -2\,\text{SIN } x\,\cos x\,dx$

$$= \frac{1}{2}\int_0^2 \sqrt{u}\,du = \boxed{\frac{2}{3}}$$

50.



$$\iint_{D_1} y\,dA + \iint_{D_2} y\,dA = \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y\,dx\,dy + \int_{-1}^{1} \int_{-1}^{y-y^3} y\,dx\,dy$$

$$= \int_0^1 y\left(y-y^3-\sqrt{y}+1\right)dy + \int_{-1}^{1} y\left(y-y^3+1\right)dy$$

$$= \underbrace{\int_0^1 y^2-y^4-y^{3/2}+y\,dy} + \underbrace{\int_{-1}^{1} y^2-y^4+y\,dy}$$

$$= \left[\tfrac{1}{3}y^3-\tfrac{1}{5}y^5-\tfrac{2}{5}y^{5/2}\right]_0^1 + \left[\tfrac{1}{3}y^3-\tfrac{1}{5}y^5+\tfrac{1}{2}y^2\right]_{-1}^{1}$$

$$= \left(\tfrac{1}{3}-\tfrac{3}{5}\right) + \left(\tfrac{2}{3}-\tfrac{2}{5}\right) = 1-1 = \boxed{0}$$